Fermilab

$\bar{p}$ Note #450

Longitudinal Phase apace in Booster to Debuncher/Accumulator Beam Transfers

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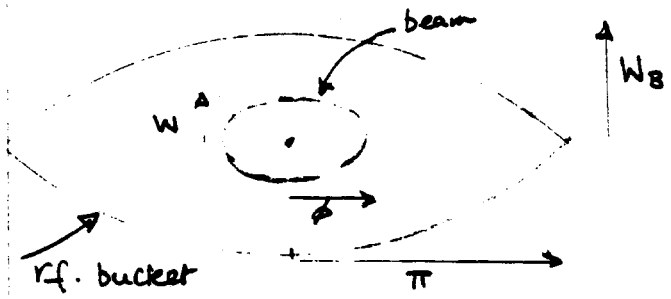
LONGITUDINAL PHASE SPACE IN BOOSTER TO DEBUNCHER/ACCUMULATOR BEAM TRANSFERS

(A) FORMULAE



β = velocity
$\omega_{rf.}$ = angular rf frequency
E_s = beam energy
V = rf voltage
e = proton charge
h = harmonic number
η = momentum compaction factor
W_B = bucket hieght
A_B = bucket area
A = beam emittance
W = beam Δp
f_s = synchrotron frequency
f_0 = revolution frequency

then

$$A_B = \frac{8\beta}{\omega_{rf.}}\left[\frac{2\,E_s\,(eV)}{\pi h \eta}\right]^{\frac{1}{2}}$$

$$W_B = \frac{A_B\,\omega_{rf}}{8}$$

where A, A_B are in eV-secs

$$W = W_b \sin\frac{\phi}{2}$$

and W, W_B are in eV

$$A = \frac{\pi W \phi}{\omega_{rf.}} \Rightarrow \phi \sin\frac{\phi}{2} = \frac{A\,\omega_{rf}}{\pi W_B}$$

$$f_s = f_0\left[\frac{h\eta(eV\cos\phi)}{2\pi E_s}\right]^{\frac{1}{2}}$$

(B) <u>Booster Beam To Accumulator</u>

Booster beam emmittance = 0.08 per bunch (measured EXP 111A)

Booster r.f. voltage at extraction = 400 kV

In the booster $h=84$, $\eta = 0.0226$, $\omega_{r.f.} = 2\pi\, 52.813 \times 10^6 = 3.3183 \times 10^8$
$E_s = 8.89 \times 10^9$, $\beta \sim 1$

$$A_B = \frac{8\beta}{\omega_{rf}} \left[\frac{2 E_s eV}{\pi h \eta}\right]^{\frac{1}{2}} = 0.8325 \text{ eV-sec}$$

$$W_B = \frac{A_B \omega_{rf}}{8} = 34.532 \text{ MeV}$$

$$A = 0.08 = \frac{\pi W \phi}{\omega_{r.f.}} \quad ; \quad W = W_B \sin\frac{\phi}{2}$$

$$\Rightarrow \quad \phi = 0.71 \text{ radians} = 0.226\pi \qquad W = 12.0 \text{ MeV}$$

Since the Accumulator $\eta = 0.024$ is about the same as the Booster to make a matched bucket to bucket transfer we would need to make the same voltage in the Accumulator as the Booster. Clearly 400kV in the Accumulator is impossible. Alternatives are

(a) If nothing can be done in the Booster ~~then~~ and the maximum voltage in the accumulator is 100kV (pulsed) and 50kV (CW) then

At 100 kV in the Accumulator

$$A_B = 0.404 \text{ eV-sec}$$

$$W_B = 16.75 \text{ MeV}$$

$$\sin\frac{\phi}{2} = \frac{W}{W_B} = \frac{12}{16.75} = 1.60 \text{ radian} \quad (.508\pi) \quad \left(\frac{\pi}{1.96} \text{ ie. } \sim 90^\circ\right)$$

$$A = \frac{\pi W \phi}{\omega_{rf}} = 0.1817 \Rightarrow \text{dilution} = 2.27$$

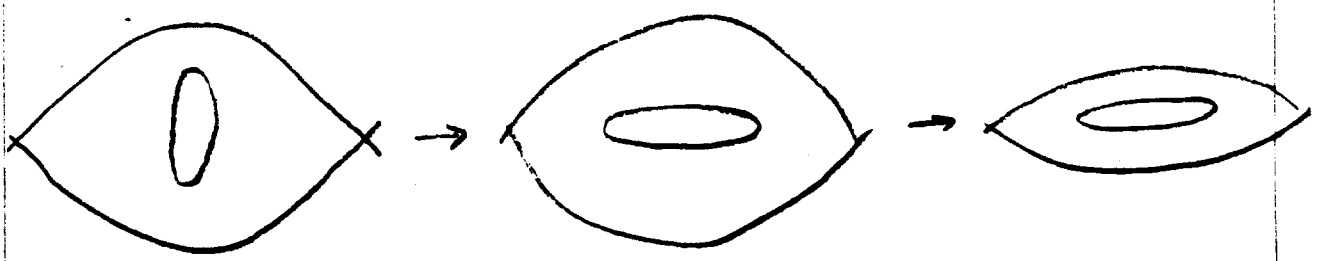
We reduce to 50kV in the Accumulator adiabatically

then A remains the same : $A_B = 0.286$, $W_B = 11.84$

$$A = 0.1817 = \frac{\pi W \phi}{\omega_{rf}} \quad \text{and} \quad W = W_B \sin\frac{\phi}{2}$$

$$\Rightarrow \phi \sin\frac{\phi}{2} = \frac{0.1817\,\omega_{rf}}{\pi \times 11.84 \times 10^6} = 1.62 \Rightarrow \phi \sim 0.62\pi$$

another scenario is to snap down from 100 kV down to a matched bucket after a $\frac{1}{4}$ turn in longitudinal phase space.



