Pbar Note #548
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8.30.93

Measurement of Transverse Emittances as a Function of Revolution Frequency and Beam Current in the Accumulator

ABSTRACT

Horizontal and vertical beam emittance profiles (emittances vs. revolution frequency) were measured in the Accumulator at the end of last machine run. The profiles were measured during regular stacking operation for almost 40 different beam currents, and were unfolded from transverse and longitudinal Schottky spectra. For large stacks, the maxima of the emittance distribution was observed to consistently coincide with the separation between the beam stack tail and core. This may be an indication of beam heating in the region on the edge of both the core and tail transverse cooling systems, and may be the source of significant beam losses. Emittance growth with beam current was also observed. This growth is quite significant in the transition region between core and tail near the maxima of the distributions, but is also significant in the stack tail region. The growth can be considered negligible in the core region.

1. PROCEDURE

In order to obtain the transverse emittance profiles, a procedure similar to the one developed at CERN by S. van der Meer [1] was followed. First, five Schottky scans are obtained: one longitudinal scan at the 126th harmonic, and four transverse scans corresponding to the $126 \pm q_{\mathrm{H,V}}$ sidebands. The emittance profiles are then unfolded from the spectra recorded on each measurement, through simple mathematical manipulation.

An application program (PA1272 on page P112) was written to automate the measurements. This program controls the spectrum analyzers, records the Schottky spectra,

computes the tunes and the emittances, and plots their profiles. A help page is available.

For each transverse direction, the program scans the upper sideband and integrates its power at pre-defined steps. Then, for each point on that sideband, it searches for the point in the lower sideband to which corresponds the same integrated power. Assuming that each particle contributes the same power to all sidebands, the two points correspond to the same revolution frequency, $f_{\rm rev}$, and their frequency is $f_+ = (n+q)f_{\rm rev}$ and $f_- = (n-q)f_{\rm rev}$. Thus, we can obtain not only the revolution frequency, but also the corresponding tune:

$$f_{\rm rev} = \frac{f_+ + f_-}{2n} \tag{1}$$

and

$$q = n\,\frac{f_+ - f_-}{f_+ + f_-}, \tag{2}$$

where, in our case, the harmonic number is $n = 126$.

The emittance ε is straightforward to obtain, knowing that it is proportional to the upper and lower sidebands' spectral power densities, Ψ_+ and Ψ_-:

$$\Psi_\pm \;\propto\; \varepsilon\,\frac{{\rm d}N}{{\rm d}f_\pm} \;=\; \frac{\varepsilon}{n \pm (q + \xi Q/\eta)}\left(\frac{{\rm d}N}{{\rm d}f_{\rm rev}}\right), \tag{3}$$

where Q is the (full) tune, $\xi = \frac{p}{Q}\frac{{\rm d}q}{{\rm d}p}$ is the chromaticity, $\eta = 1/\gamma^2 - 1/\gamma_T^2 = \frac{p}{f_{\rm rev}}\frac{{\rm d}f_{\rm rev}}{{\rm d}p}$ and $\frac{{\rm d}N}{{\rm d}f_{\rm rev}}$ is to be taken from the longitudinal spectrum. Therefore:

$$\varepsilon \;\propto\; \frac{n}{\left(\frac{{\rm d}N}{{\rm d}f_{\rm rev}}\right)}\,\frac{\Psi_+\Psi_-}{\Psi_+ + \Psi_-}. \tag{4}$$

Errors associated with each spectrum are estimated by averaging the noise in the neighborhood of each point on the spectrum. The total error associated with each point in the profile is then obtained through the following relation:

$$\frac{\Delta\varepsilon}{\varepsilon} = \left[\frac{\Delta\left(\frac{{\rm d}N}{{\rm d}f_{\rm rev}}\right)}{\frac{{\rm d}N}{{\rm d}f_{\rm rev}}}\right] + \frac{1}{\Psi_+ + \Psi_-}\left\{\Psi_-\left[\frac{\Delta\Psi_+}{\Psi_+}\right] + \Psi_+\left[\frac{\Delta\Psi_-}{\Psi_-}\right]\right\} \tag{5}$$

2. CALIBRATION

Calibration of the emittances — i.e, determination of the proportionality constants in expression (4) — is done by comparing the emittances at the core center with the values read from the devices A:EMITHN and A:EMITVN. Values of 0.100 for the horizontal

emittances and 0.065 for the vertical emittances, were chosen. Comparisons performed at different beam currents show that the agreement is satisfactory within the accuracy available, although a more direct calibration could provide better accuracy.

The tune profiles (tunes vs. revolution frequency) were also compared to an actual tune measurement and a good agreement was found in the region were the two profiles overlap (compare Figure 1 to Figures 2).

3. MEASUREMENTS AND RESULTS

Emittance profiles were measured for 38 different beam currents in the Accumulator, ranging from low (9 mA) to high currents (128 mA). Except for 4 measurements, the stacking rates were always above 3 mA/h. A sample of profiles can be found on Figures 2.

For small stacks the emittance decreases with the revolution frequency, emittances in the stack tail region being higher than those in the core region. However, as the current increases, a peak starts to appear in the distribution, which becomes the predominant feature for large stacks. The frequency of this peak is consistently around 628,930 Hz, which coincides with the separation between the beam core and the stack tail. In the plots, where the beam profile is also represented, the peak coincides with the inflection point of the beam distribution. This may be an indication that the cooling systems (core and tail) are actually heating the beam in this region. This is possibly due to an undesirable combination of the effects of the two systems in this overlapping region.

4. EMITTANCE AS A FUNCTION OF STACK SIZE

In order to have a more qualitative idea of the emittance growth with beam current, the emittance was taken at three points on each profile and then plotted as a function of the current. The frequencies chosen were 628,900 Hz (stack tail), 628,930 Hz (peak) and 628,953 Hz (core). The plots are shown on Figures 3 (a) to 3 (f).

