Control of Transverse Multibunch Instabilities in the First Stage of the VLHC

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VLHC mini-workshop
SLAC, March 21-23, 2001.

Content

VLHC Parameters

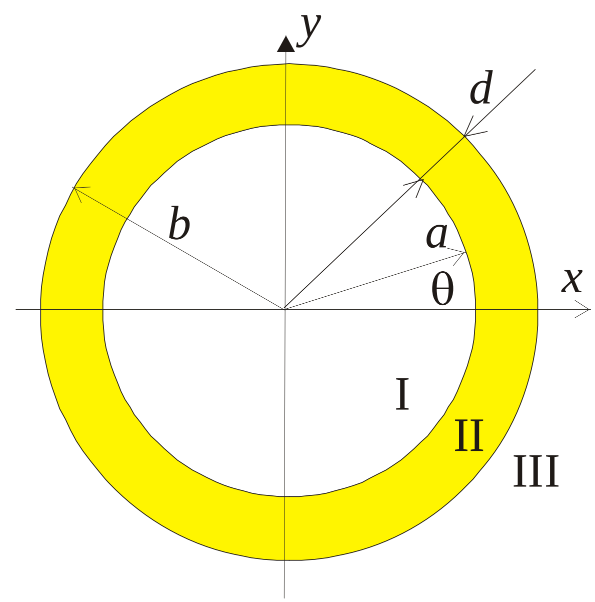
		Stage 1	Stage 2
Beam energy	E_p	20 TeV	87.5 TeV
Luminosity	L	10^{34} cm^{-2}s^{-1}	$2\cdot10^{34}$ cm^{-2}s^{-1}
Magnetic field	B_0	1.96 T	10 T
Injection energy	E_{inj}	0.9 TeV	10 TeV
Bunch spacing	τ_b	18.9 ns	
Circumference	$C=2\pi R$	232 km	
Revolution frequency	f_0	1294 Hz	
Number of bunches	N_b	36943	
RF frequency	f_{RF}	477.9 MHz	
Betatron tunes	$\nu_\perp$	~214	
Momentum compaction	α	$2.1\cdot10^{-5}$	
Beta-function at IP	β^*	30 cm	50 cm
Head-on beam-beam tune shift per IP	ξ	$1.8\cdot10^{-2}$	$1.8\cdot10^{-2}$
Number of particles per bunch	N	$2.5\cdot10^{10}$	
Beam current	I_b	0.190 A	0.175 A
RF voltage per turn, top/injection energy	V_0	50/50 MeV	50/50 MeV
Synchrotron frequency, top/injection energy	f_s	2.32/10.9 Hz	1.1/3.28 Hz
Rms momentum spread, top/injection energy	σ_p	$1.5/14.9\cdot10^{-4}$	$0.5/2.4\cdot10^{-4}$
Rms bunch length, top/injection energy	σ_s	6.6/14.2 cm	4.5/7.8 cm

2. Transverse Impedance for Vacuum Chamber with Thin Wall

Impedance calculation for round vacuum chamber

$$k=\frac{1-i}{\delta}\quad,\quad \delta=\frac{c}{\sqrt{2\pi\sigma\omega}}\quad,$$

$$Z_0=\frac{4\pi}{c}\approx 377\,\Omega\quad.$$



Solutions for the vector potential in different regions

$$A_z(r,\theta,t)=\begin{bmatrix}A_I\\A_{II}\\A_{III}\end{bmatrix}=\begin{bmatrix}\dfrac{2I_0x_0}{cr}+C_1r\\[2ex] C_2e^{k(r-a)}+C_3e^{-k(r-a)}\\[2ex] \dfrac{C_4}{r}\end{bmatrix}e^{i\omega t}\cos\theta$$

I_0 – the beam current
x_0 – amplitude of transverse beam motion

Matching solutions at boundaries: $\left.\dfrac{dA}{d\theta}\right|_r=\left.\dfrac{dA}{d\theta}\right|_{r+0}$; $\left.\dfrac{dA}{dr}\right|_r=\left.\dfrac{dA}{dr}\right|_{r+0}$ yields four linear equations for coefficients C_i . The solution is:

$$C_1 = \frac{2I_0 x_0}{ca^2}\left[2\frac{e^{kd}(bk+1)+e^{-kd}(bk-1)}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)}-1\right] \quad ,$$

$$C_2 = \frac{4I_0 x_0}{ca}\frac{e^{-kd}(bk-1)}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)} \quad ,$$

$$C_3 = \frac{4I_0 x_0}{ca}\frac{e^{kd}(bk+1)}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)} \quad ,$$

$$C_4 = \frac{8I_0 x_0}{ca}\frac{b^2 k}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)} \quad .$$

