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P-Bar Notes #525

A Study of the Effects of Betatron Coupling the Accumulator using a Tracking Program

GOALS

This report will summarize some tracking simulations I performed to demonstrate that in a strongly coupled machine all or most of the transverse energy associated with betatron motion can at times reside in a single dimension. In this case the maximum excursion from the closed orbit is larger than what is expected from the known emittances, and can cause beam loss when it otherwise is not expected.

THE MODEL

This study was performed using the lattice design program DIMAD and our current state of the art lattice for the accumulator. This model includes all dipoles, quadrupoles, and sextupoles, as well as an accurate description of the fringe fields from the dipoles. I adjusted the values of SEX10 and SEX12 to give chromaticities of about +1.5 and -1.0, and the tunes were adjusted to be exactly 6.612 and 8.612 on the central orbit. These chromaticities are somewhat larger than our current operating point. Two sqew quadrupoles were added to the lattice in approximately the same position as our sqew quads, and were given a strength equal to the difference in their settings between the injection and core orbit. There is evidence that there is a strong sqew sextupole field in the accumulator, see PBAR note 427. This was not added to the lattice, but explains why we cannot remove the coupling at more than one momentum in the accumulator.

With the coupling turned on in the model I launched a number of test particles from position A10. The lattice parameters at this point are: betax=6.851, betay=7.224, alfax=0., alfay=-0.0058. This is approximately a dispersionless location in the lattice; this subject will be discussed in more detail below.

Figures 1 and 2 show the x and y position of the test particles every other revolution around the machine for particles launched with an initial displacement in x. For each revolution the emittance in each dimension is calculated from the known lattice parameters using positions and angles given by the tracking program. Note that in a coupled machine the emittances and lattice functions projected into an x-y coordinate system are an ill defined concept, but in this case the dominant motion on the timescale of a few revolutions is still uncoupled betatron motion, so I expect the calculated emittance to adequately describe the particle's motion over the time period of a few turns.

Close to the central orbit the coupling is essentially 100%, while farther away the coupling is strong but not quite 100%. In both cases the energy oscillates from one dimension to the other with a half-period of about 60 turns. Note that the sum of what I call the emittances is roughly an invariant, but the "emittance" in each dimension varies as a function of time.

Figure 3 shows a typical measurement of injection oscillations on the accumulator injection orbit taken with the BPM system. The coupling is also about 100%, and the half-period of energy transfer from one dimension to the other is about 50 turns, in excellent agreement with the model.

CHECKOUT

To show that the program could track particles without emittance blowup I turned the coupling off and launched a number of test particles at various momenta. Figures 4-6 show scatter plots of x-y positions at A10 for a large number of turns with test particles launched with offsets in a single dimension, and Figures 7-9 show the results for particles launched with initial offset in two dimensions. These results agree with what is expected for normal non-coupled motion.

For higher dp/p it was necessary to correct for the finite dispersion of the lattice at A10. This point should be dispersionless, but the addition of higher magnetic elements adds a small dependence of closed orbit on dp/p. The closed orbit locations found using the tracking program are displayed in Table I. The dispersive effects are non-linear, and are important at high dp/p..

TABLE I

Tunes used for tracking test particles

dp/p	Qx	Qy	Closed orbit location (meters)
-.02	.5821779	.6313082	-0.006392
-.008	.6000711	.6197233	-0.001982
.0001	.6121491	.6119035	-0.000009
.0003	.6124473	.6117104	+0.000023
.001	.6134911	.6110346	+0.000137
.002	.6149822	.6100692	+0.000278
.005	.6194556	.6071729	+0.000569
.0105	.6276566	.6018632	+0.000598
.02	.6418221	.5926918	-0.001800

SINGLE PARTICLE MOTION WITH COUPLING

Figures 10-12 show the population in x-y phase space for several particles launched with about "14 pi" of emittance in each plane. Comparing these distributions to Figure 7 it is clear that the maximum excursion of the particles is increased with coupling turned on. Figures 13 and 14 show this effect in more detail. In each case the particles were launched with "14 pi" of emittance in each plane, and in each case the the initial transverse energy oscillates from one plane to the other with a period of about 60 turns. The energy transferal for the case shown in Figure 14 is nearly 100% complete, therefore the physical aperture required to hold such a particle is twice as large in phase space as it need be without coupling.

TRACKING STATISTICAL ENSEMBLES OF PARTICLES

Several ensembles of particles with gaussian distributions in betatron phase space were generated and tracked for 400 turns in the above model. The generated beams had about 4 pi of emittance in each dimension, roughly equal to the expected size of pbars from the debuncher. The ensembles were tracked both with and without coupling to ensure that coupling was indeed the cause of the increased beam sizes.

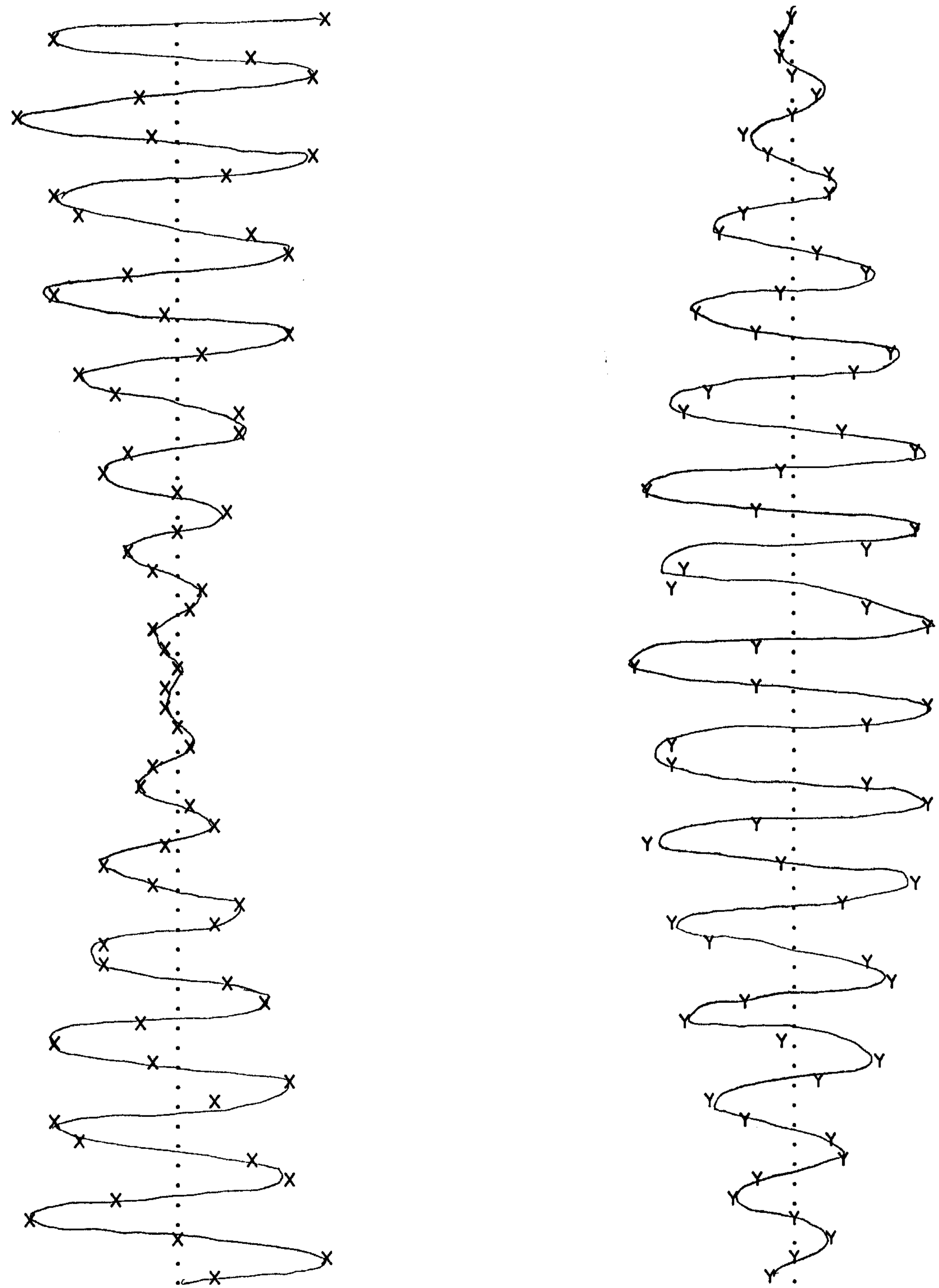
For each particle on each turn the emittance was calculated from the known lattice parameters using positions and angles given by the tracking program. Again, I am using the term "emittance" rather loosely here to mean the amplitude of betatron motion of a particle over the time scale of a few turns. For each plane the maximum "emittance" found after 400 turns was saved. Figures 16-20 show histograms of both the maximum "emittance" and the injected particle emittances. The arrows on each distribution show the 90% point of the tail. For small dp/p the effective beam size of coupled beams determined from these marks is roughly twice that of the injected beam, and the distributions appear to be hollow. At dp/p = .5% the coupling is still visible, but its contribution to the beam size is small.