Figures 3 (d) and 3 (f) indicate that the core emittance is essentially independent from the stack size (flat distributions of emittance vs. current). The same does not happen, however, with the stack tail emittances, which grow linearly with the current, at rates of 34×10^{-3} mm.mrad/mA horizontally and 9×10^{-3} mm.mrad/mA vertically, at 628,900 Hz (Figures 3 (a) and 3 (b)). At the distribution peak, the emittance growth is even more pronounced, at 71×10^{-3} mm.mrad/mA horizontally and 32×10^{-3} mm.mrad/mA vertically (Figures 3 (c) and 3 (d)).

5. REFERENCE

[1] S. van der Meer, "Diagnostics with Schottky Noise," in *Proceedings of the Joint US-CERN School on Beam Observation, Diagnosis and Correction,* Capri, 1988. CERN preprint *CERN/PS/88-60 (AR).*

6. FIGURE CAPTIONS

Figure 1: Measurement of the tunes across the Accumulator aperture (M. Church).

Figure 2: Tune and emittance profiles (horizontal and vertical) as plotted by P112. On the left side of each plot, the upper curve corresponds to the horizontal distribution and the lower one to the vertical distribution; the two are completely uncoupled. On the right side, the horizontal and vertical emittance distributions always cross each other at some point in the stack tail; at the core, the horizontal emittances are always higher than the vertical ones. Note that plots 2 (b) and 2 (h) have different scales than the other plots.

Figure 3: Emittances as a function of beam current, for three different revolution frequencies: (a) and (b) 628,900 Hz (stack tail); (c) and (d) 628,930 Hz (peak); (e) and (f) 628,953 Hz (core).

Console Location 21, 16-FEB-1993 13:10
Accumulator Corrected Tunes vs Momentum

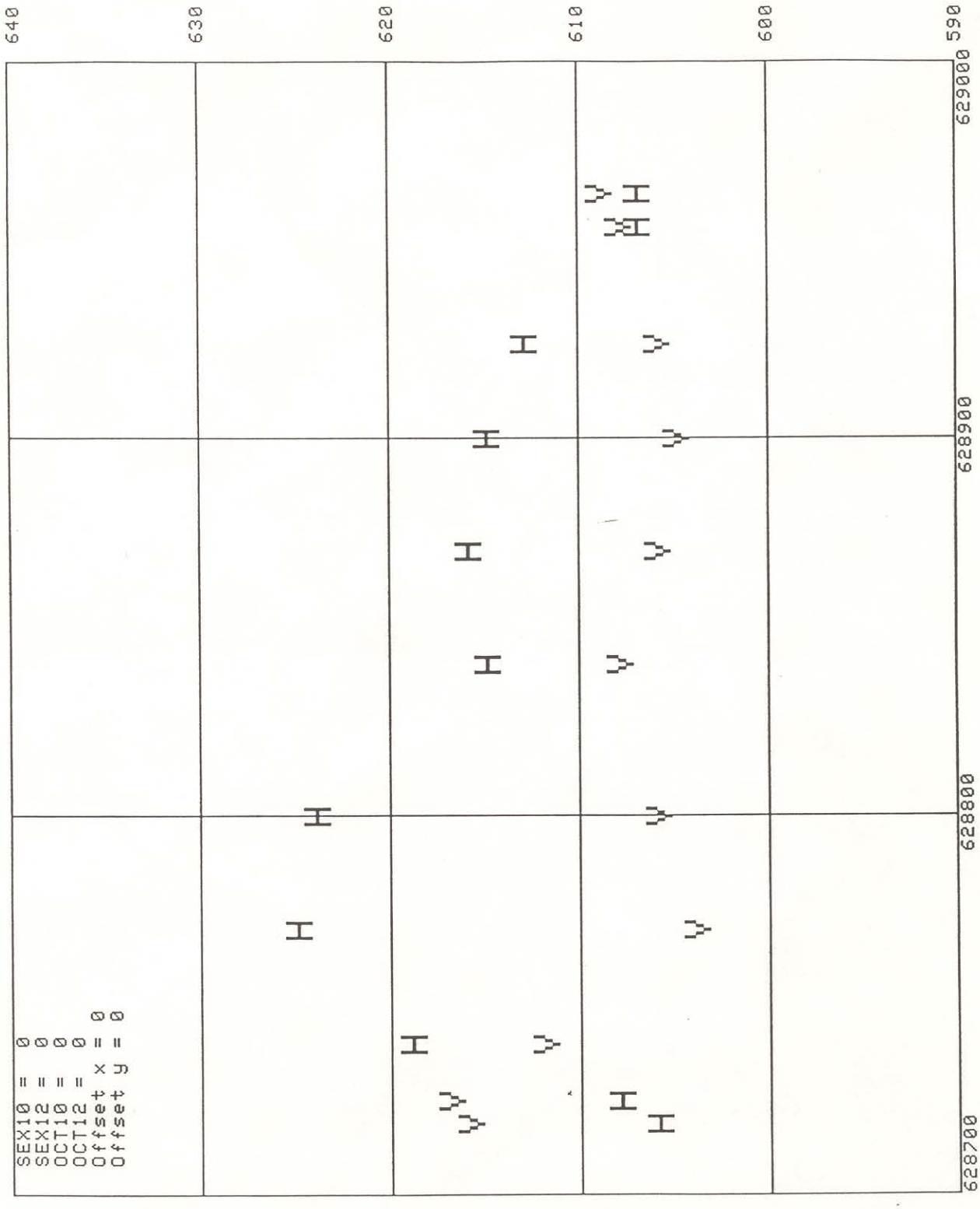
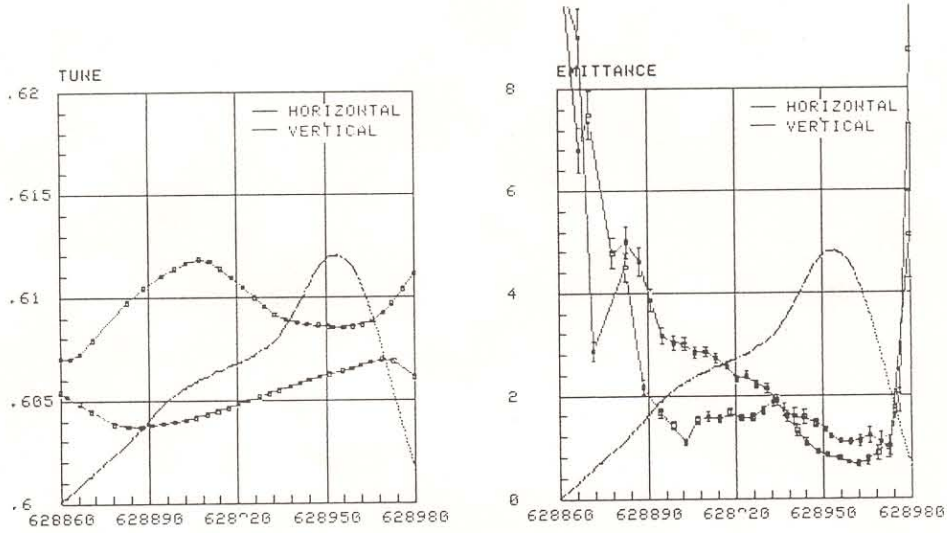