Taking into account contribution from electric field of the beam we obtain an expression for the vacuum chamber transverse impedance (case of positive frequency):

$$\boxed{Z_\perp = -i\left(\frac{\beta C_1}{I_0 x_0}+\frac{2}{ca^2\beta}\right) = -i\frac{Z_0}{2\pi a^2\beta}\left[2\frac{\beta^2 e^{kd}(1+kb)-e^{-kd}(1-kb)}{e^{kd}(1+ka)(1+kb)-e^{-kd}(1-ka)(1-kb)}+1-\beta^2\right] \qquad (1)}$$

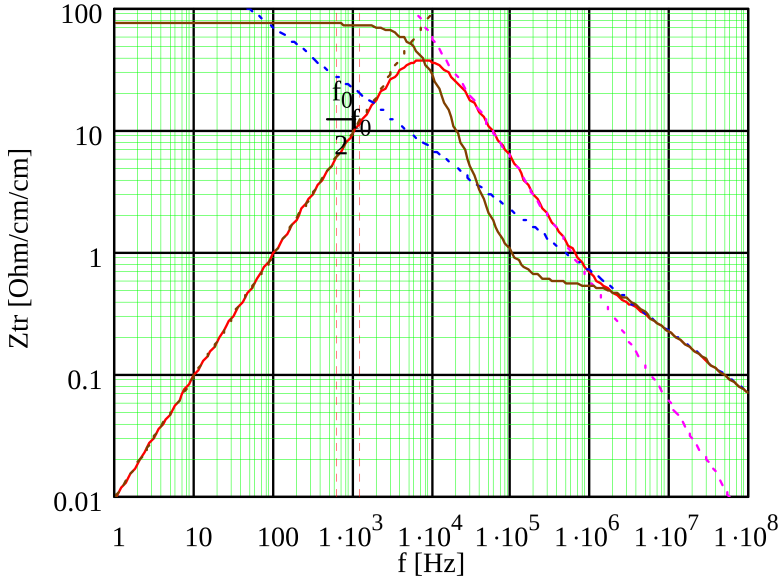
There are following asymptotes for the transverse impedance ($\omega > 0$):

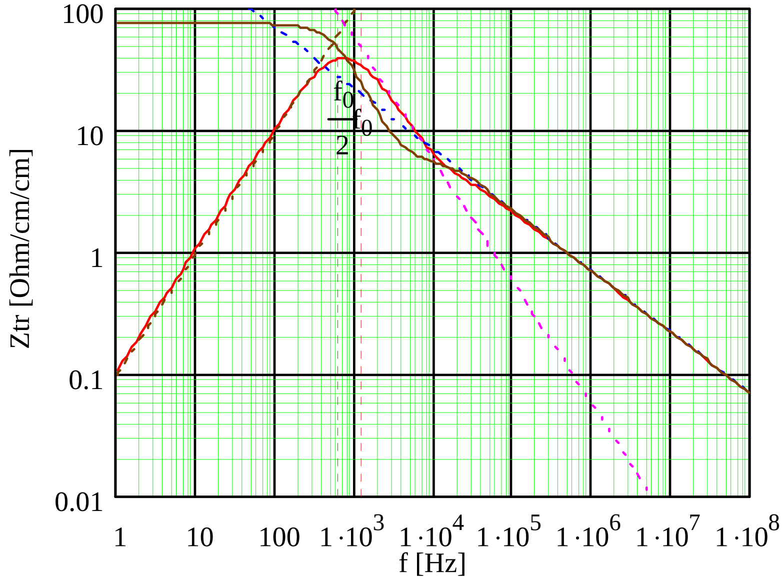
$$Z_\perp = Z_0 \begin{cases} \dfrac{c(1-i)}{2\pi a^3 \sqrt{2\pi\sigma_R\omega}} & , \quad \delta \le d \\ \dfrac{c^2}{4\pi^2\sigma_R\omega a^3 d} - i(\ldots) & , \quad \sqrt{ad} \ge \delta \ge d \\ \dfrac{\sigma_R\omega d}{c^2 a} - i\dfrac{1}{2\pi a^2\beta} & , \quad \delta \ge \sqrt{ad} \end{cases}$$