These results agree remarkably well with the measured beam sizes in PBAR note 519. At the injection orbit on 1/13/92 the tunes were split by .008, at dp/p = .5% in this model the tunes are split by .012. This result says that the measured beam size at this point should be 10-30% larger than the uncoupled beam size. At the central orbit the coupling was seen to be severe, the measured emittance distribution is larger than at injection, and it's distribution in phase space is hollow as it is in this model. The beam size falls as the beam is carried away from the central orbit, and then close to the core the cooling systems cool the beam.

To eliminate this beam size blowup we will need to either split the tunes by more than .012, compensate for the sqew fields, or eliminate the abnormally large sqewed focusing fields in the accumulator.

Figure 1

NTURNS	ε_x	ε_y
0	14.595	0.000
2	14.692	0.034
4	13.906	0.076
6	14.211	0.242
8	14.210	0.436
10	13.630	0.642
12	13.601	1.071
14	12.998	1.221
16	12.507	1.983
18	12.148	2.019
20	12.178	2.931
22	10.672	3.168
24	10.528	3.989
26	10.066	4.404
28	9.360	5.132
30	8.720	5.853
32	7.766	6.204
34	7.451	7.447
36	6.481	7.474
38	6.412	8.630
40	4.985	8.953
42	4.729	9.930
44	4.031	10.168
46	3.675	11.017
48	2.929	11.530
50	2.210	11.836
52	2.082	12.697
54	1.447	12.766
56	1.301	13.354
58	0.655	13.524
60	0.622	14.009
62	0.281	13.987
64	0.264	14.274
66	0.062	14.315
68	0.123	14.351
70	0.080	14.250
72	0.269	14.299
74	0.281	13.984
76	0.719	13.858
78	0.864	13.439
80	1.286	13.339
82	1.484	12.736
84	2.014	12.331
86	2.786	12.019
88	2.919	11.142
90	3.714	11.005
92	3.907	10.060
94	5.234	9.770
96	5.369	8.638
98	6.215	8.585
100	6.585	7.472
102	7.572	6.925
104	8.320	6.375
106	8.709	5.449
108	9.590	5.129
110	9.649	4.175
112	11.124	3.955
114	11.285	2.888
116	11.690	2.856
118	12.067	2.006
120	13.058	1.707
122	13.144	1.268
124	13.439	0.881
126	14.001	0.664
128	13.706	0.312



$\Delta P/P = .01\%$

$X_0 = 10$ mm

$X_0' = 0$

$Y_0 = 0$

$Y_0' = 0$

Figure 2

$\Delta p/p = +.1\%$

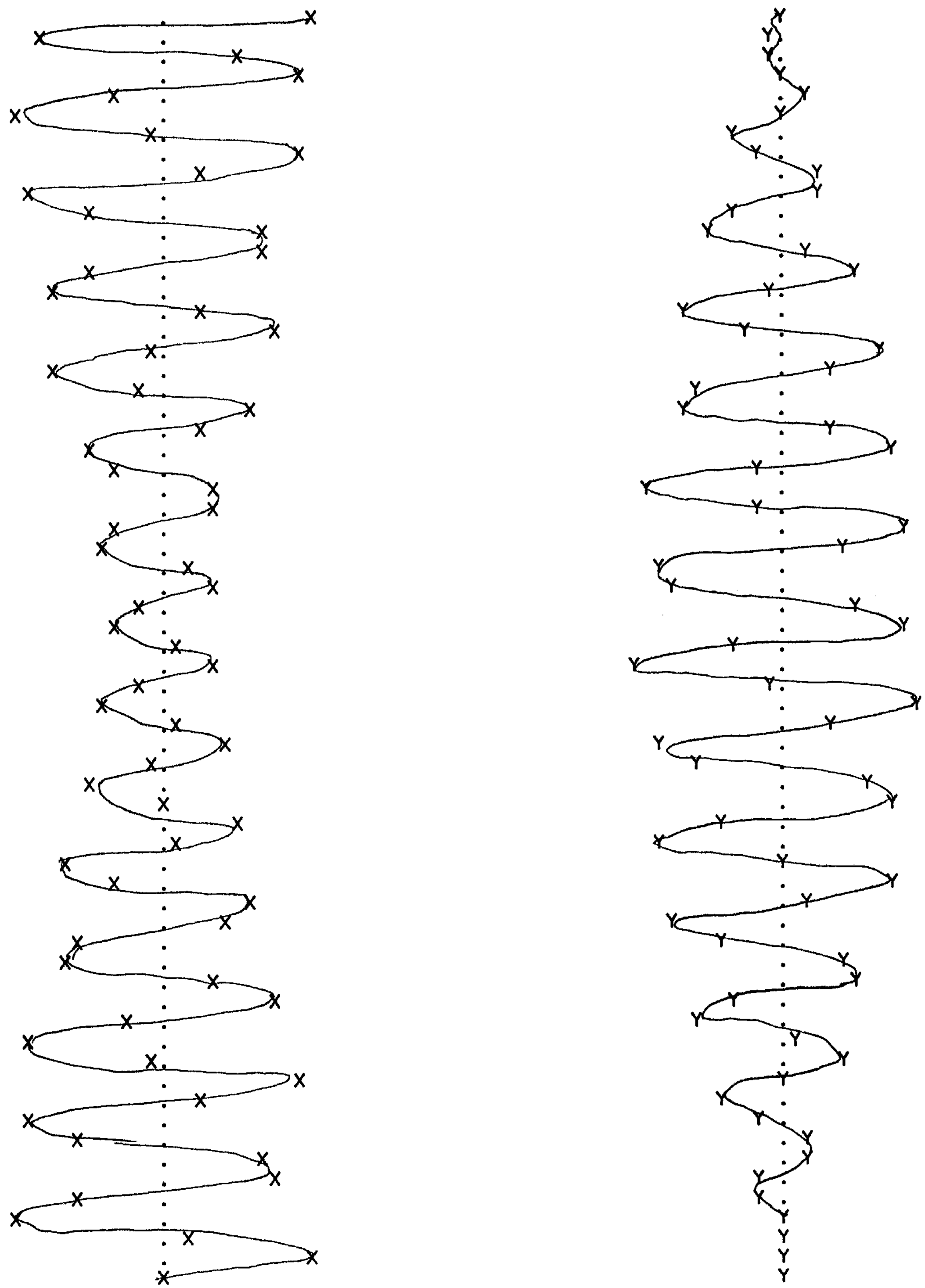
$X_0 = 10$ mm

$X'_0 = 0$

$Y_0 = 0$

$Y'_0 = 0$

NTURNS	ε_x	ε_y
0	14.585	0.000
2	14.528	0.032
4	13.561	0.071
6	14.087	0.241
8	13.844	0.415
10	13.704	0.632
12	13.057	1.028
14	13.069	1.199
16	12.198	1.912
18	12.072	1.962
20	11.991	2.845
22	10.611	3.016
24	10.381	3.914
26	9.947	4.166
28	9.624	4.979
30	8.376	5.527
32	8.027	5.986
34	7.520	6.982
36	6.821	7.138
38	6.451	8.117
40	5.411	8.315
42	5.169	9.406
44	4.476	9.352
46	4.403	10.276
48	3.407	10.491
50	3.141	10.981
52	2.905	11.365
54	2.651	11.675
56	2.233	11.916
58	1.899	11.979
60	2.126	12.462
62	1.787	12.215
64	1.893	12.447
66	1.665	12.204
68	2.199	12.399
70	2.014	11.857
72	2.458	12.072
74	2.424	11.437
76	3.062	11.316
78	3.370	10.758
80	3.771	10.608
82	4.153	9.905
84	4.501	9.392
86	5.639	9.101
88	5.675	8.168
90	6.500	7.998
92	6.569	6.997
94	8.107	6.806
96	8.189	5.661
98	8.756	5.632
100	9.363	4.588
102	10.081	4.184
104	10.851	3.586
106	10.951	2.989
108	11.973	2.598
110	11.580	1.919
112	13.045	1.768
114	13.093	1.046
116	13.140	1.015
118	13.317	0.497
120	14.157	0.412
122	14.240	0.147
124	13.668	0.083
126	14.566	0.006
128	13.887	0.039