Figure 1

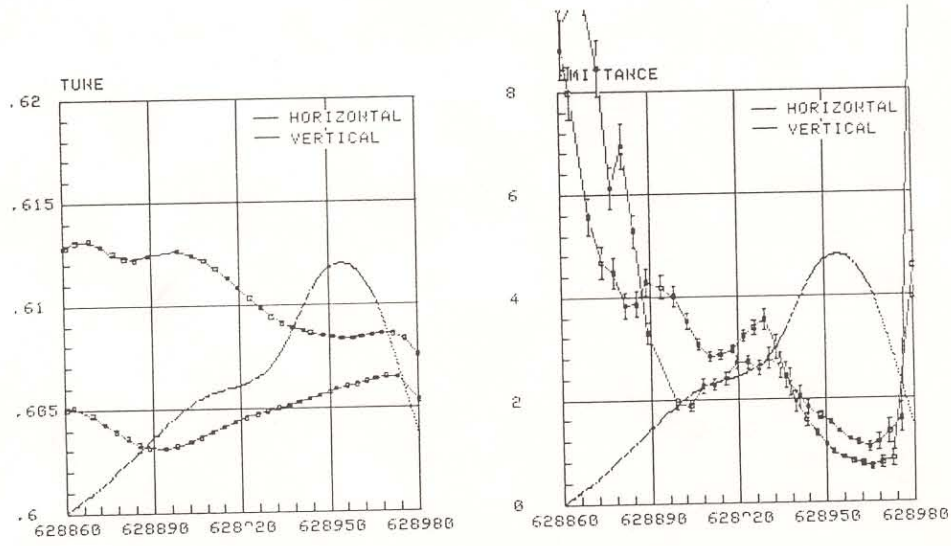


ACCUMULATOR

05/14/93 1426

BEAM CURRENT	8.88 mA
HARMONIC NUMBER	126
REV FREQUENCY	628952 Hz
DELTAP/P	.3047 %
HORIZONTAL TUNE	.6086
VERTICAL TUNE	.606
HORZ CHROMATICITY	.6915
VERT CHROMATICITY	-.671
STACKING RATE	3.80 mA/h

Figure 2 (a)

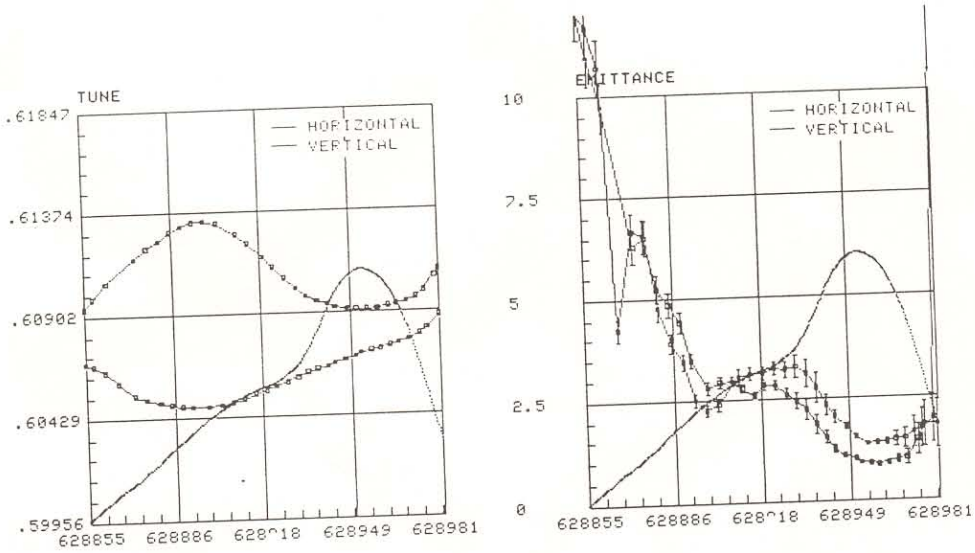


ACCUMULATOR

05/14/93 1811

BEAM CURRENT	21.11 mA
HARMONIC NUMBER	126
REV FREQUENCY	628956 Hz
DELTAP/P	.228 %
HORIZONTAL TUNE	.6085
VERTICAL TUNE	.6058
HORZ CHROMATICITY	.4798
VERT CHROMATICITY	-.6476
STACKING RATE	4.05 mA/h