- There is no simple expression for imaginary part of the impedance in the case $\sqrt{ad} \ge \delta \ge d$ and Eq. (1) has to be used
- To simplify the solution the expression for the vector potential inside conductor was chosen for the flat geometry (instead of Bessel functions of imaginary argument) and therefore the result is not quite accurate if the wall is sufficiently thick.
  - In particular the equation yields incorrect value for imaginary part of the impedance at very small frequencies, $\delta \ge \sqrt{ad}$
    - The following correction term is applied to fix the problem

$$\Delta Z_\perp = -i\frac{Z_0}{2\pi a^2}\frac{d}{(a+d)\left(1-k^4 d^2 a^2\right)}$$

    - Note that although this term corrects asymptotic behavior of the impedance at low frequencies it does not change the impedance at frequencies of interest of VLHC.





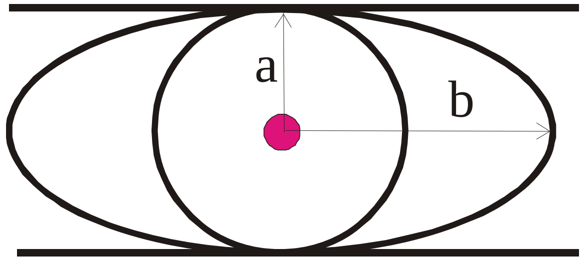
d = 0.1 mm d = 1 mm

Real (red) and imaginary (brown) parts of the transverse impedance and asymptotes for the real part of the impedance as function of frequency for round vacuum chamber: Al, 300 K, ρ = 2.74 μΩ·cm, a = 9 mm.

- Skin-layer thickness is 3.3 mm at $f_0/2$=646 Hz.
  - Peak is achieved when $\delta^2 \approx ad$ and its value does not depend on vacuum chamber thickness

Transverse Impedance of Elliptic Vacuum Chamber

- Impedance of flat vacuum chamber is about half of the round vacuum chamber impedance. For an estimate we will introduce the effective radius of the vacuum chamber:



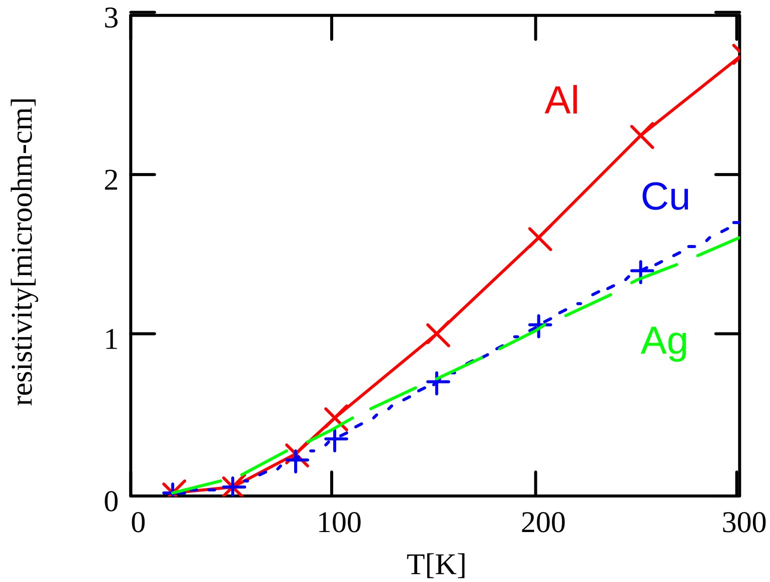
$$a_{eff} = \left(\frac{2a^3 b^3}{a^3 + b^3} \right)^{1/3}$$

so that we would get correct result for cases $a = b$, $a \gg b$ and $a \ll b$.

- For impedance estimates we will substitute a_{eff} instead of a into Eq.(1)

Vacuum Chamber Conductivity

- Vacuum chamber cooling significantly reduces transverse impedances

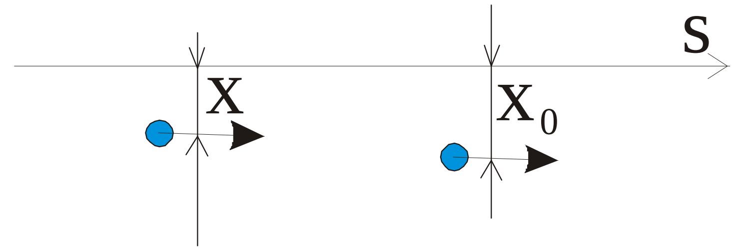


Detuning Wake

- For round vacuum chamber the beam excites dipole field behind him. That corresponds to normal wake field *W*(*s*).
- For non-round vacuum chamber there are quadrupole field and higher multipoles. Contribution corresponding to quadrupole field is described by detuning wake *D*(*s*):