CONSOLE LOCATION 08; AP10 26-JAN-1990 22:51
Pbar BPM Turn-by-turn display

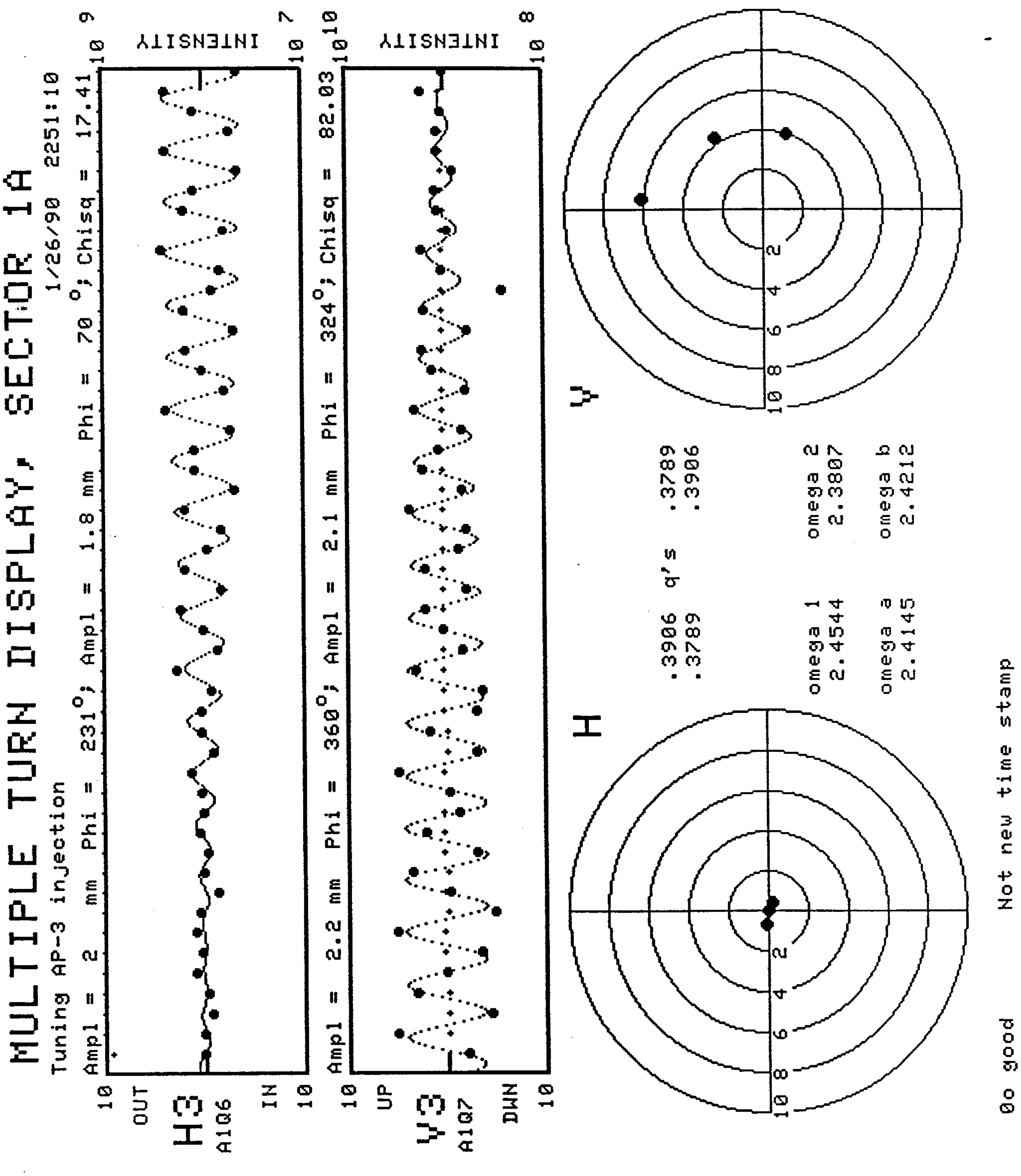


Figure 3

Figure 4

$\Delta P/P = .01\%$

$X_0 = 10$ MM

$X_0' = Y_0' = Y_0 = \emptyset$

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OPERATION LIST ,

  TRACKING

1 1Ø 1 1ØØØ
Ø.Ø1ØØ17 Ø Ø.ØØ Ø Ø Ø.ØØØ1
Ø 13 -.Ø4 .Ø4 Ø Ø -.Ø4 .Ø4 Ø Ø 4Ø 4Ø,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   46Ø(LSL     )  DURING TURN  1ØØØ
                                           PHYSICAL PHASE SPACE

 Ø.4ØØE-Ø1   +    !    !    !    !    !    !    !   +

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             -              1111111111              -

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-Ø.4ØØE-Ø1   +    !    !    !    !    !    !    !   +
          -Ø.4ØØE-Ø1                              Ø.4ØØE-Ø1
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Figure 5

$\Delta p/p = .1\%$

$y_0 = 10$ mm

$x_0 = x_0' = y_0' = \emptyset$

```
OPERATION LIST ,

  TRACKING

1 -2 1 1000
0.00017 0 0.01 0 0 0.001
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   460(LSL     )  DURING TURN  1000

                                                    PHYSICAL PHASE SPACE

 0.400E-01   +    !    !    !    !    !    !    !    +


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             -                  11                   -
                                11
                                11
                                 1
                                 1
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             -                                       -


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-0.400E-01   +    !    !    !    !    !    !    !    +
         -0.400E-01                               0.400E-01
```

Figure 6

$\Delta P/P = .1\%$

$X_0 = 10$ mm

$X_0' = Y_0' = Y_0 = \emptyset$

```
OPERATION LIST ,

  TRACKING

1 -2 1 1000
0.01017 0 0.00 0 0 0.001
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   460(LSL     )  DURING TURN  1000
                                                PHYSICAL PHASE SPACE

 0.400E-01   +    !    !    !    !    !    !    !   +

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             -             1111111111               -

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-0.400E-01   +    !    !    !    !    !    !    !   +
        -0.400E-01                                0.400E-01
```

Figure 7

$\Delta p/p = .1\%$

$Y_0 = X_0 = 10\,mm$

$X_0' = Y_0' = 0$

```
OPERATION LIST ,

  TRACKING

1 -2 1 10000
0.01017 0 0.01 0 0 0.001
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   460(LSL     )  DURING TURN 10000
                                            PHYSICAL PHASE SPACE

 0.400E-01   +    !    !    !    !    !    !    !   +

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                            1111111111
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-0.400E-01   +    !    !    !    !    !    !    !   +
         -0.400E-01                               0.400E-01
```

Figure 8

$\Delta P/P = .4\%$

$X_0 = Y_0 = 10\,mm$

$X_0' = Y_0' = 0$

```
OPERATION LIST ,

  TRACKING

1 -2 1 10000
0.0107 0 0.01 0 0 0.004
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   460(LSL      )  DURING TURN 10000
                                          PHYSICAL PHASE SPACE

 0.400E-01   +    !    !    !    !    !    !    !   +


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             -              11111111111            -
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-0.400E-01   +    !    !    !    !    !    !    !   +
        -0.400E-01                                0.400E-01
```

Figure 9

$\Delta p/p = 1\%$

$X_0 = Y_0 = 10$ mm

$X_0' = Y_0' = 0$

```
OPERATION LIST ,

  TRACKING

1 -2 1 10000
0.01174 0 0.01 0 0 0.01
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   460(LSL     )  DURING TURN 10000
                                             PHYSICAL PHASE SPACE

 0.400E-01   +    !    !    !    !    !    !    !   +


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             -            111111111111             -
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                          111111111111
                          111111111111
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-0.400E-01   +    !    !    !    !    !    !    !   +
         -0.400E-01                                0.400E-01
```

