

Effects of Transverse Coupling on Transverse Beam size, Simulation and Measurements

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ABSTRACT

The equations of motion for particles in an accelerator lattice show that a larger physical aperture is required to hold a beam of constant invariant emittance if there is transverse coupling of the tunes. The results of a tracking simulation of particle motion in the Fermilab accumulator ring are discussed, and results are shown from beam tests carried out in the accumulator to demonstrate this effect.

$$\delta = (Q_x - Q_z) / R$$
$$\kappa = (Q_x + Q_z) / 2R$$
$$\eta = ((\kappa/\kappa)^2 + \delta^2)^{1/2}$$

In the limit that the coupling strength $k=0$ the motion of particles is described by the following two equations:

$$x = Ae^{i\kappa_x s}$$
$$z = Be^{i\kappa_z s}$$

I. INTRODUCTION

It is well known that transverse coupling in an accelerator lattice can cause the transfer of betatron motion from one transverse plane to another. This effect is easy to demonstrate with modern day beam position monitor (BPM) measurements by injecting beam with a large transverse betatron motion in one dimension. Our present instrumentation does not, however, shed full light on the motion of particles injected with initial transverse motion in both planes.

The operating points of most accelerators are close to the coupling resonance, so it is important to understand the behavior of coupled beams. This report contains a discussion of three methods of exploring this problem, and will show that choosing an operating point without coupling can lead to a smaller physical beam size than a coupled operating point. First, I will summarize an analytical solution to this problem found in the literature. Second, I will show the results of a tracking simulation of this effect. Third, I will show measurements of the beam size with and without coupling in the accumulator storage ring.

For this study we are interested in coupling from skew multipole fields, so for the equations below I have set $b=0$, i.e. no velocity coupling. In the limit that the coupling fields are small w.r.t the normal focusing fields, i.e. when k/κ is much less than 1, and when the difference in tunes δ is very small, an approximate solution for the motion of the particle is given by Equations 10 and 11 from Reference 1. These equations are reproduced below for the special case when the motion of the particle becomes 100% coupled, i.e. as the fractional coupling strength k/κ becomes much larger than the fractional difference in tunes δ .

Let's consider a particle in an accelerator with zero coupling described by the above two equations. At time $s=0$ we turn on the coupling fields to a strength where the motion is 100% coupled. This is similar to injecting a beam of particles into a coupled lattice. Initially the motion must still be described by the above equations. In this case equations 10 and 11 from Reference 1 can be written as shown below, where I have chosen the new constants A' and B' to match the boundary condition at $s=0$.

II. ANALYTICAL DISCUSSION OF COUPLED MOTION

There have been many discussions in the literature of coupled particle motion in accelerator lattices. One of the simplest is that of P.J. Bryant^[1], from which I will quote several results below. In Reference 1 the author derives an approximate solution to the equations of motion:

$$x'' + (Q_x/R)^2 x = bz' - kz$$
$$z'' + (Q_z/R)^2 z = bx' - kx$$

Where:

- Q_x is the tune in x
- Q_z is the tune in z.
- $\kappa_x = Q_x / R$
- $\kappa_z = Q_z / R$
- $k = (1/B\rho)(dB_x/dx)_0 = -(1/B\rho)(dB_z/dz)_0$

$$x = e^{i\kappa_x s} [(A+B)e^{(i/2)(\eta-\delta)s} - (B-A)e^{-(i/2)(\eta+\delta)s}] / 2$$
$$z = e^{i\kappa_z s} [(B-A)e^{-(i/2)(\eta-\delta)s} + (A+B)e^{(i/2)(\eta+\delta)s}] / 2$$

The above equations describe a particle moving with a fast sinusoidal betatron motion, and with amplitude modulated by the slower functions X and Z described by equations 13 and 14 from reference 1 after correcting for a sign error.

$$|X|^2 = [|A|^2 + |B|^2 - (|A|^2 + |B|^2 - 2AB^*) \cos(\eta s)] / 2$$
$$|Z|^2 = [|A|^2 + |B|^2 + (|A|^2 + |B|^2 - 2AB^*) \cos(\eta s)] / 2$$

It is clear from the above equations that the amplitude of the coupled motion in general has larger excursions in each dimension than the original uncoupled motion. This means the physical size of the beam is larger in the presence of coupling.