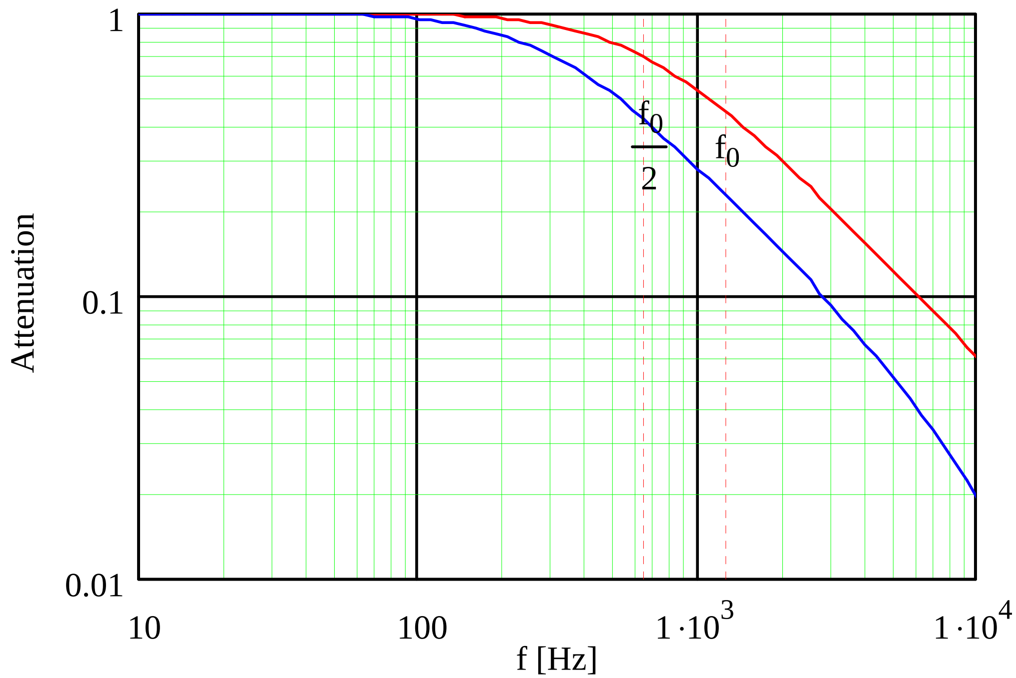
$$\int_L F_x ds' = e^2\left(x_0 W_x(s) + xD(s)\right)$$

$$\int_L F_y ds' = e^2\left(y_0 W_y(s) - yD(s)\right)$$



*A.Burov, V.Danilov “Suppression of transverse instabilities by asymmetries in the chamber geometry,” PRL, **82**, 1999, p.2286.

- Detuning wake changes betatron tunes for tail particles and can cause non-linear resonances.
  - For round vacuum chamber
    - $D(s) = 0$.
  - For two parallel plates
    - $W_x(s) = W_x(s) = -D_x(s) \approx \dfrac{R}{a^3}\sqrt{\dfrac{c}{\sigma_R s}}$



Attenuation of the beam magnetic field by the elliptic aluminum vacuum chamber at 300 K, *a* = 9 mm, *b* = 14 mm; red line - *d*=1 mm, blue line - *d*=2 mm

- There is only about 30% attenuation for thickness of 1 mm at the first unstable betatron sideband
  - That implies that the beam sees everything around vacuum chamber and can acquire additional contribution to the impedance from the surroundings if appropriate measures are not taken.
- 2 mm vacuum chamber attenuates the beam field by more than 2 times
  - That is better but still requires careful attitude to what can be located in near vicinity of vacuum chamber

<u>Multibunch Transverse Instability due to Finite Wall Resistivity</u>

Estimate of tune shift and instability increment due to vacuum chamber impedance

$$W_\perp(s) = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega Z_\perp(\omega) e^{-i\omega s/\mathrm{v}},\ Z_\perp(\omega) = -\frac{i}{\mathrm{v}}\int_0^{\infty} ds W_\perp(s) e^{i\omega s/\mathrm{v}}$$

Let the beam position for mode *n* of continuous beam be as $x(t,\theta) = x_0 e^{i((q-n)\omega_o t - n\theta)}$

