Synthesis of Band Pass Filters and Equalizers using Microwave FIR Techniques
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Abstract It is desired to design a passive bandpass filter with both a linear phase and flat magnitude response within the band and also has steep skirts. Using the properties of both coupled lines and elementary FIR (Finite Impulse Response) signal processing techniques can produce a filter of adequate phase response and magnitude control. The design procedure will first be described and then a sample filter will then be synthesized and results shown.


The design of the stochastic cooling system at Fermilab uses antennas to pickup signals from the antiprotons circulating in the accelerators of the Antiproton Source. The signals in the Debuncher are very weak and the S/N (signal to noise ratio) is consequently low. Any broadband amplification causes the noise floor of the out-of-band signal to consequently rise. From S/N requirements it is required, therefore, to reduce the noise floor of the out-of-band signals. A band pass filter will lower the noise floor and should not, by design, significantly change the signal from the Pbar source. This paper will discuss the design of a bandpass filter using FIR techniques.

a. Ideal Band Pass Filter Characteristics

An ideal band pass filter, \( G(\omega) \), is depicted in the frequency domain in Fig. 1. An analysis of the filter in Fig. 1 is performed in the time domain by doing an inverse Fourier transform and results in:

\[
g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega = \frac{2}{\pi} \sin(\omega_o t) \sin(\omega_\Delta t),
\]

where

\[
\omega_\Delta = \omega_h - \omega_l \quad \text{and} \quad \omega_o = \frac{\omega_h + \omega_l}{2}.
\]
Figure 1. The response of an ideal bandpass filter. The center frequency of the filter is 4.25 GHz, and its bandwidth is 400 MHz.

A plot of Eq. 1 for $\omega_o = 2\pi(4.25 \times 10^9)$ /sec and $\omega_\Lambda = 2\pi(400 \times 10^6)$ /sec is shown in Fig. 2. The parameter $\omega_o$ is the center frequency of the filter, $\omega_\Lambda$ is the upper cutoff frequency and equals $2\pi(4.45 \times 10^9)$ /sec, and likewise $\omega_i$ is the lower cutoff frequency and is equal to $2\pi(4.05 \times 10^9)$ /sec. Figure 2 and Eq. 1 show that the filter is realized in the time domain as a multiplication of two sinusoidal sources of different frequencies. One frequency is the center frequency of the filter; the other frequency is the bandwidth of the filter.

b. Ideal Coupled Lines

A thorough examination of the response of an ideal coupled line reveals some interesting properties that can be utilized for general filter synthesis. It is necessary to first understand the time domain response of a couple line section to make the transition toward synthesizing a filter in the frequency domain. To begin, the ports and conventions
Figure 2. Truncated time domain response of the ideal band pass filter depicted in Fig. 1. The truncation includes only the first three low frequency lobes.

of a single coupled line section is depicted in Fig. 3. The time domain response of a single coupled line section is shown in Eq. 2.

\[
s_{41}(t) = \frac{C}{2} \left( \delta(t) - \delta\left(t - \frac{2L}{c}\right) \right)
\]

where the \( \delta(t) \) functions are Dirac impulse functions which have the property that \( \int_{0^{-}}^{0^{+}} \delta(t) dt = 1 \). The important parameters in Eq. 2 are \( c \), the speed of light in the medium of the coupled lines, \( L \), the length of the coupled lines, and \( C \), the level of coupling between the lines. It is important to note that the response in Eq. 2 does not
address dispersion which is intrinsic to many common physical topologies such as, for example, microstrip.

**c. Construction of the FIR filter.**

Use of coupled lines for filter synthesis is performed by first appropriately sampling the desired response in the time domain and then by demanding an appropriate delay between each coupled line section. This procedure is described in detail below and has been documented thoroughly in the literature in the design of FIR (Finite Impulse Response) filters [1]. The specifics of the design procedure found in the literature most commonly describes digital filter designs rather than microwave designs.