III. SIMULATIONS WITH A TRACKING PROGRAM

To study this effect I performed a tracking calculation using the lattice design program DIMAD. This model includes all dipoles, quadrupoles, and sextupoles, as well as an accurate

† Operated by the Universities Research Association, Inc under contract with the U.S. Department of Energy.

description of the fringe fields from the dipoles. The values of the sextupoles were adjusted to give chromaticities close to our measured values, and the tunes were adjusted to be 6.612 and 8.612 on the central orbit. To add coupling to this model I added two skew quadrupoles with a strength equal their operating point when used to remove coupling during normal operations.

With the skew quads turned on I launched test particles from a nominally dispersionless location in the lattice with a small amount of $dp/p = 0.02\%$, an energy where the coupling in the lattice is essentially 100%. Figure 1 shows the x and y position of the test particle every other revolution around the machine for particles launched with an initial displacement in x. The motion in the model is very similar to the motion actually measured by the BPM system shown in Figure 2.

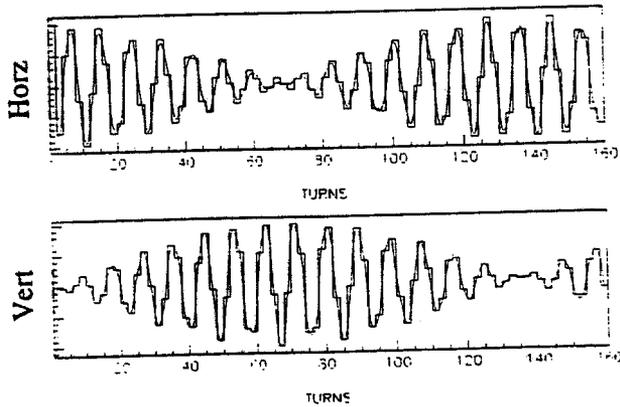


Figure 1. Simulation of horizontal and vertical motion of a particle launched with initial horizontal displacement.

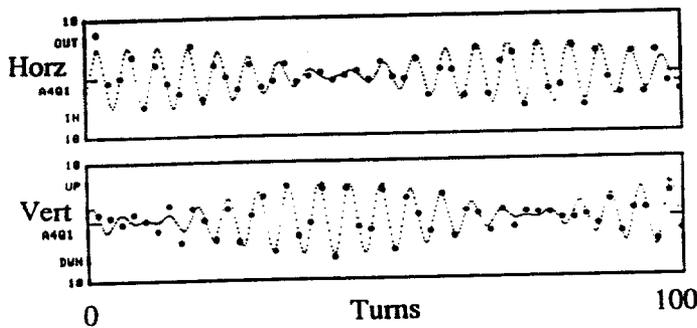


Figure 2. Measured horizontal and vertical motion of a particle injected with initial horizontal displacement.

IV. TRACKING STATISTICAL ENSEMBLES OF PARTICLES

Several ensembles of particles with gaussian distributions in betatron phase space were generated and tracked for 400 turns in the above model. The generated beams had a 95% emittance of 4π in each dimension, roughly equal to the expected size of pbars from the debuncher during normal stacking operations. The ensembles were tracked both with and without coupling to ensure that coupling was indeed the cause of the increased beam sizes.

On each turn the betatron amplitude of each particle was calculated using the known lattice parameters and the position and angle given by the tracking program. The maximum betatron amplitude for each particle was saved and entered into

a histogram after 400 turns, see Figures 3 and 4. Each figure shows two distributions, one for an uncoupled lattice and one for a coupled lattice. The curves are 5th order polynomials meant to guide the eye. The arrows on each distribution show the 90% point of the tail. The effective beam size of coupled beams is roughly twice that of the uncoupled beam, and the distributions appear to be hollow in phase space.