Then, the force acting on the reference particle is

$$F_0(t) = \frac{eI_b}{\mathrm{v}}\int_0^s W(s) x_0 e^{-i((q-n)\omega_0 s/\mathrm{v})} ds = ieI_b Z_\perp(-(q-n)\omega_0) x_0$$

$$\Rightarrow \quad \Delta\nu_n = -i\frac{eI_b R^2}{2P\mathrm{v}\nu} Z_\perp(-(q-n)\omega_0)$$

The exact result for the betatron tune shift of *n*-th mode for the bunched beam

$$\Delta\nu_n = -i\frac{eI_b R^2}{2P\mathrm{v}\nu}\sum_{k=-\infty}^{\infty} Z_\perp(-\omega_{nk}) \quad , \qquad (2)$$

$$\omega_{nk} = \omega_0([q] + n + Nk)$$

[q] – the fractional part of the betatron tune ν

N – the number of bunches, $n \in [-N/2, (N-1)/2]$

$Z_\perp(\omega)$ – the transverse impedance per unit length averaged over the ring

The Dimensionless Increment

$$\boxed{\frac{\lambda_n}{f_0} \equiv 2\pi\,\mathrm{Im}(\Delta\nu) = -\frac{\pi e I_b R^2}{P\mathrm{v}\,\nu}\mathrm{Re}\left(\sum_{k=-\infty}^{\infty} Z_\perp(-\omega_{nk})\right) \qquad (3)}$$

- The definition of the transverse impedance yields that impedance at a negative frequency is related to the impedance at positive frequency through its complex conjugated value

$$Z_\perp(-\omega) = -Z_\perp^{\,*}(\omega)$$

or

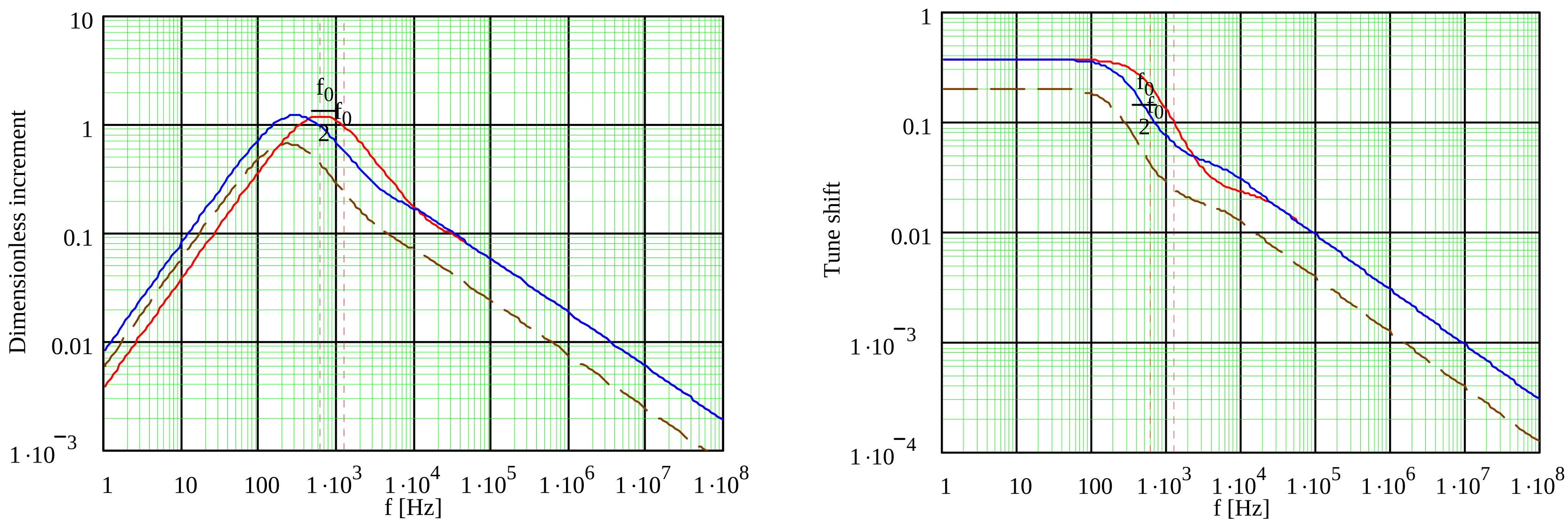
$$\mathrm{Re}\left(Z_\perp(-\omega)\right) = -\mathrm{Re}\left(Z_\perp(\omega)\right)$$
$$\mathrm{Im}\left(Z_\perp(-\omega)\right) = \mathrm{Im}\left(Z_\perp(\omega)\right)$$