Close examination of Fig. 2 shows that it is constructed of a series of individual pieces—namely each individual piece is a positive lobe and a negative lobe, and each positive and negative lobe combination is shown as a red box in Fig. 4. Each positive and negative lobe combination is likewise connected to another positive and negative lobe combination. Each set of positive and negative lobe combinations are connected to neighboring sets of positive and negative lobe combinations by a short time delay. This combination of positive and negative lobes is described pictorially in Fig. 4. The first positive and negative lobe combination is shown in the leftmost red box in Fig. 4. The
magnitude of each impulse is simply the area under the curve defined by the zero crossings of the time domain response. For the filter in Fig. 4, the length of each positive/negative lobe combination is simply \( \frac{\lambda}{4} \), which is described by half of the length of “Line 1,” “Line 2,” or “Line 3” in Fig. 4. The delay between each coupled line combinations to another coupled line combination is \( \frac{\lambda}{2} \), which is the length “Delay 1,” or “Delay 2” in Fig. 4.

It is desired, therefore, to take the individual subsections from the time domain response of the ideal bandpass filter of Fig. 2 and compare them to the time domain

**Figure 4.** A schematic of 3 coupled line sections to reproduce a time domain response. Each coupled line is contained within one red box. The length of each coupled line is determined by the lengths “Line 1,” “Line 2,” and “Line 3.” The time delay between each coupled line is determined by the lengths “Delay 1,” and “Delay 2.” The level of coupling is determined by the magnitude of the peaks. In this figure, the oscillation frequency is constant while the magnitude changes.

**Figure 5.** A short schematic of the FIR synthesis technique. The boxes labeled with C’s depict coupled line sections of varying levels of coupling and of identical length. The boxes labeled with D’s depict equal delay length sections. The boxes labeled \( Z_0 \) depict terminations.
response of a coupled line. This comparison and construction technique is described below.

Imagine making a train of alternating coupled lines and delay sections as depicted in Fig. 5. Proper termination of the train is required to stop reflections at the output. The goal is to design a series of coupled lines and match the impulse response of Fig. 6 to Fig. 2. To match the impulse response requires the designer to control the levels of coupling, the length of each coupled line section, and finally the delay between each coupled line.

For example, integrating Eq. 1 from \( t = 0 \) to \( t = \frac{\pi}{\omega_0} \) defines a partial-level of coupling for one particular coupled line section. The next coupled line section is determined by integrating Eq. 1 from \( t = \frac{2\pi}{\omega_0} \) to \( t = \frac{3\pi}{\omega_0} \). Between the coupled line sections requires a meander line, or time delay line, which allows the proper time spacing from one coupled line section to the next. The length of each meander section for Eq. 1 is only 90° delay, since a delay is incurred by the 90° delay of the coupled line itself. A 90° phase delay is simply \( \frac{\lambda_\omega}{4} \) of length.

The overall physical length of the filter is dictated by the skirt requirements of the filter. The skirt of the filter is defined as the steepness of the transition from frequencies that are in-band to frequencies that are out-of-band. Quite simply, the FIR filter will have a steeper skirt by adding more low frequency lobes. A low frequency lobe is an entire section defined between the zero crossings of the \( \sin(\omega_\Delta t) \) function in Eq. 1. The technique described in this paper, therefore, requires a minimum length requirement to realize a bandpass filter by including only the main lobe in Fig. 2, or from \( t = -\frac{\pi}{\omega_\Delta} \) to \( t = \frac{\pi}{\omega_\Delta} \). Including more LFLs (low frequency lobes) increases the steepness of the skirts
and adds to the total time for the signal to be filtered.

The main contribution to the group delay through the filter is in the LFLs before the main LFL and in the main LFL itself. The LFLs do not have to be symmetrically added to produce steep skirts, although LFLs placed unsymmetrically will suffer a higher phase distortion for frequencies in the bandpass region of the filter. Including LFLs after the main LFL, however, do not add significant group delay since the majority of the energy spectrum of the filter is concentrated within the main LFL. For example, including the lobes from \( -\frac{\pi}{\omega_\lambda} \leq t \leq \frac{2\pi}{\omega_\lambda} \) can be performed satisfactorily if a certain

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**Figure 6.** An expanded view of the impulse response shown in Fig. 2. Note the phase change undergone about the point \( t=-2.5 \) nsec.
amount of phase distortion is allowed in the final design and if group delay requirements
are tight.

