Scenarios for the Accumulator 4-8 GHz betatron core cooling systems are described. Included are signal and noise calculations, optimum gain values and settings, and total power levels necessary. Comparisons are made to the Run I system performance with both the coaxial line and laser link from A10 to A30.

1. Accumulator Betatron Core Cooling Systems

The Accumulator has betatron core cooling systems in both the horizontal and vertical plane. The pickups are located in the A10 straight section, with the horizontal pickup tank upstream of the vertical pickup tank. There are 32 pickups in the frequency range 4-8 GHz. At this time, we do not have a good measurement of the pickup sensitivity. It is believed that there is approximately 10 dB of falloff in pickup response across the frequency band.

In previous running periods, the signal has been communicated from the A10-2 stubroom to the A30 stubroom over half inch coax lines. These lines had insertion losses from 18 to 28 dB over the frequency range and up to 360° of phase dispersion. Measurements made during Run I showed the signal going below the noise floor for frequencies above 6 GHz. For Run II, the coaxial lines are being replaced by a free space optical laser link. The optical link has flat amplitude and phase response (±1 dB and ±10° over the 4-8 GHz band) with large insertion loss of approximately 45 dB and approximately 35 dB of dynamic range.

2. Signal and Noise Calculations

The induced current on a pickup is equal to \( \frac{2 S D_{\text{RMS}}}{h} \), where \( S \) is the pickup sensitivity, \( h \) the aperture (in m) and \( D_{\text{RMS}} \) the RMS dipole moment. The dipole moment is a function of the beam current, beam emittance, \( \beta \) function, and bandwidth of the pickup. Including all effects, the induced current \( I_{\text{PU}} \) can be expressed as

\[
I_{\text{PU}} = \frac{S}{h} \sqrt{\frac{2eW\epsilon\beta I_{\text{beam}}}{6\pi}},
\]

where \( e \) is the charge of an antiproton, \( \epsilon \) is the beam emittance, \( \beta \) is the value of the betatron oscillation function, \( W \) is the bandwidth and \( I_{\text{beam}} \) is the beam current. For the calculations in this note, I will use \( \epsilon = 3\pi \text{ mm mr} \), \( \beta = 8 \text{ m} \), \( W = 4 \text{ GHz} \), \( h = 0.033 \text{ m} \) and \( S = 0.25 \) (flat across the frequency bandwidth). With 32 pickups and a beam current of 100 mA, the total signal power is 94 picowatts.
The front end noise power can be expressed as \( kTW \), where \( k \) is the Boltzmann constant \( (1.38 \times 10^{-23} \text{ joules/Kelvin}) \), \( T \) is the front end temperature (in this case, 308 K) and \( W \) the system bandwidth = 4 GHz. The front end thermal noise power is 17 picowatts.

3. Optimum Gain and Power

For a betatron cooling system, the cooling time can be expressed as

\[
\frac{1}{\tau} = \frac{W}{N} [2g - g^2(M + U)],
\]

where \( M \) is the mixing factor, \( U \) is the Noise to Signal ratio, \( N \) is the number of particles in the beam, and \( g \) is the correction applied to the beam. The cooling time is minimized when \( g = 1/(M+U) \). The mixing factor \( M \) is a function of the frequency band \((W, f_{\text{max}}\text{ and } f_{\text{min}})\), the momentum width \( \Delta p/p \), the revolution frequency \( f_0 \), and the phase slip factor \( \eta \):

\[
M = \frac{\ln(f_{\text{max}}/f_{\text{min}})}{2W\eta \Delta p \frac{1}{p} f_0}.
\]

For the upgraded Accumulator, \( f_{\text{max}} = 8 \text{ GHz}, f_{\text{min}} = 4 \text{ GHz}, \eta = 0.012, \text{ and } f_0=628910 \text{ Hz}. \) For these calculations, I will use \( \Delta p/p = 5.4 \times 10^{-4} \) (a half width \( \Delta p = 4.8 \text{ MeV based on stacking simulations} \). For these values, \( M = 8.4 \). The noise to signal power is a function of the beam current and is equal to 0.18 at the pickups. Figure 1 shows how \( U \) depends upon beam current. For \( I_{\text{beam}} > 2 \text{ mA}, U < M \) and for \( I_{\text{beam}} > 17 \text{ mA}, U< 1 \). In the calculation of optimum gain, the core betatron systems are dominated by the mixing factor, not the noise/signal ratio.

For 100 mA beam current, \( g = 1 / (M + U) = 0.117 \). The necessary kicker voltage to make this correction is

\[
KV = \frac{p_{\text{beam}}gh}{2l_k} \sqrt{\frac{\epsilon W}{6\pi\beta N f_0}},
\]

where \( l_k \) is the kicker length. For 100 mA, optimum gain implies a kicker voltage of 34V, a total signal power of 11.6 W, and an additional 2 W of noise power at the kicker.
In Figure 2, I show the signal power (symbols) and total power (shaded regions) for optimum gain with the above assumptions. Included on this figure are similar calculations for the Run I core betatron cooling system. With 1 TWT (limited to less than about 50 W for good performance), optimum gain was attainable for beam currents above 110 mA in the Run I setup and will only be attainable for beam currents above 30 mA in Run II. This is somewhat optimistic as the noise calculation only includes front end thermal noise. If the system has a total noise figure of 2 dB, the total power drops below 50 W for beam currents greater than 35 mA.

To achieve optimum gain, the core betatron systems need to have large total gain. For 100 mA, the required total gain from pickup to kicker is approximately 111 dB. Figure 3 shows the necessary electronic gain required to achieve optimum gain. The Run II betatron cooling systems should have between 140 and 150 dB of total gain, so for beam currents above 30 mA (where the TWT power is reasonable), the system should have enough total gain to reach optimum.
Figure 2: Signal and Total Kicker Power (Watts) vs. Beam Current (mA) necessary to achieve optimum gain.

Figure 3: Total Electronic gain necessary to achieve optimum gain.