- At low frequencies impedance is completely determined by the wall resistivity.
  - ➢ For $\omega > 0$ real part of the impedance is positive $\Rightarrow$ coherent motion is damped
    - $\omega_n = \omega_0\left([q]+n\right) \quad , \quad n = 0...(N-1)/2$
  - ➢ For $\omega < 0$ real part of the impedance is negative $\Rightarrow$ coherent motion is unstable
    - $|\omega_n| = \omega_0\left(n-[q]\right) \quad , \quad n = 1...N/2$

  $\Rightarrow$ If the impedance grows with frequency decrease preferable fractional tune is in the range [0 – 0.5]

Asymptotics for the dimensionless increment

$$\frac{\lambda_n}{f_0} = -\frac{\pi e I_b R^2}{P \mathrm{v} \nu} Z_0 \, sign(\omega_n) \begin{cases} \dfrac{c}{2\pi a^3 \sqrt{2\pi \sigma_R |\omega_n|}} & , \quad \delta \le d \\ \dfrac{c^2}{4\pi^2 \sigma_R |\omega_n| a^3 d} & , \quad \sqrt{ad} \ge \delta \ge d \\ \dfrac{\sigma_R |\omega_n| d}{c^2 a} & , \quad \delta \ge \sqrt{ad} \end{cases}$$



Dependence of the dimensionless decrement (left) and the betatron tune shift (right) on the mode frequency for elliptic aluminum vacuum chamber at 300 K, a = 9 mm, b = 14 mm;
red line - d=1 mm, blue line - d=2 mm; brown dashed line - a = 12 mm, b = 20 mm, d=2 mm.

- 9 mm vacuum chamber yields instability growth time about one revolution
  - Increase of wall thickness from 1 mm to 2 mm decreases
    - the growth time by about 1.5 times
    - the coherent tune shift by about 2 times to $\Delta\nu \sim 0.1$
- To decrease decrement to about 2 revolutions one needs to increase vacuum chamber size to about 12×20 mm
  - That also decreases the coherent tune shift to $\Delta\nu \sim 0.03$ (30 deg per turn)

Coherent and incoherent tune shifts

- The interaction of the beam with round vacuum chamber
  - causes instabilities and changes frequencies of coherent beam motion
  - but does not change incoherent particle frequencies and cannot cause non-linear resonance
- In the case of elliptic vacuum chamber detuning wake is not zero. That means that the interaction of the beam with vacuum chamber creates additional focusing fields which changes incoherent particle tunes
  - In the case of flat vacuum chamber and uniformly filled ring the frequency of all particles is shifted by the same amount: $$\Delta\nu_{ic0} = -\frac{\pi^2}{24}\frac{eI_b R^2}{mc^3\gamma\beta^3 a^2\nu}$$

    $\Delta\nu_{ic0}$= –0.205 for half-gap a=9 mm. It is equal to half of the coherent tune shift at zero frequency. $\Delta\nu_{ic0}$ can be compensated by betatron frequency adjustments proportional to the beam current
  - If the ring is partially filled then there is tune variations proportional to "AC" components in the beam current
    - For flat vacuum chamber the wake and the detuning wake are equal and therefore tune variations can be computed using results obtained for coherent betatron tune shift

$$\Delta\nu_{ic_n} = \mathrm{Re}(\Delta\nu) = -\frac{eI_{b\omega_n}R^2}{2\,P\mathrm{v}\nu}\,\mathrm{Im}\left(\sum_{k=-\infty}^{\infty} Z_\perp(\omega_n)\right) \quad , \quad \omega_n = n\omega_0$$

    - To compensate tune variations along the beam the modulation of lattice betatron tune on revolution frequency is required

- Interaction of the beam DC current with iron of the dipoles causes additional incoherent tune shift

$$\Delta\nu_{icB} = -\frac{\pi^2}{12}\frac{eI_bR^2}{mc^3\gamma\beta\, a_d{}^2\nu}\eta$$

a_d – is the dipole half-gap

η – fraction of the orbit with dipoles

That tune shift is almost two times higher than the incoherent tune shift due to vacuum chamber $\Delta\nu_{ic0}$($\Delta\nu_{icB}$= –0.332 for a_d=10 mm)

- If the ring is partially filled then there are tune variations proportional to "AC" components in the beam current. Their amplitude will depend on the thickness of vacuum chamber and dipole laminations.