Figure 2 (b)

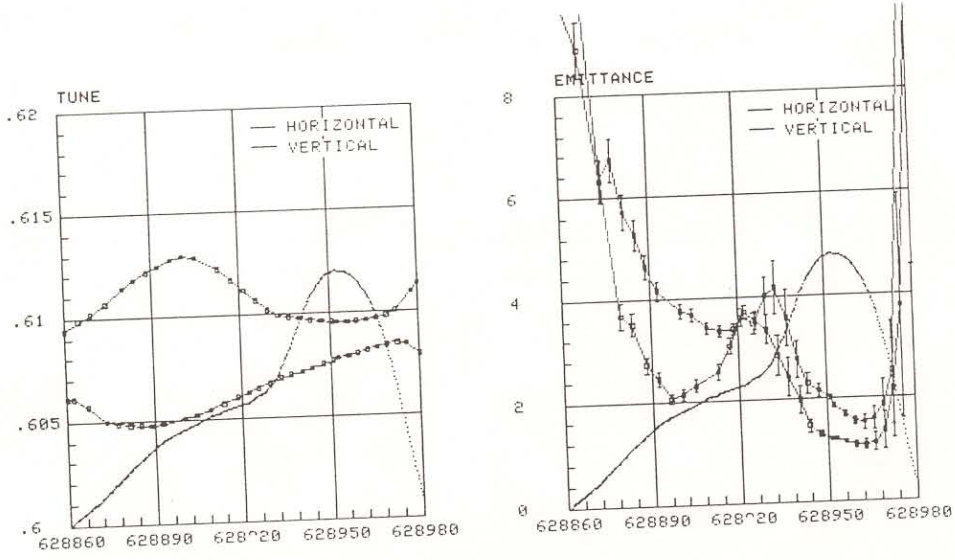


ACCUMULATOR

05/07/93 2034

BEAM CURRENT	43.37 mA
HARMONIC NUMBER	126
REV FREQUENCY	628955 Hz
DELTAP/P	.2061 %
HORIZONTAL TUNE	.6092
VERTICAL TUNE	.607
HORZ CHROMATICITY	.2249
VERT CHROMATICITY	-.6657
STACKING RATE	4.05 mA/h

Figure 2 (c)

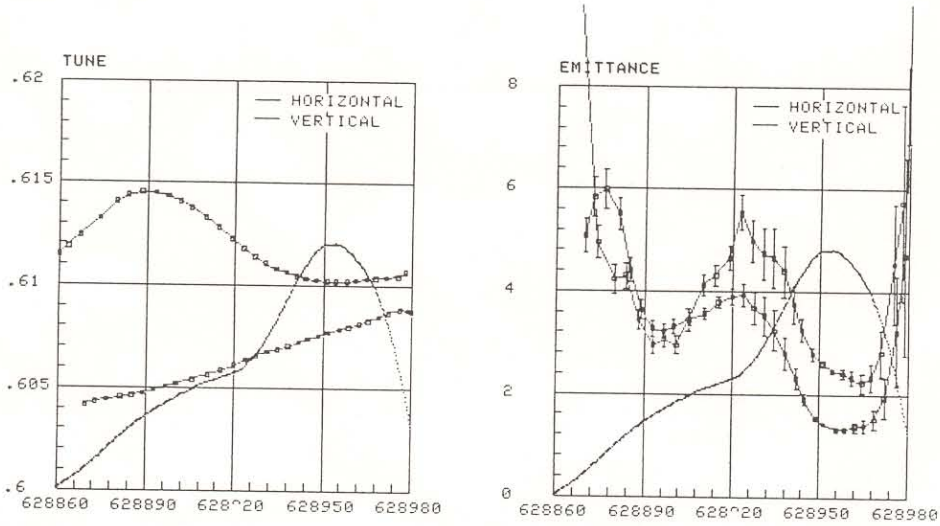


ACCUMULATOR

05/11/93 1817

BEAM CURRENT	59.35 mA
HARMONIC NUMBER	126
REV FREQUENCY	628957 Hz
DELTAP/P	.1863 %
HORIZONTAL TUNE	.6095
VERTICAL TUNE	.6077
HORZ CHROMATICITY	.0497
VERT CHROMATICITY	-.6641
STACKING RATE	4.12 mA/h

Figure 2 (d)

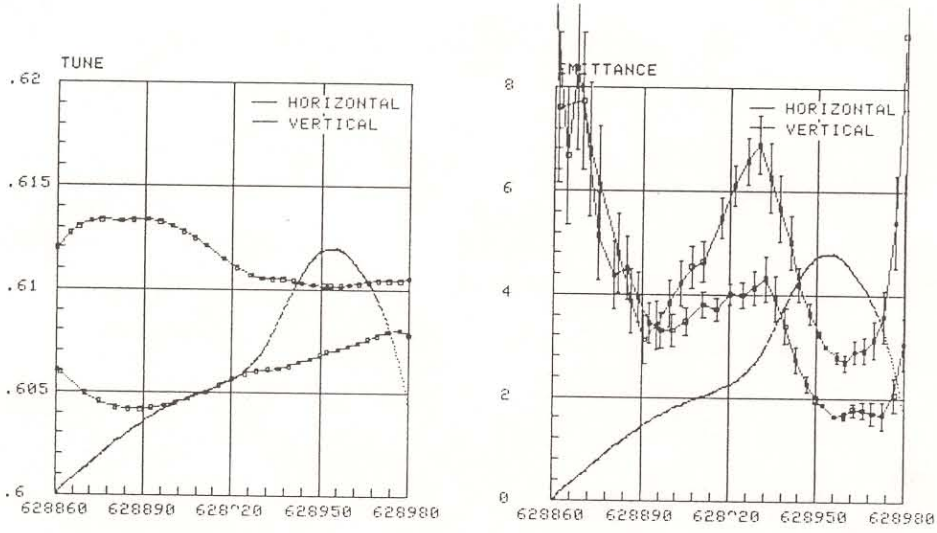


ACCUMULATOR

05/19/93 1116

BEAM CURRENT	76.55 mA
STACKING RATE	3.624 mA/h
HARMONIC NUMBER	126
REV FREQUENCY	628952 Hz
DELTAP/P	.2149 %
HORIZONTAL TUNE	.6102
VERTICAL TUNE	.6076
HORZ CHROMATICITY	.176
VERT CHROMATICITY	-.6909

