The beam sweeping magnet kickers are two pairs of conductors placed 90 degrees apart inside a circular magnetic yoke. Each pair has the same excitation current in the opposite directions, and the two currents are a sine and a cosine in phase.

To estimate the magnetic forces on the kickers due to the excitation currents, we make the following simplifications:

1. The four conductors are all parallel to one another.
2. There are no relative motions within the system.
3. The magnetic yoke has $\mu = \infty$, and the fields are zero near the outside inner surface of the yoke.

Fig. 1 shows the cross section of the conductors. Conductors 1 and 3 have equal and opposite currents, conductors 2 and 4 have equal and opposite currents, and the current $I_1$ leads $I_2$ by 90 degrees. We can write $I_1$ as $I_{\text{peak}} \cdot \sin \theta$, $I_2$ as $I_{\text{peak}} \cdot \cos(\theta)$, $I_3$ as $-I_{\text{peak}} \cdot \sin(\theta)$, and $I_4$ as $-I_{\text{peak}} \cdot \cos(\theta)$, where $I_{\text{peak}}$ is 10,000 amps. We now wish to calculate the net forces on a conductor due to the other three conductors and the magnetic yoke.

We make the simplifying assumption that the magnetic permittivity, $\mu$, is infinite; in reality $\mu$ is about 60 and very little B-field escapes to the outside. Assuming that the B-field is zero right outside the yoke, we can then replace the yoke with 4 image currents in the same directions at a distance $a$ so that the B-field does cancel at the outer circumference of the yoke (Fig. 2). The problem is now reduced to summing up all the forces on one conductor due to the other 7 currents. As shown in Fig. 1, for example for conductor 1, we decompose the forces on it into $F_x$ (radially outward) and $F_y$ (tangential). Then,

$$F_{1x} = F_{21(x)} + F_{31(x)} + F_{41(x)} + F_{131(x)} + F_{231(x)} + F_{331(x)} + F_{431(x)} =$$

$$\frac{\mu_0 l}{2\pi r} \cdot I_{\text{peak}}^2 \cdot \left( - \sin \theta \cos \theta \sin \beta + \frac{1}{r^2} \sin^2 \theta + \sin \theta \cos \theta \sin \beta + \right.$$

$$\frac{r}{a} \sin^2 \theta - \frac{r}{b} \sin \theta \cos \theta \sin \alpha + \frac{r}{a + \sqrt{2}r} \sin^2 \theta + \frac{r}{b} \sin \theta \cos \theta \sin \alpha \bigg),$$

and
\[
F_{1y} = F_{21(y)} + F_{31(y)} + F_{41(y)} + F_{11(y)} + F_{221(y)} + F_{321(y)} + F_{421(y)} = \\
\frac{\mu_0 l^2}{2\pi r} \cdot I_{peak}^2 \cdot \left(-\sin \theta \cos \theta \cos \beta + 0 - \sin \theta \cos \theta \cos \beta + 0 - \right) \\
\frac{r}{l} \sin \theta \cos \theta \cos \alpha + 0 - \frac{r}{b} \sin \theta \cos \theta \cos \alpha),
\]
where \( \theta = \omega t \) is the reference phase angle of all currents, \( l \) is the length of the conductor, \( l = 27.9\text{cm}, r = 2.00\text{cm}, a = 4.98\text{cm}, b = 6.55\text{cm}, \beta = 45^\circ \) and \( \alpha = 12.5^\circ \). Then,

\[F_{1x} = 1.37 \sin^2 \theta,\]
and

\[F_{1y} = -2.00 \sin \theta \cos \theta,\]

where a positive \( F_x \) indicates an radially outward force, and a positive \( F_y \) indicates a force causing a counter-clockwise rotation. Similar analyses for the other three conductors yield

\[F_{3x} = 1.37 \sin^2 \theta,\]
\[F_{4x} = 1.37 \cos^2 \theta,\]
\[F_{1y} = F_{3y} = -2.00 \sin \theta \cos \theta,\]
and
\[F_{2y} = F_{4y} = 2.00 \sin \theta \cos \theta.\]

Fig. 3 shows the radial and tangential forces through a complete current cycle. We draw the following conclusion about the behavior of the conductors:

- **Torques caused by the tangential forces about the central Z-axis cancel, hence there is no tendency for the circuit to rotate about the Z-axis.**
- **There is no radially inward force on any conductors during the entire cycle.**
- **Peak radial outward force \( \approx 8.0 \text{ lb/in} \) on each conductor.**
- **Peak tangential force \( \approx 5.2 \text{ lb/in} \) on each conductor.**
Figure 1: Cross section of conductors; $r$ is 2 cm.

Figure 2: Equivalent current distribution in which the magnetic shield ($\mu = \infty$) is replaced with 4 image currents.
Figure 3: The forces on conductors during a full cycle of current pulse.