4. Feedback System for Suppression of the Transverse Multibunch Instability

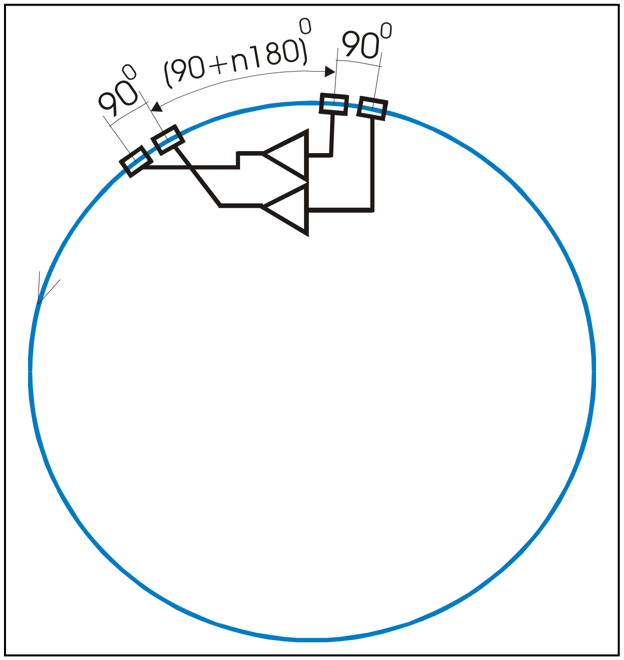
Standard system

$$\mathbf{V}_n = \mathbf{V}_0 \exp\left(i\omega \frac{T}{N} n \right)$$

$$\mathbf{V}_{n+N} = \begin{bmatrix} \cos(\mu_0) & \sin(\mu_0) \\ -\sin(\mu_0) & \cos(\mu_0) \end{bmatrix} \begin{bmatrix} 1-k(\omega) & 0 \\ 0 & 1-k(\omega) \end{bmatrix} \mathbf{V}_n$$

$$k(\omega) = \sum_{j=0}^{\infty} g\left(\frac{T}{N} j \right) \exp\left(-\frac{i\omega T}{N} j \right)$$