Figure 10

$\Delta p/p = .1\%$

$X_0 = Y_0 = 10$ mm

$X_0' = Y_0' = \emptyset$

```
OPERATION LIST ,

  TRACKING

1 -2 1 10000
0.01017 0 0.01 0 0 0.001
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   460(LSL     )  DURING TURN 10000
                                            PHYSICAL PHASE SPACE

 0.400E-01   +    !    !    !    !    !    !    !    +


             -                                       -


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                                    11
                                   1111
             -                    111111             -
                                  11111
                                 11111
                                111111
                               111111
             -                 111111                -
                              111111
                              11111
                             11111
                             11111
             -               1111                    -
                              11


             -                                       -


             -                                       -


-0.400E-01   +    !    !    !    !    !    !    !    +
         -0.400E-01                               0.400E-01
```

Figure 11

$\Delta p/p = .1\%$

$X_0 = 10mm$

$Y_0 = -10mm$

$X_0' = Y_0' = 0$

OPERATION LIST ,

TRACKING

```
1 -2 1 10000
0.01017 0 -0.01 0 0 0.001
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,
```

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT 460(LSL) DURING TURN 10000

PHYSICAL PHASE SPACE

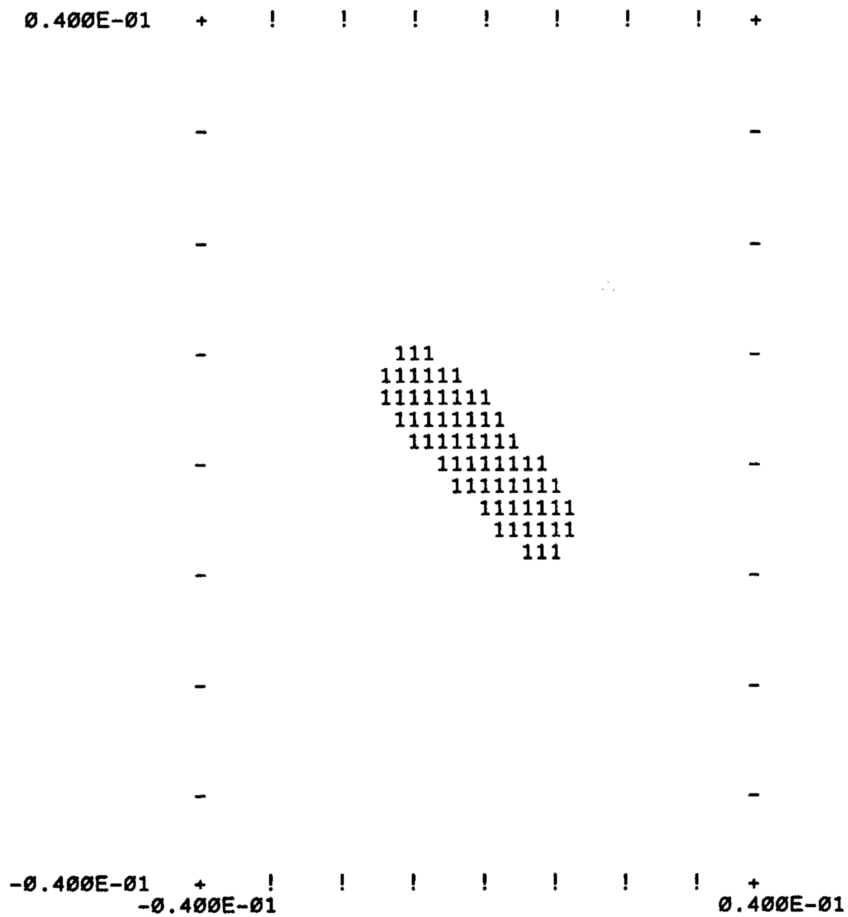


Figure 12

$\Delta p/p = .1\%$

$X_0 = 0$

$Y_0 = 10$ mm

$Y_0' = 0$

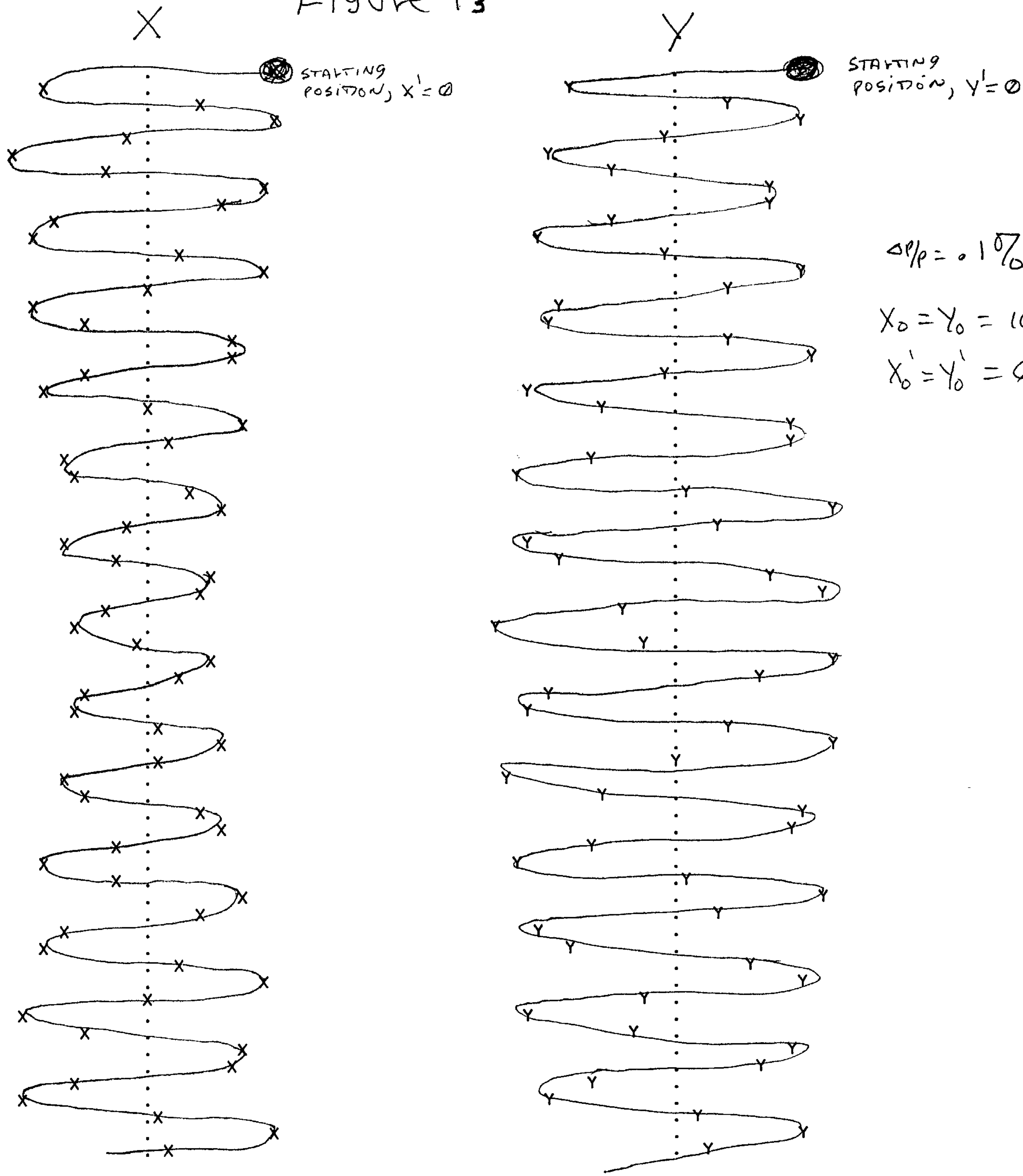
$X_0' = -.0014287$

(CALCULATED TO GIVE SAME MAXIMUM EXCURSION IN X)

```
OPERATION LIST ,

 TRACKING

1 2 1 500
0.00012755 -0.0014287 0.01 0 0 0.001
0 13 -.04 .04 0 0 -.04 .04 0 0 40 40,

PLOTS OF PARTICLE POSITIONS AFTER ELEMENT   460(LSL     )  DURING TURN   500
                                             PHYSICAL PHASE SPACE

 0.400E-01   +    !    !    !    !    !    !    !    +

             -                                       -

             -                                       -

                          111
                          1111
             -           1111111                     -
                         11111111
                        1 1 11 1111
                       1 1 11 11111
                       111 1  1  1
             -        111 11 11 11                   -
                     111111 1 111
                      1111111 11
                       111111111
             -           11 111                      -
                  .       1111
                           11

             -                                       -

             -                                       -

-0.400E-01   +    !    !    !    !    !    !    !    +
         -0.400E-01                               0.400E-01
```