Figure 2 (e)

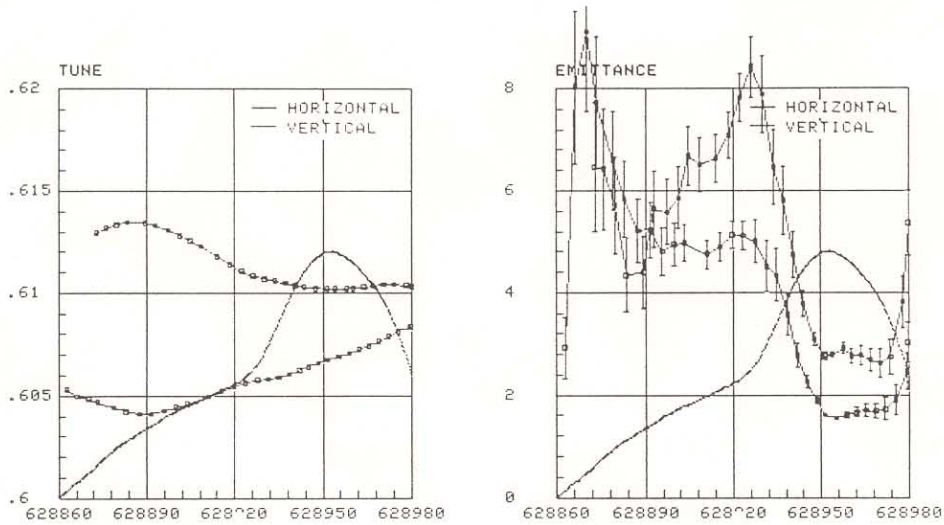


ACCUMULATOR

05/20/93 1530

BEAM CURRENT	94.08 mA
STACKING RATE	3.638 mA/h
HARMONIC NUMBER	126
REV FREQUENCY	628951 Hz
DELTAP/P	.2127 %
HORIZONTAL TUNE	.6102
VERTICAL TUNE	.6069
HORZ CHROMATICITY	.0224
VERT CHROMATICITY	-.6873

Figure 2 (f)

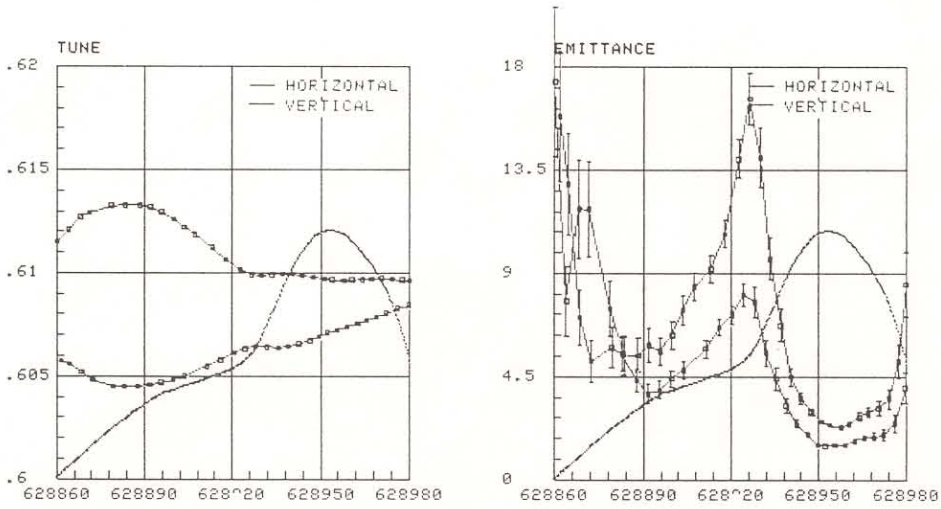


ACCUMULATOR

05/21/93 2140

BEAM CURRENT	108.7 mA
STACKING RATE	2.924 mA/h
HARMONIC NUMBER	126
REV FREQUENCY	628950 Hz
DELTAP/P	.2214 %
HORIZONTAL TUNE	.6102
VERTICAL TUNE	.6067
HORZ CHROMATICITY	.051
VERT CHROMATICITY	-.7504

Figure 2 (g)



ACCUMULATOR

05/26/93 1202

BEAM CURRENT	127.7 mA
STACKING RATE	3.122 mA/h
HARMONIC NUMBER	126
REV FREQUENCY	628952 Hz
DELTAP/P	.217 %
HORIZONTAL TUNE	.6097
VERTICAL TUNE	.607
HORZ CHROMATICITY	.0179
VERT CHROMATICITY	-.5886

Figure 2 (h)