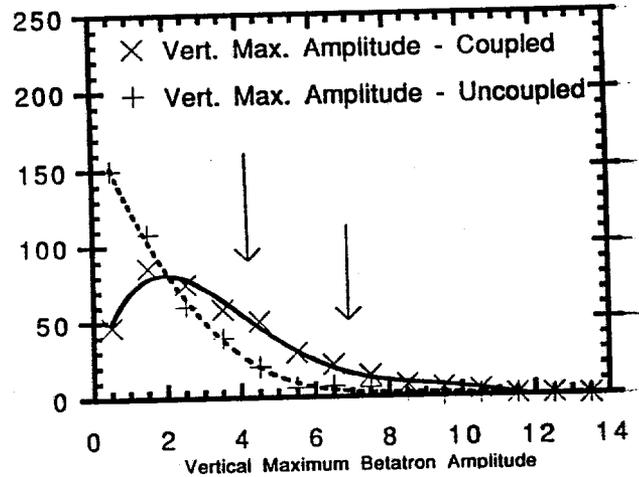


Figure 3. Density of particles as a function of maximum vertical betatron motion with 4π gaussian input beam. The two curves shown are for coupled and uncoupled lattices. The arrows point to the approximate point that contains 90% of the beam.

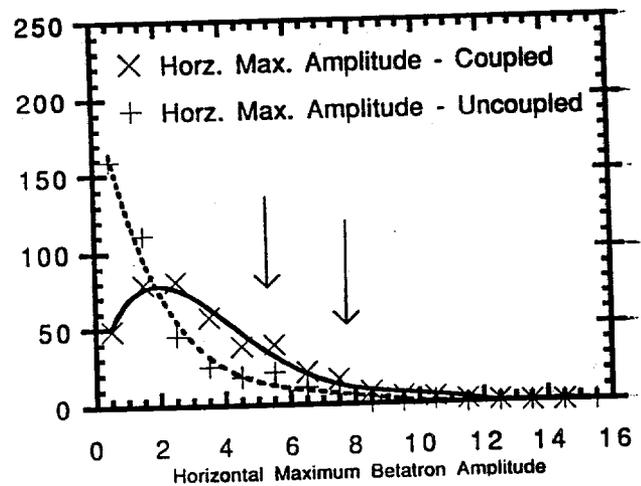


Figure 4. Density of particles as a function of maximum horizontal betatron motion with 4π gaussian input beam. The two curves shown are for coupled and uncoupled lattices. The arrows point to the approximate point that contains 90% of the beam.

V. MEASUREMENTS WITH BEAM

To confirm this effect measurements were made with 8 GeV/c protons injected into the accumulator storage ring. Protons were first injected directly onto the closed orbit and the scraper was used to determine the location of closed orbit, as is shown in Figure 5. The closed orbit location with the skew

quad on is a little difficult to determine from this small reproduction due to the low density of particles with zero betatron amplitude, but is shown by the arrow in the figure. There is very good agreement on the closed orbit location with and without coupling, showing that the skew quad does not steer the beam.

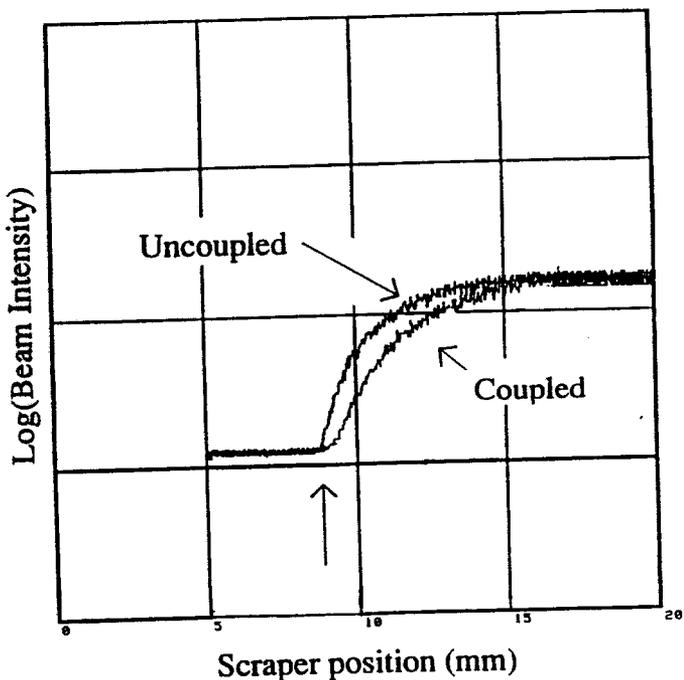


Figure 5. Log(intensity) as a function of scraper location.. The two curves shown are for coupled and uncoupled lattices. The arrow points to the end of the coupled distribution. The closed orbit location is the same with and without coupling, showing that the skew quad does not steer the beam.