A final consideration in the design of a FIR filter is the implementation of LFLs.

Examination of Fig. 2 shows that at t=±2.5 nsec, the response of the filter undergoes a
180° phase change. This just implies that a longer meander section is required and
therefore instead of using a 90° meander, a 180° meander is required. This point will be
discussed again later in this paper.

d. Conclusion - Theory

In conclusion, a general design may be realized by:

• finding the center frequency, or period of the filter. This period determines both
  the length of the coupled lines and also the length of the meander lines.

• integrating over the positive and/or negative periods of a particular spike cycle to
determine the coupling for each individual couplers.

• adding meander sections between the coupled lines to accommodate the next
  series of spikes.

• designing the couplers to have the correct input impedance.

• placing matched loads on the output ports.

II. Specific Design of a Microwave FIR Filter

The design requirements for designing an FIR bandpass filters are its center
frequency, its bandwidth, and the medium with which to design the filter. It was decided
that the filter should be designed using a stripline topology. It is also required that the
filter have a good match (i.e. $s_{11} = -20$ dB) over a broad band. Even having a modest
insertion loss of -7 dB allows each coupled line section to be weakly coupled (on the
order of -20 dB). Since each coupled line is weakly coupling implies that no gross discontinuities in line width exist from coupling section to delay section and to the next coupled line section, thus making it easier to have better matches between coupled lines. Note that the requirement of weak coupling of individual couplers does not necessarily imply that the level of the output signal is limited. The author has seen stripline FIR filters which were designed to have only -5 dB insertion loss within the band under the conditions that a sufficient number of LFLs are present and through the use of a low-loss dielectric.

A salient point for the design of the Pbar filters are the requirement that the skirts be sharp and that the in-band phase be flat. A sharp skirt requires that more than one low frequency lobe of Eq. 1 be included into the design.

The specific results for the design of a stripline FIR filter, which is to have the frequency response, is shown in Table 1. The data in Table 1 is coupled with the schematic of Fig. 6 to generate a layout. It was determined from theoretical results and timing criteria that only the first lobes outside of the main lobe are required. The initial design was optimized using Hewlett-Packard Touchstone Series IV package. The design was optimized to have minimal reflections and a flat in-band response. The design can also be optimized to have a particular level of coupling to compensate for the different pickup antennae. In particular, the design of Table 1 has a -7.5 dB insertion loss within the bandpass region of the filter. A plot of the simulation of the design is shown in Fig. 7.

III. Measurement Results and Interpretations
Several different BPF’s were designed and fabricated for the debuncher stochastic cooling system. I designed the filters using circuit board material from Arlon-CuClad LX-04503355 material. This particular circuit board has a dielectric constant of 2.33±0.02, and has a loss tangent of 0.002. The board height is 45 mils. In a stripline configuration, one can assume that the board height is 96 mils (one must include the bonding film height for proper ground-ground height) The results for each filter were relatively

Figure 7. The schematic representation for the realization of the time domain response of Fig. 2. Note the presence of a new box labeled D180. This is simply a 180° delay as opposed to the usual 90° delay. Note also that the filter realization is mirrored about the center delay section located between the two C19 couplers.
similar so only the results for band 1-lower\(^1\) will be presented.

A picture of the inside of the circuit board is shown in Fig. 9 on FR-4. Figure 9 shows the important parts of the design - namely the 90 degree meander lines from coupled line to coupled line, it shows the 180 degree meander lines from one LFL to the next LFL, and it shows the coupling change between the zero crossings of the time domain response.