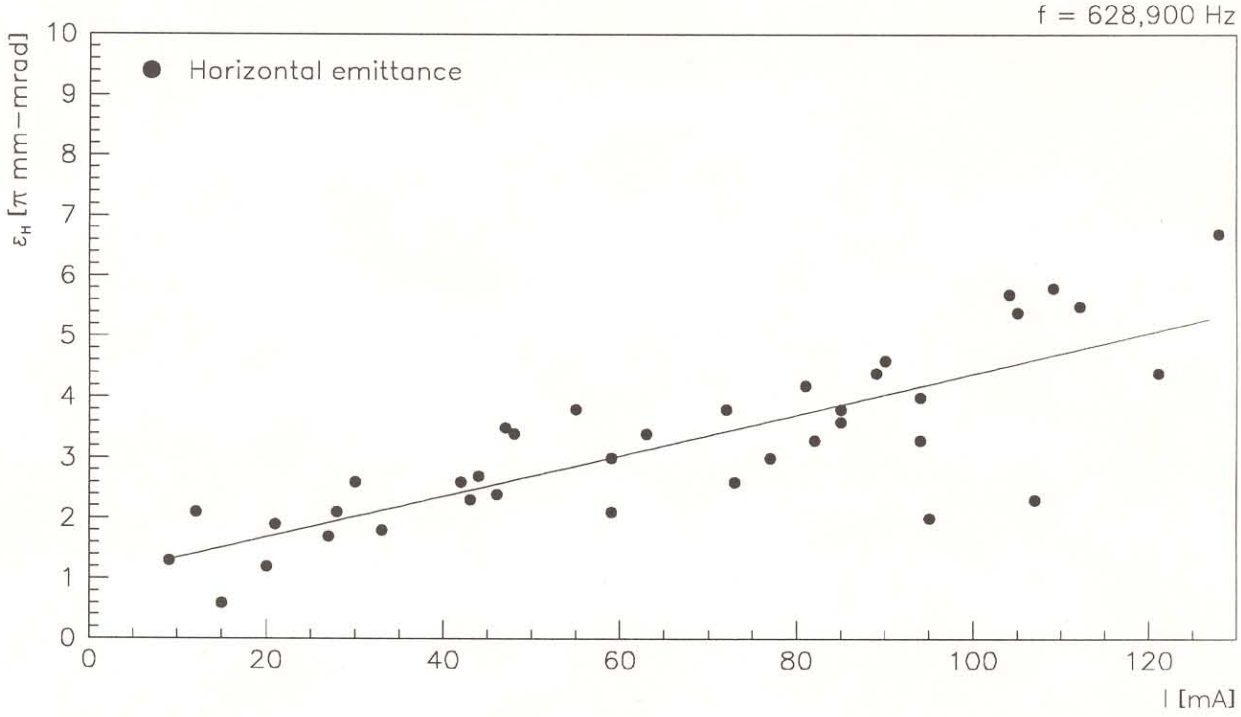


Figure 3 (a)

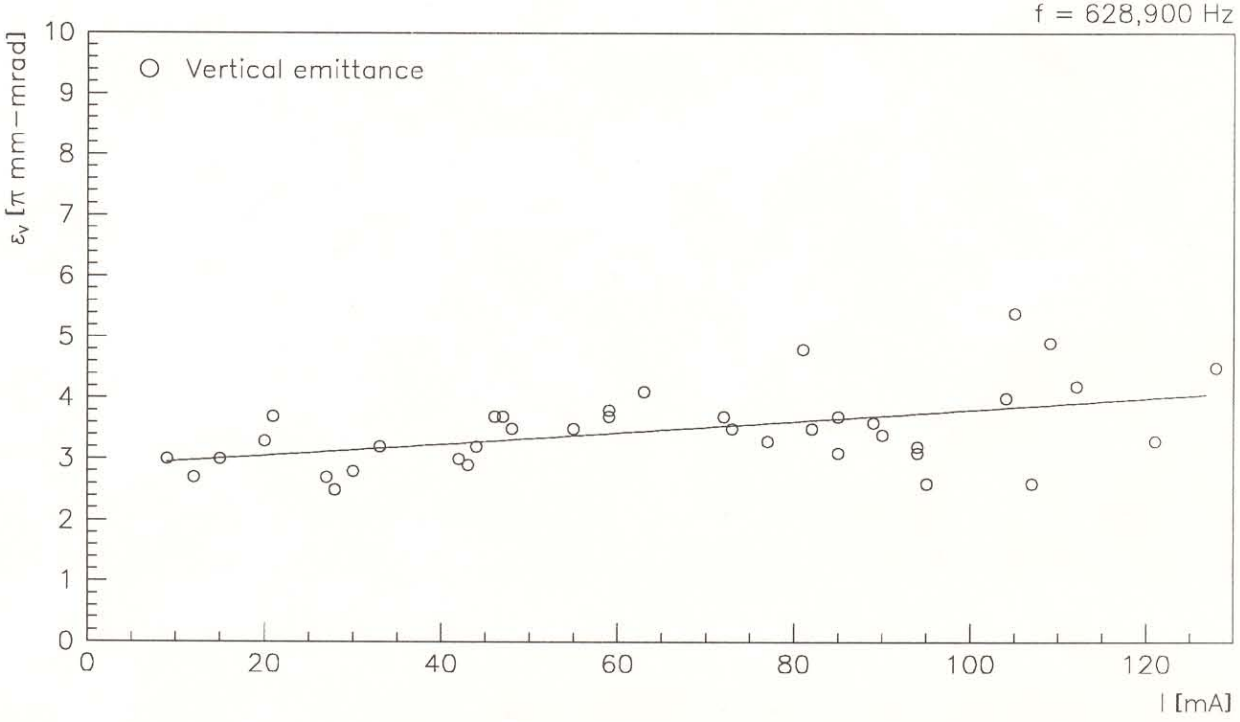


Figure 3 (b)