Figure 13

NTURNS	$\mathcal{E}_x$	$\mathcal{E}_y$
0	14.585	13.841
2	14.688	13.487
4	12.914	12.418
6	14.634	13.546
8	13.508	12.450
10	14.809	13.381
12	13.809	12.971
14	13.725	13.252
16	13.115	13.415
18	13.523	13.439
20	14.014	14.420
22	11.689	13.687
24	13.042	15.249
26	11.383	14.465
28	12.181	15.607
30	11.465	16.086
32	10.081	15.905
34	10.269	17.222
36	9.570	16.839
38	10.071	18.204
40	8.121	18.014
42	8.477	19.088
44	7.325	18.981
46	7.569	19.688
48	7.411	20.556
50	5.650	20.054
52	6.373	21.597
54	5.251	20.865
56	5.880	21.931
58	5.056	22.081
60	4.460	22.038
62	4.576	22.626
64	4.278	22.336
66	4.926	23.073
68	3.855	22.437
70	4.391	23.114
72	3.909	22.479
74	4.515	22.669
76	5.049	22.872
78	4.080	21.785
80	5.274	22.641
82	4.650	21.424
84	6.000	21.760
86	6.078	21.204
88	5.835	20.582
90	6.681	20.548
92	6.779	19.706
94	8.349	19.918
96	7.594	18.472
98	8.814	19.002
100	8.349	17.652
102	9.808	17.582
104	10.711	17.374
106	9.600	15.927
108	11.501	16.633
110	10.696	15.184
112	12.625	15.569
114	12.271	14.586
116	12.415	14.493
118	12.558	14.020
120	13.315	13.617
122	14.415	14.022
124	12.734	12.675
126	14.621	13.792
128	13.110	12.543



$\Delta p/p = .1\%$

$X_0 = Y_0 = 10$ mm

$X_0' = Y_0' = 0$

Figure 14

$\Delta p/p = .1\%$

$X_0 = \emptyset$

$Y_0 = 10$ mm

$Y_0' = \emptyset$

$X_0' = -.0014287$

NTURNS	ε_x	ε_y
0	13.984	13.841
2	12.482	14.055
4	12.245	16.068
6	10.808	16.533
8	10.229	18.276
10	8.278	19.511
12	6.819	20.161
14	6.294	22.220
16	5.051	22.309
18	4.624	23.847
20	2.811	24.350
22	2.602	25.643
24	1.737	25.695
26	1.613	26.592
28	0.763	26.935
30	0.522	27.127
32	0.445	27.431
34	0.455	27.545
36	0.334	27.261
38	0.690	27.114
40	1.192	26.789
42	1.453	26.415
44	1.919	25.699
46	2.485	24.970
48	4.223	24.526
50	4.141	22.846
52	5.614	22.910
54	5.849	21.040
56	8.272	20.389
58	8.982	18.641
60	9.862	18.096
62	11.171	16.324
64	12.271	15.074
66	14.824	14.300
68	14.744	11.972
70	16.915	11.935
72	16.476	9.773
74	19.929	9.254
76	20.398	7.450
78	20.337	6.962
80	22.090	5.662
82	22.998	4.723
84	24.448	4.351
86	23.843	2.952
88	25.945	3.137
90	24.112	2.083
92	26.588	2.124
94	26.877	1.800
96	25.019	1.578
98	25.909	1.983
100	26.207	1.690
102	26.246	2.539
104	24.167	2.554
106	25.353	3.366
108	23.078	3.961
110	23.352	4.555
112	23.136	5.931
114	20.302	6.174
116	19.972	8.258
118	19.151	8.215
120	18.815	10.285
122	15.738	11.135
124	15.472	12.395
126	13.938	13.882
128	12.966	14.787