The measurement results of this technique, however, were not very good. The results of the measurement for band 1 lower are presented in Figure 10. I had 3 boards

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\(^1\)Band 1-lower is defined to be from 4.05 GHz to 4.45 GHz.
fabricated and I wished to compare the results with the hope of getting measurements that are repeatable. The results of Fig. 10 were observed repeatedly with every design. Controlling the line width to better than 0.5 mils did not control the center frequency and magnitude flatness. Controlling the dielectric constant any better did not control the center frequency or magnitude flatness. Using different board stock (Arlon-CuClad LX-03103355 material – this material is 31 mils thick, dielectric constant of 2.33, and loss tangent of 0.002) all suffered from similar poor results. It became clear that the models that were used had to be better precision for the measurements to get better. Since the boards were so large (on the order of 33 inches long), it was impossible to simulate the boards electromagnetically.

I then decided to make the boards simpler and to only include the main lobe into the design. Elimination of the LFL in the design would, of course, make the response broader-band which would make tolerances on the design align with the tolerances of the modeling technique used by the design software. Additionally, I decided to use the thinner Arlon material. Using thinner material demands that the fringe fields of the

**Figure 10.** The measurement of the Band 1 lower. The center frequency could not be controlled. The magnitude could not be well controlled in-band. The lobes out of band could also not be well controlled.
**Figure 11.** The main lobe FIR filter schematic. One can clearly see that there are via holes for binding the top and bottom ground planes together. One can also clearly see that the coupling goes from minimal to maximal to minimal as one would expect from the time domain sinc function response.

stripline to evanesce in a shorter distance than in the thicker material. The layout of the board is depicted in Fig.11. The actual board is shown in Fig. 12. The delay through the board of Fig. 12 is around 3.1 nsec. The board requires two barrel 50Ω loads and requires special SMA connectors - the MACOM #2070-5029-02.

The results of the measurements for this board are shown in Figs. 13 and 14. It is evident by these graphs that even using only the main lobe for designing the filter, this technique suffers from stability problems related to the models of the microwave design toolbox. The trace widths were carefully measured to be withing 0.5 mils. The board heights were consistently manufactured to within 0.5 mils. The phase ripple in-band is still not perfect and the magnitude is not very flat.

In one application, however, we have seen that...
the FIR technique does work adequately. Dave McGinnis designed an equalizer, using ideal coupled lines, for the stacktail cooling 2-4 GHz cooling system. Dave McGinnis, of Pbar, gave the initial design to Ed Cullerton, of RF&I, and Ed was given the task to realized the design. I assisted Ed with some of the problems associated with a large board, coupled lines, and the layout. Ed did not do any optimization from the ideal coupled lines that Dave McGinnis used to the models that Touchstone uses for stripline. The results are shown in Figs. 15 and 16. A picture of the installed board is shown in Fig. 17.
The results of the filter equalizer show that the FIR technique can be a viable technique with broader band designs. Clearly, the designs with the LFL’s had problems since they required an excellent model. When a broader band model was used, the precision of the model became less of an issue and the results became better. The results with the stacktail equalizer show similar results.

IV. Conclusion.

The technique presented in this paper is a simple and quick method to designing filters. The stripline layout has shown its potential in vacuum, low temperature, and noisy environments. It is required, however, that one knows a priori, the dielectric constant as well as the stripline ground spacing to a relatively high degree of precision, and the model of the stripline to a
very high precision. As long as the material is easily found and can be manufactured to the required precision, a seemingly wide array of filters may be designed.

It is important to note that the use of the main frequency lobe for the debuncher is still not an optimal design. Keypoints to consider when choosing the FIR technique:

- The board size can be large and this drives up the costs for manufacturing to around $2000/board.
- Launches to the filter are extremely important and can change the results dramatically.
- Extremely high precision connectors may be required (based on the board heights) and these connectors will be required to connect to a load - all requiring significantly more money and not-so-robust of a design.

The author would like to kindly thank Steve Deibele, Dave McGinnis, Ed Cullerton, and Ralph Pasquinelli for their helpful comments and support. I also wish to thank Don Poll for his technical expertise with both software and hardware.

**Bibliography.**