Phase Advance of a Simple Damper

Consider a simple damper system shown in Figure 1. The phase advance between pickup and kicker is \( \phi_k \). Assume a particle passes through the pickup at time \( t=0 \) with a complex betatron amplitude:

\[
\tilde{A}_{pu} = A_0 \quad (1)
\]

When the particle passes through the kicker, the betatron amplitude of the particle will be:

\[
\tilde{A}_{kr} = A_0 e^{j\phi_k} \quad (2)
\]

Assume that the damper is timed so that the delay through the electronics matches the time of flight for the particle between pickup to kicker. The difference in phase between the kicker betatron amplitude and the pickup amplitude must be:

\[
\arg(\tilde{A}_{kr}) - \arg(\tilde{A}_{pu}) = (2n + 1) \frac{\pi}{2} \quad (3)
\]

\[
\phi_k = (2n + 1) \frac{\pi}{2} \quad (4)
\]

because the pickup measures the position and the kicker corrects an angle.
Phase Advance of a Damper with a Notch Filter

Now consider a damper system with two pickups right next to each other as shown in Figure 2. This is electrically equivalent to a damper system with a notch filter. The first pickup is the same as shown in Figure 1 but the second pickup signal is delayed by 1 turn. The amplifier takes the difference between the first pickup and the second pickup. The signal that arrived at the amplifier from the first pickup is when the beam went through the pickups at \( t = 0 \). The betatron amplitude of the particle is:

\[
\tilde{A}_{pu1} = A_o
\]  

(5)

The signal that arrives at the amplifier from the second pickup is when the beam passed through the pickups at \( t = -T_{rev} \). The betatron amplitude of the particle at \( t = -T_{rev} \) is:

\[
\tilde{A}_{pu2} = A_o e^{-j2\pi Q}
\]  

(6)

where \( Q \) is the betatron tune. At the output of the amplifier, the effective betatron amplitude of the signal is:

\[
\tilde{A}_{pu} = A_o - A_o e^{-j2\pi Q}
\]  

(7)

\[
\tilde{A}_{pu} = 2A_o \sin(\pi Q)e^{j\left(\frac{\pi}{2} - \pi Q\right)}
\]  

(8)

When the particle passes through the kicker, the betatron amplitude of the particle will be:

\[
\tilde{A}_{kr} = A_o e^{j\phi_k}
\]  

(9)

Using the requirement stated in Eqn. 3:

\[
\phi_k + 2\pi \left(\frac{Q - \frac{1}{2}}{2}\right) = (2n + 1)\frac{\pi}{2}
\]  

(10)
Figure 2. Electrical Equivalent of damper system with a notch filter