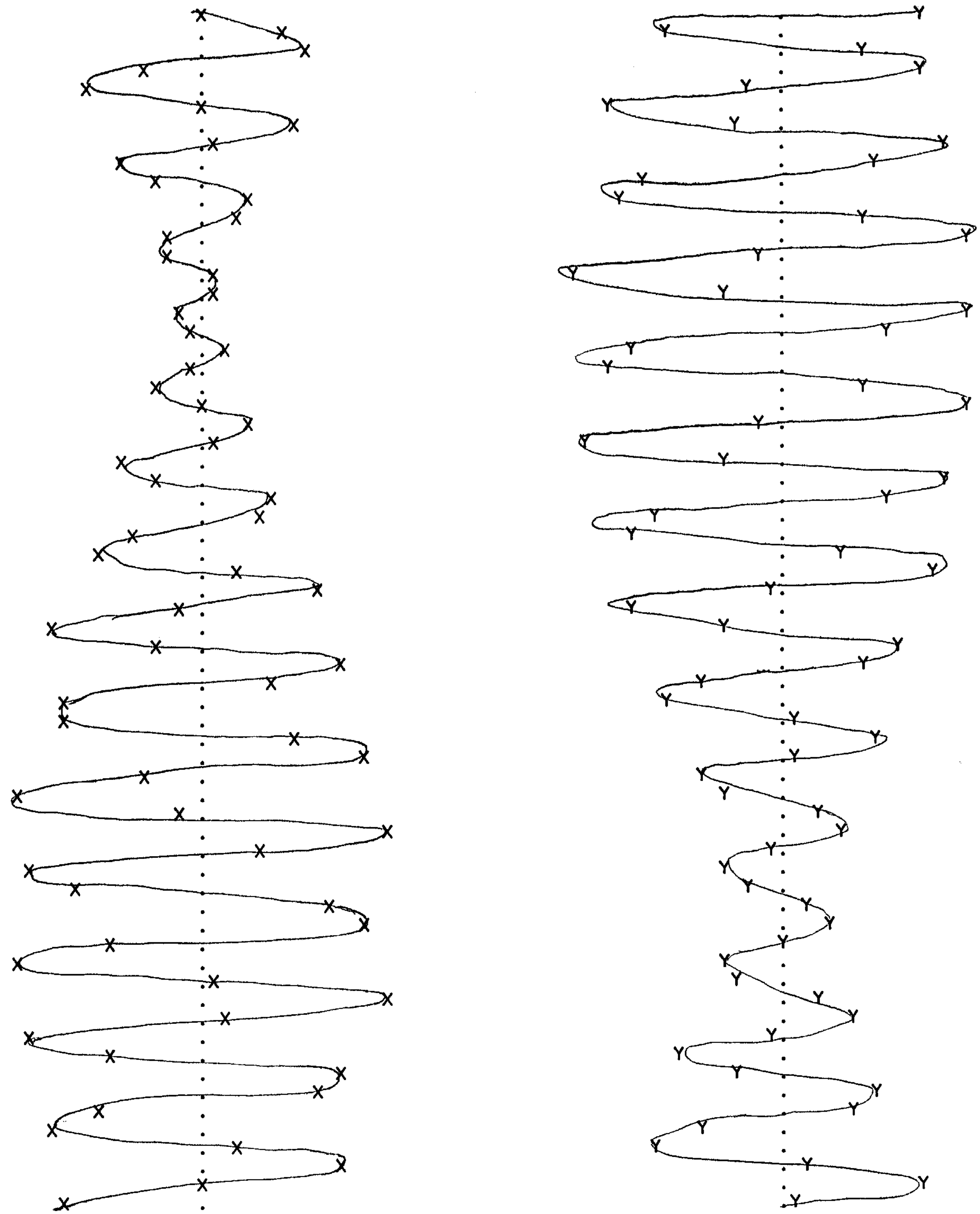
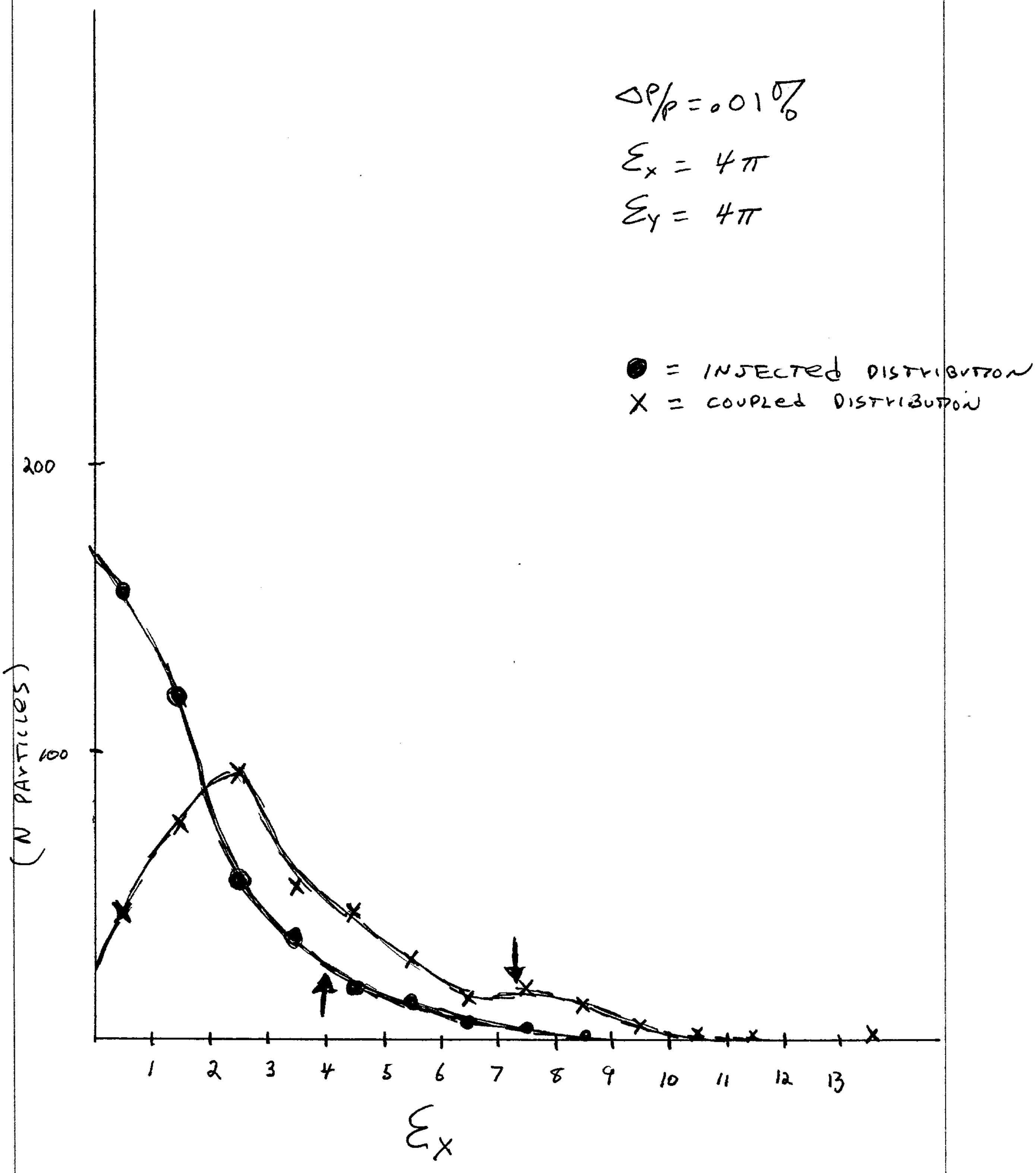
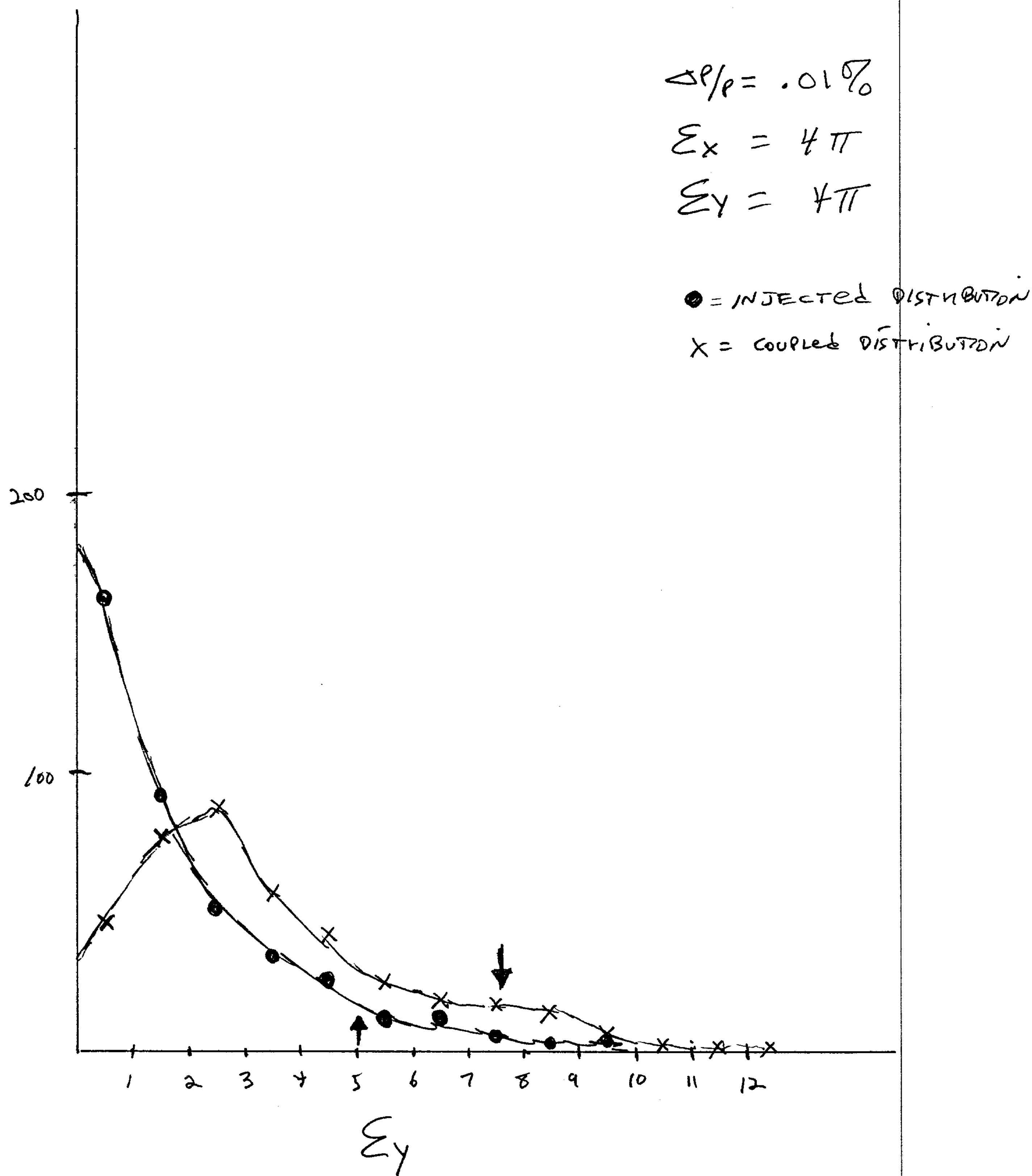