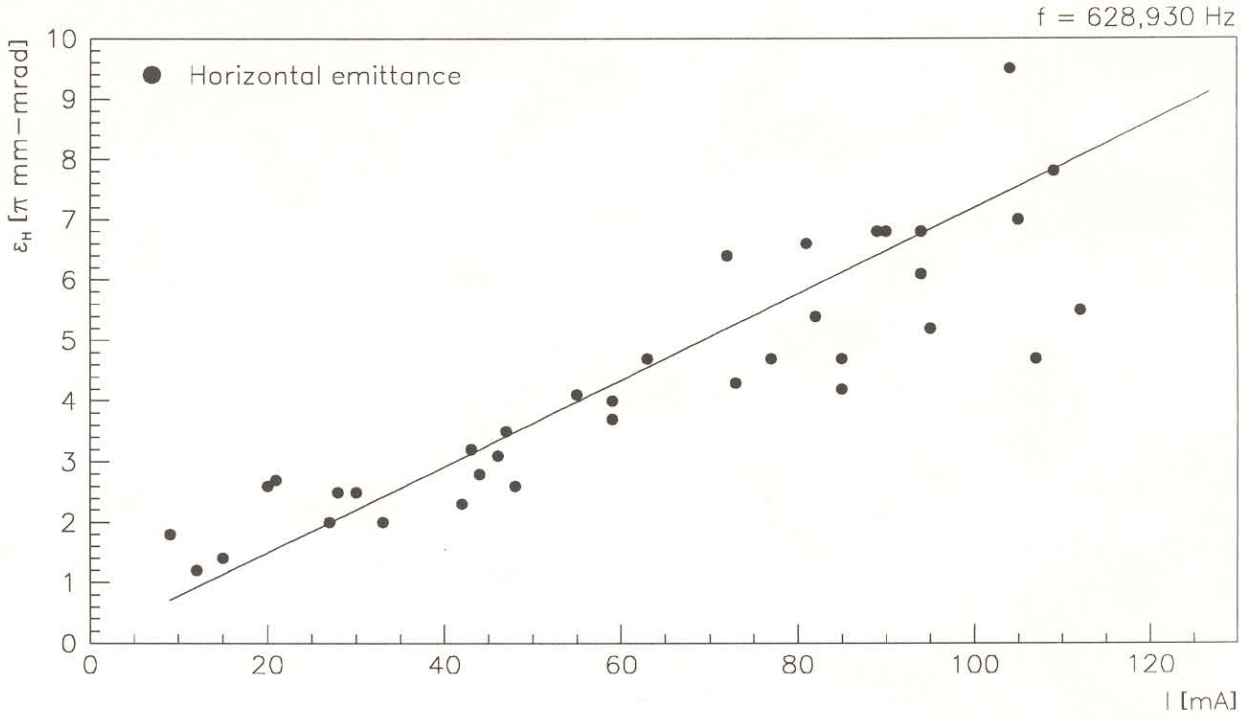


Figure 3 (c)

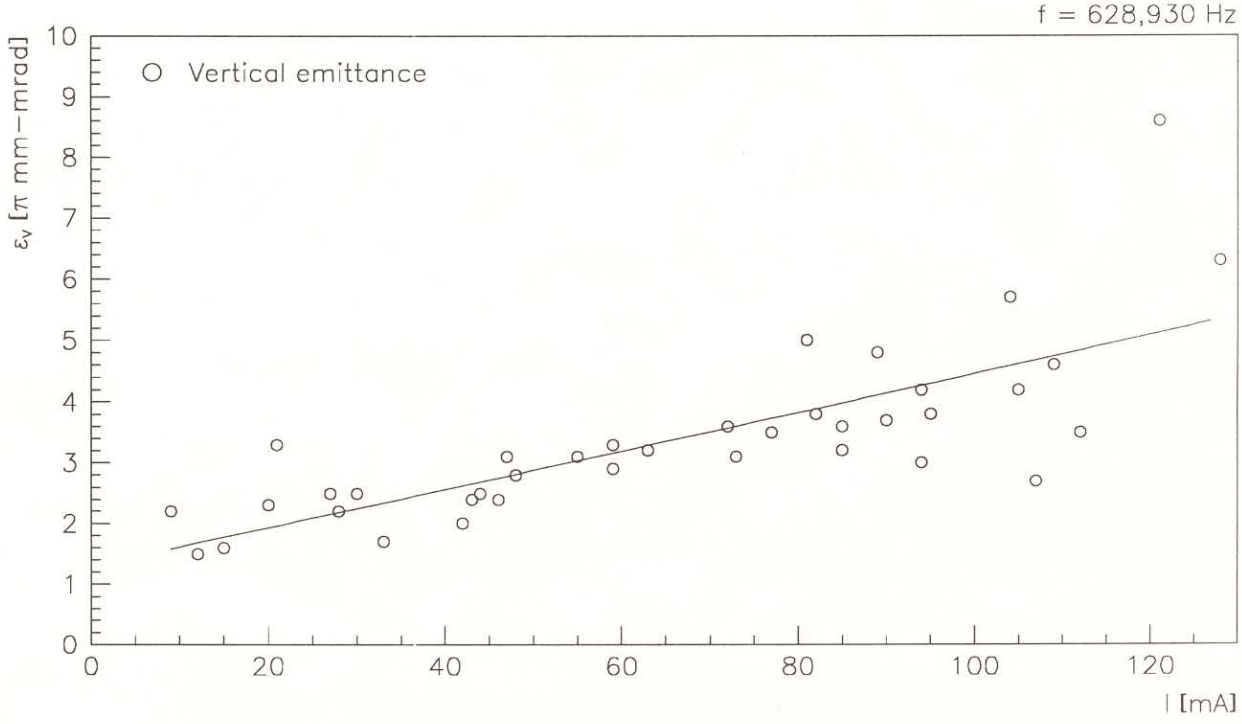
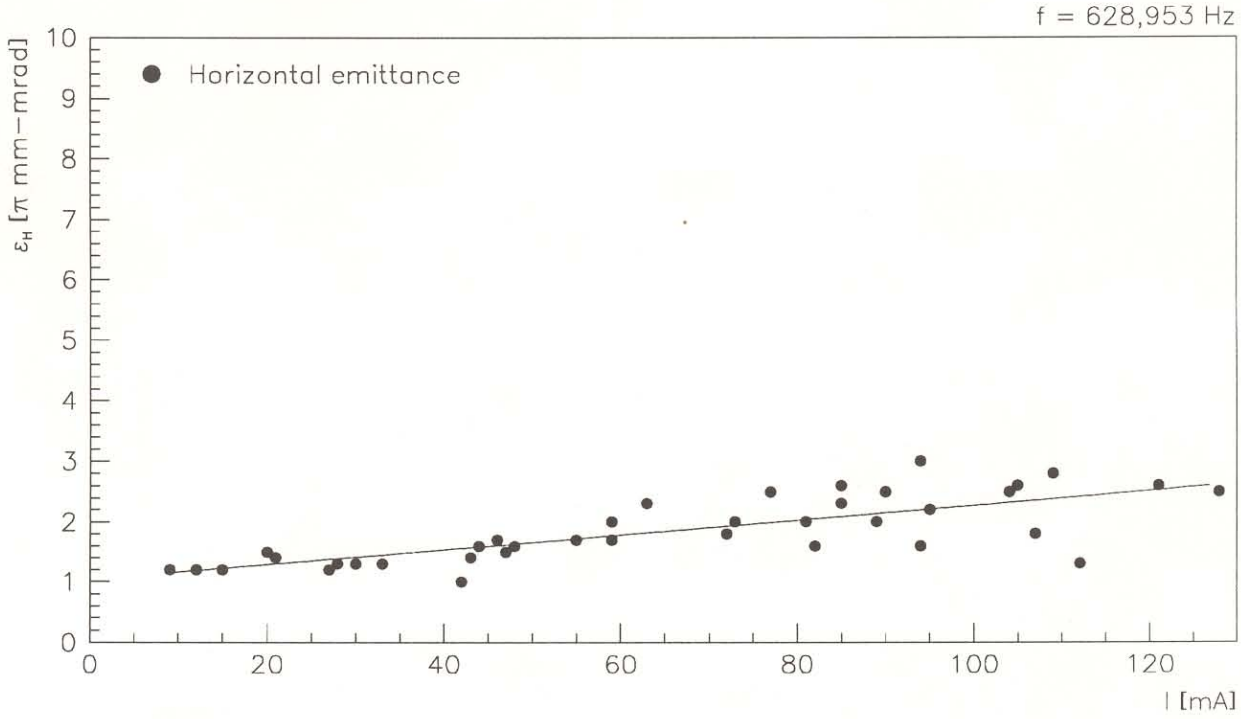