If the beam spot can fit inside a 100 kV bucket, then ideally there will be no dilution in the beam phase space

Coming out of the Booster $W = 12$ MeV
$\phi = 0.71$ radians

at 100 kV in the Accumulator

$$A_B = 0.404 \text{ eV-sec}, \quad W_B = 16.75 \text{ MeV} \;\|\; A = 0.08 \text{ eV-sec}$$

A ±12 MeV beam can fit into this bucket.

What is the rf voltage that the r.f. should be reduced to, to form the matched bucket?
at 100 kV

(1) $W = 12.0$, then $\sin\frac{\phi}{2} = \frac{12}{16.75} \Rightarrow \phi = 0.508\pi$

(2) $\phi = 0.71$ then $W = W_B \sin\frac{\phi}{2} \Rightarrow W = 5.82$ MeV

$\Rightarrow$ Matched bucket is that which contain ± 5.82 MeV in $\phi = 0.508\pi$

$$\Rightarrow W_B = \frac{5.82}{\sin\left(\frac{0.508\pi}{2}\right)} = 8.13 \text{ MeV} \qquad eV = \frac{W_B^2 \pi h \eta}{2 E_s} = 24 \text{ kV}$$

at 100 kV synchrotron frequency $= f_0 \left[\frac{h \eta eV \cos\phi}{2\pi E_s}\right]^{\frac{1}{2}} = 1140$ hz

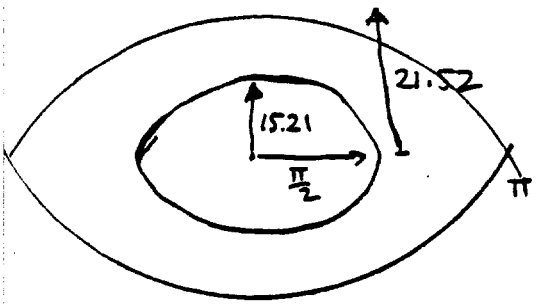
$\Rightarrow \frac{1}{4}$ turn happens in ~ 220 μseconds.

(C) Booster Beam To Debuncher

We would like to capture beam in the $h=4$ system (DRF3) in the debuncher

The maximum voltage that DRF3 can generate is 1.8 kV

For a ~~bucket~~ beam that is $\pm\frac{\pi}{2}$



$$A_B = \frac{8}{\omega_{rf}}\left[\frac{2E_s(eV)}{\pi h \eta}\right]^{\frac{1}{2}}$$

$\omega_{rf} = 1.483\times10^{-}$
$\eta = 0.0055$
$h = 4$

$$= 11.608$$

$$W_B = 21.52 \text{ MeV} \quad \left(\frac{A_B\omega_{rf}}{8}\right)$$

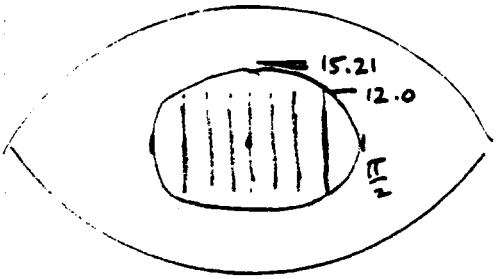
$$W = 15.21 \text{ MeV} \quad \left(W_B \sin\frac{\phi}{2}\right)$$

$$A = \frac{\pi W \phi}{\omega_{rf}} = 5.06 \text{ eV-sec.}$$

Incoming into the debuncher from the booster are bunches that are ± 12 MeV and $\frac{0.71}{\pi} \times 18.93$ ns wide

$= 4.28$ ns wide (at $A=0.08$

$V_{rf} = 400$ in the buc



How far in ϕ for a $h=4$ bucket can be go before the 53 MHz bunches no longer sit inside the desired beam $h=4$ bunch?

For an ellipse $W' = W\cos\alpha$, $\phi' = \phi\sin\alpha$ to trace out the circumference

$$\Rightarrow \cos\alpha = \frac{W'}{W} = \frac{12.0}{15.21} \doteq$$

$$\Rightarrow \phi' = \frac{\pi}{2}\sin\alpha = 0.965$$

$$\Rightarrow \text{\# of 53MHz bunches} = \frac{0.965}{\pi} \times 22.5 = 6.91$$

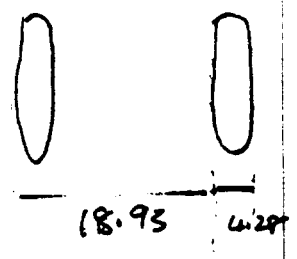
22.5 53 MHz = 1 $h=4$ bunch.

Width of beam = 18.93 ns / 53 MHz bunch.

If we can time the $h=4$ bucket so that the beam alway arrives in time then we can capture about 7 rf. buckets from the booster

If there is a ± 50 ns jitter in the timing of the $h=4$ bucket relative to the beam

then we can only use $130 - 100 = 30$ nsec of the beam

This corresponds to 2 r.f. (53 MHz) buckets.

(ie. $18.93 + 4.28$ ns < 30 ns)