Figure 15



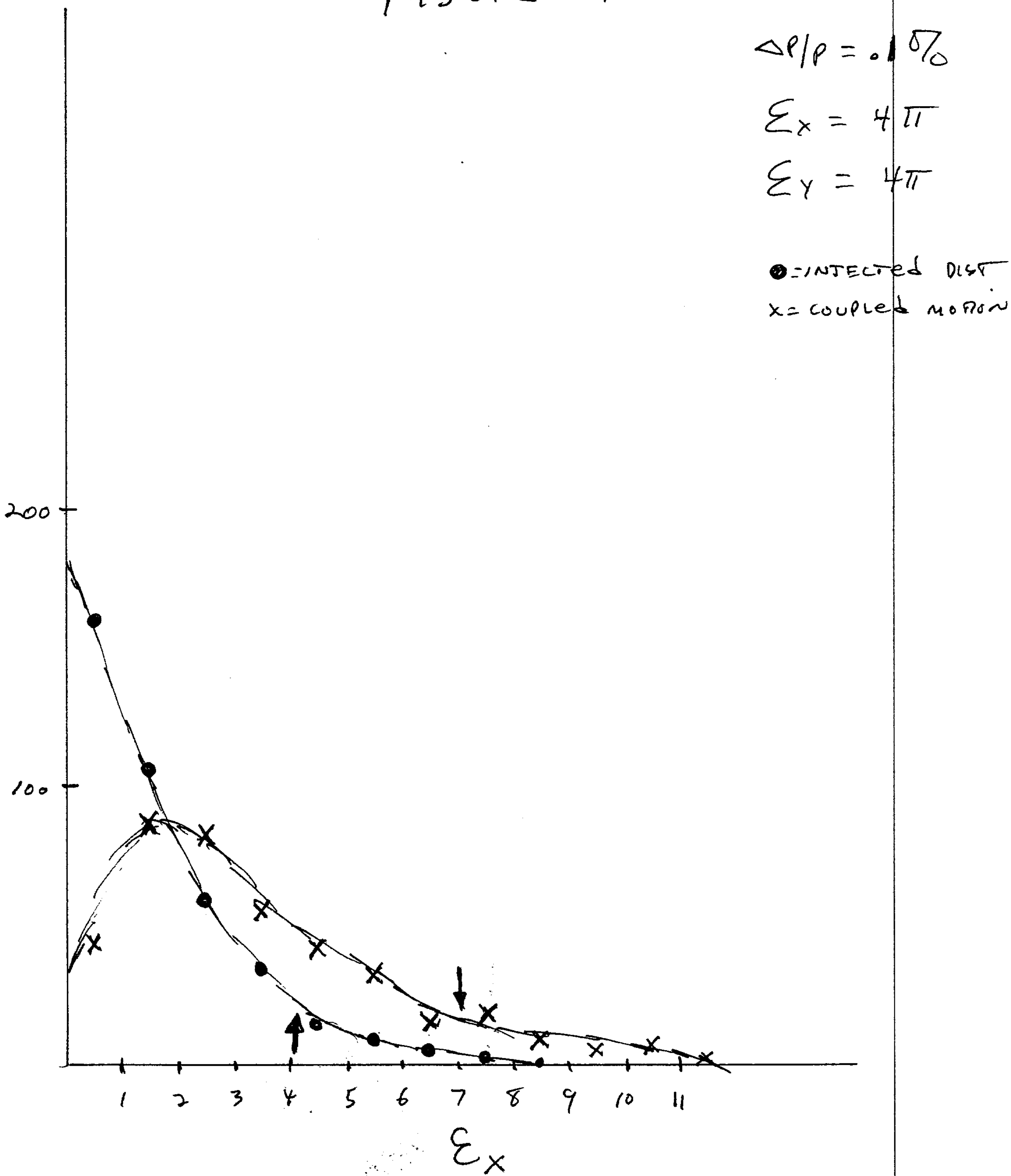
ΔP/P = .01%
$\mathcal{E}_x = 4\pi$
$\mathcal{E}_y = 4\pi$
● = INJECTED DISTRIBUTION
X = COUPLED DISTRIBUTION
200
100
(N PARTICLES)
1
2
3
4
5
6
7
8
9
10
11
12
13
$\mathcal{E}_x$

Figure 16



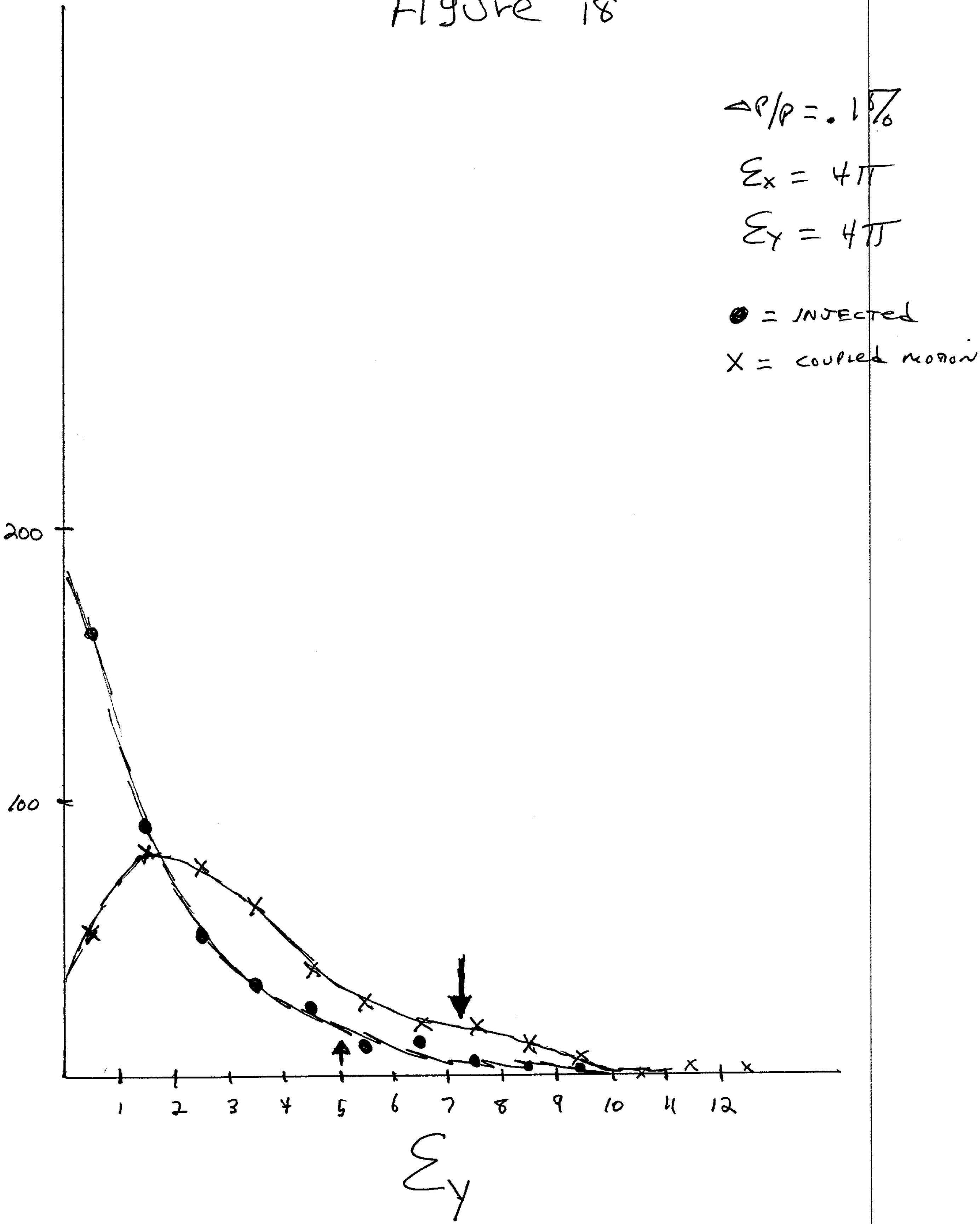
$\Delta p/p = .01\%$
$\mathcal{E}_x = 4\pi$
$\mathcal{E}_y = 4\pi$
● = INJECTED DISTRIBUTION
X = COUPLED DISTRIBUTION
200
100
1
2
3
4
5
6
7
8
9
10
11
12
$\mathcal{E}_y$