Figure 3 (d)



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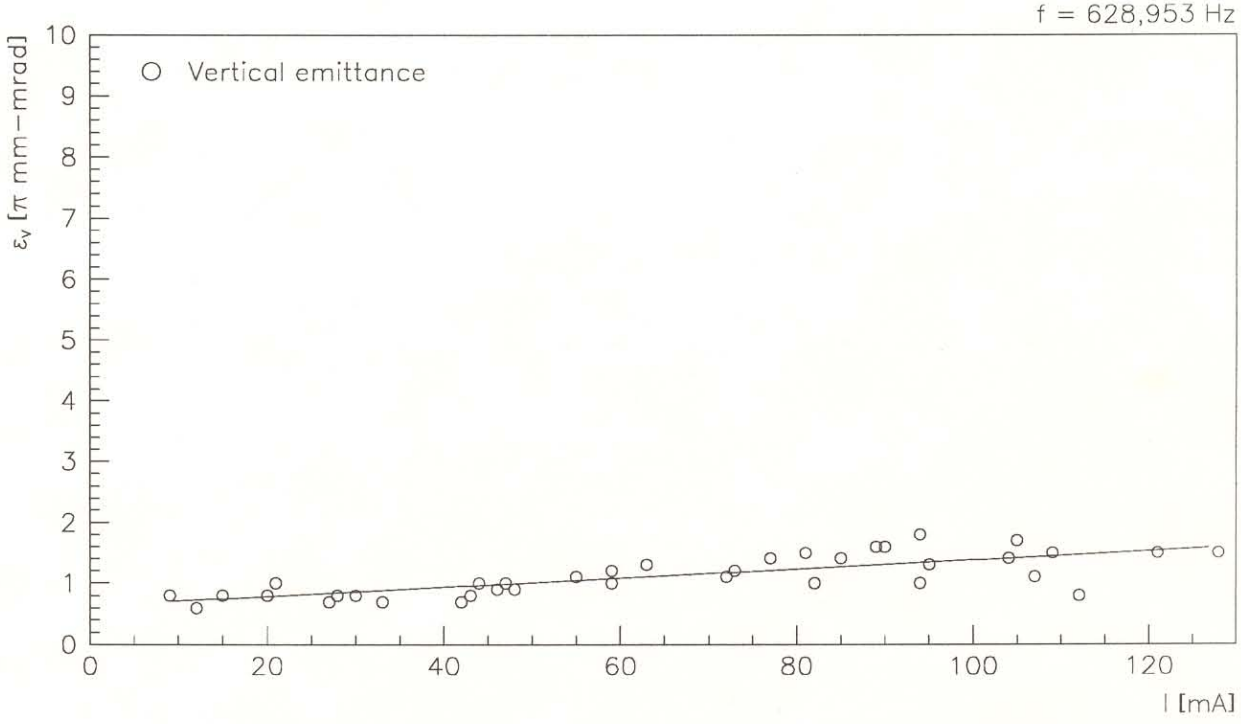

Figure 3 (f)