Protons were then injected into the accumulator with deliberate injection oscillations in both x and y dimensions. This ensured that the emittance of the beam in the accumulator was uniform from pulse to pulse rather than being determined by factors outside of our control, and made the input emittances large enough for an accurate measurement with the scraper. Figure 6 shows the beam intensity on a linear scale as a function of scraper location for coupled and uncoupled lattice configurations. The arrows in the figure point to the point where 50% of the beam remains for each case. The beam size is clearly larger in a coupled lattice.

To make a quantitative measurement we determined the point where 50% of the beam was scraped away, which is much better determined than the 90% point. We then made a number of measurements summarized in Table I of the maximum betatron amplitude in both dimensions with and without coupling. In all cases the maximum betatron amplitude measured without coupling is smaller than that measured with coupling, giving clear confirmation of the effect described above.

VI. CONCLUSION

This paper has presented both analytical and experimental evidence that beams in a coupled lattice have a larger physical

size than expected from their invariant emittance. In a machine where aperture restrictions are important decoupling the tunes can make a significant improvement. The increased effective beam size may also limit the luminosity of colliding beams. On the other hand, increased beam size may be a benefit in some conditions, for example when ion trapping instabilities are limiting the performance as in the PBAR source. The addition of coupling to the accumulator lattice has been used for most of the last year as one of the more reproducible tools for controlling instabilities caused by ion instabilities.

TABLE I

Measured maximum betatron amplitudes for coupled and uncoupled lattice. The numbers given are the maximum amplitude in π -mm-mr that contains 50% of the beam.

Uncoupled		Coupled	
ϵ_x	ϵ_y	ϵ_x	ϵ_y
1.77	1.44	2.80	2.44
2.06	1.38	2.77	
1.59	1.22	2.60	
1.86			
Average =		1.82	1.35
		2.72	2.44

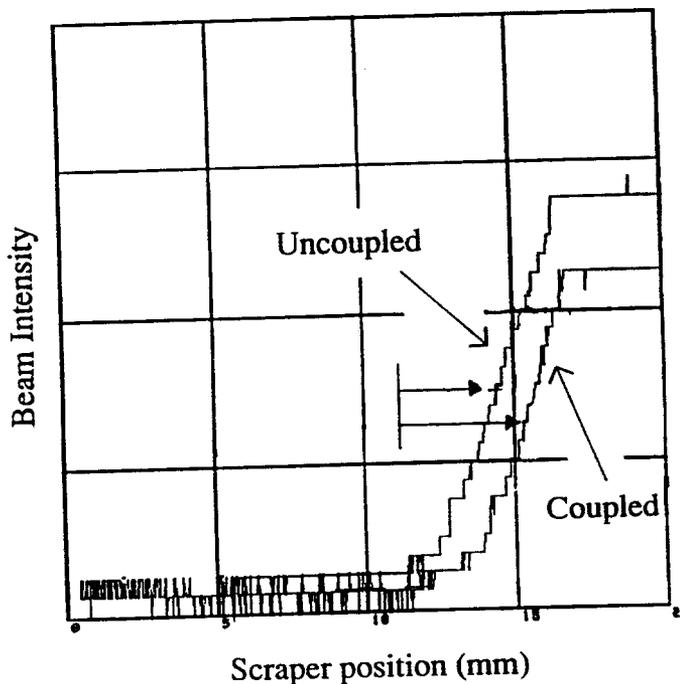


Figure 6. Beam intensity as a function of scraper location.. The two curves shown are for coupled and uncoupled lattices.

VII. REFERENCES

- [1] P.J. Bryant, "A Simple Theory For Weak Betatron Coupling", Proc. of the Cern Accelerator School, Salamanca Spain, September 19-30 1988, p50-62, edited by S. Turner.