Figure 17



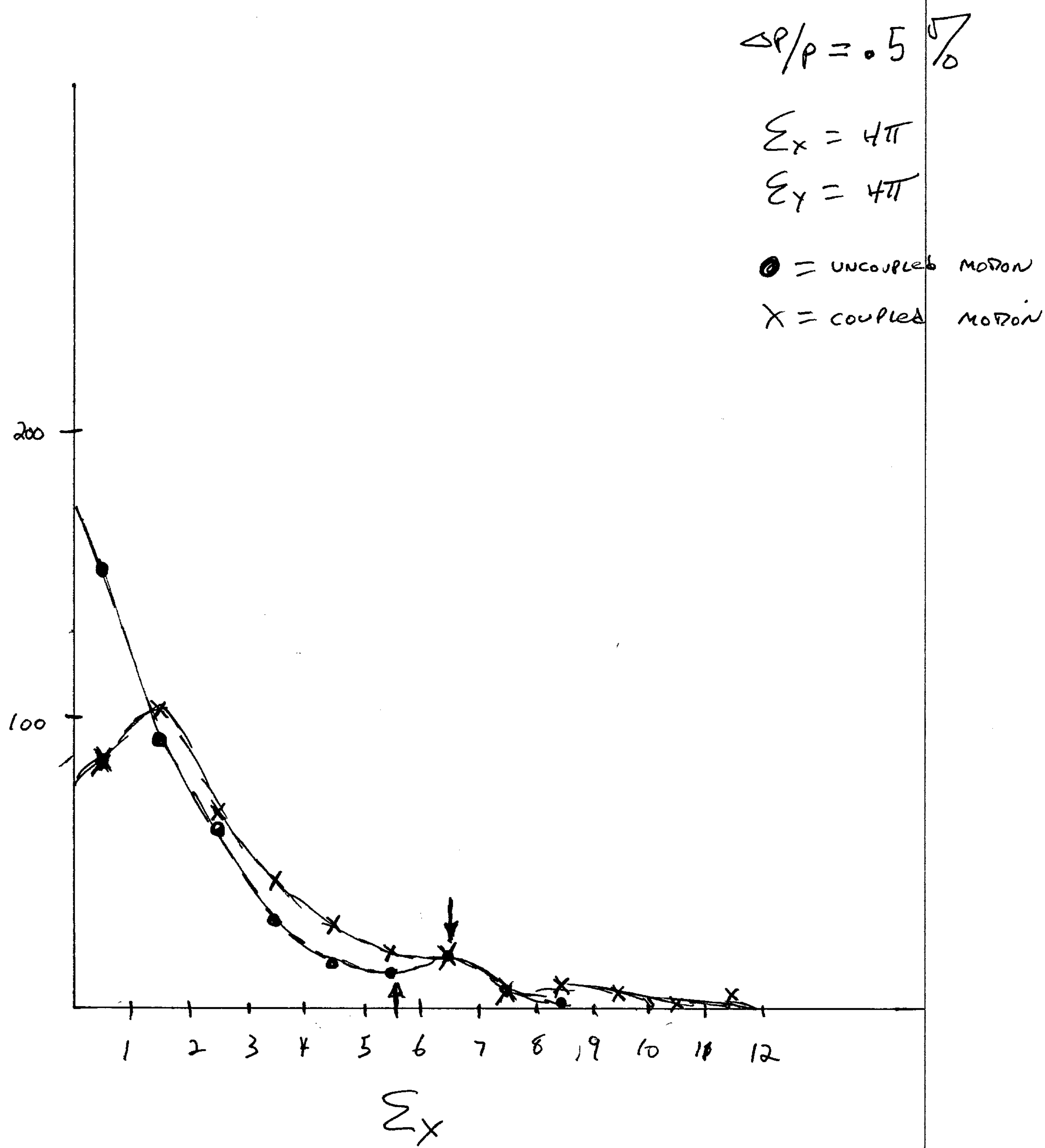
$\Delta p/p = .1\%$
$\mathcal{E}_x = 4\pi$
$\mathcal{E}_y = 4\pi$
● = INJECTED DIST
x = COUPLED MOTION
200
100
1
2
3
4
5
6
7
8
9
10
11
$\mathcal{E}_x$

Figure 18



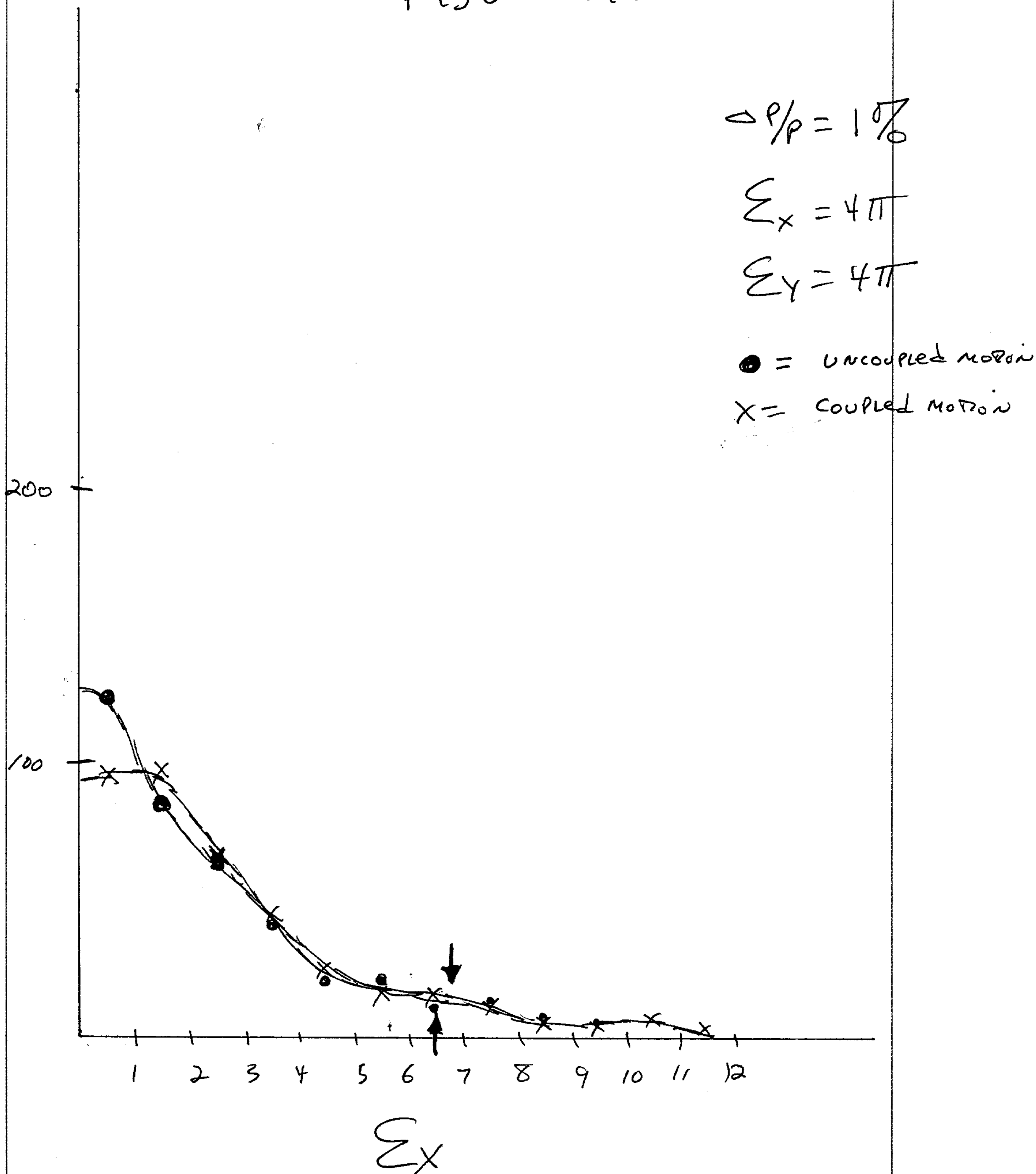
Δp/p = .1%
$\mathcal{E}_x = 4\pi$
$\mathcal{E}_y = 4\pi$
● = INJECTED
X = COUPLED MOTION
200
100
1 2 3 4 5 6 7 8 9 10 11 12
$\mathcal{E}_y$

Figure 19



$\Delta p/p = .5\%$
$\mathcal{E}_x = 4\pi$
$\mathcal{E}_y = 4\pi$
● = UNCOUPLED MOTION
X = COUPLED MOTION
200
100
1
2
3
4
5
6
7
8
9
10
11
12
$\mathcal{E}_x$

Figure 20



$\Delta P/P = 1\%$
$\mathcal{E}_x = 4\pi$
$\mathcal{E}_y = 4\pi$
● = UNCOUPLED MOTION
X = COUPLED MOTION
200
100
1 2 3 4 5 6 7 8 9 10 11 12
$\mathcal{E}_x$