N – number of bunches
T – revolution period
$g(t)$ – response function



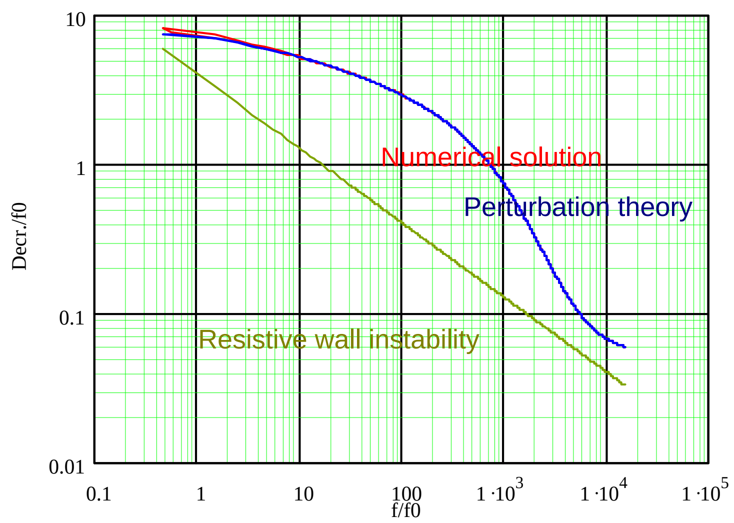
Perturbation theory solution

$$\Lambda_n \equiv \exp(i\omega_n T) = (1 - k(\tilde{\omega}_n))\exp(i\mu_0)$$

$$\tilde{\omega}_n T = \mu_0 + 2\pi n$$

Damping decrement: $\lambda_n = -\ln(|\Lambda_n|)/T$

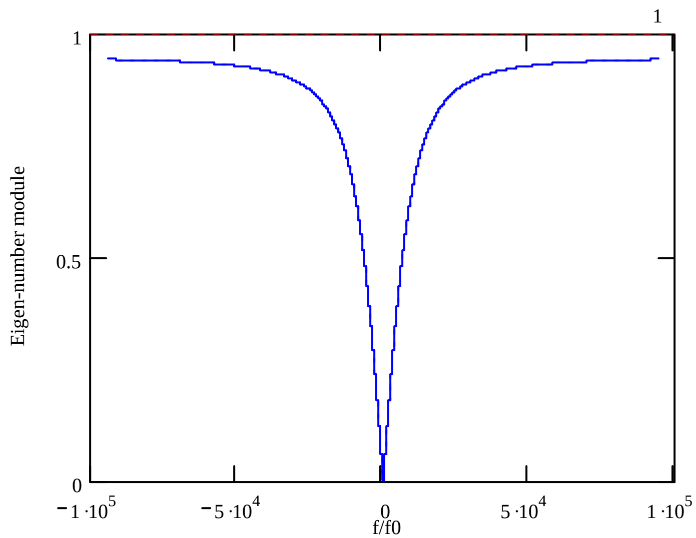
Integrating type narrow band system

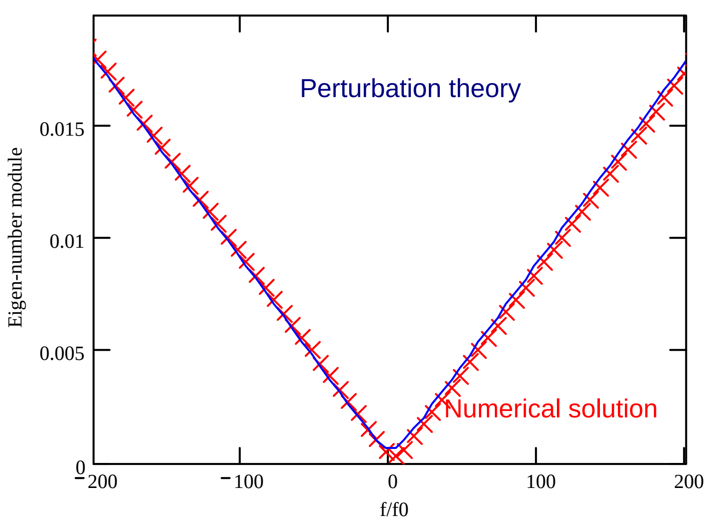


$$g(t) = g_0 \exp\left(-\frac{t}{\tau}\right)$$

$$k(\omega) = \frac{g_0}{1 - \exp\left(-\frac{T}{N}\left(\frac{1}{\tau} + i\omega\right)\right)}$$

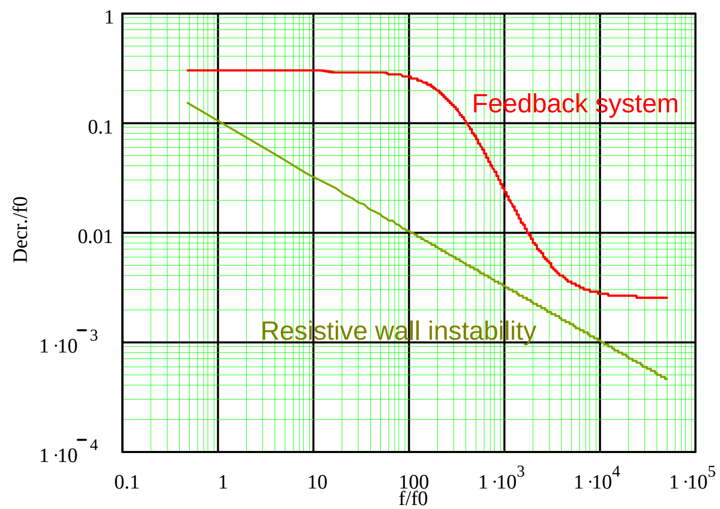
g_0=0.1, τ/T=9.485·10^{-5}, μ_0=2π0.46, N=10^5





- There is no principal limits to reach damping faster than the revolution time
- Narrow band system can be stable in the entire frequency range
- Technical problems
  - ➢ Signal transfer from BPMs to kickers
    - ♦ Forward transfer
      - ❖ Expensive tunnel for signal transfer
    - ♦ One turn delay
      - ❖ Large betatron phase shift after one turn
      - ❖ Large delay ⇒ digital system
    - ♦ Practical system needs subtraction of orbit offset
      - ❖ Slow feedback to nullify BPMs
      - ❖ Or notch filter at revolution frequency which will reduce maximum achievable gain
  - ➢ Limits of the system gain
    - ♦ Final accuracy of electronics
    - ♦ Emittance growth due to BPM noise
  - ➢ Kicker voltage
    - ♦ 5 m, ±10 kV kicker suppresses 2.5 mm injection oscillations at 1 turn

Practical system for the case of aluminium vacuum chamber at 78 K



g_0=0.005, τ/T=5·10^{-4}, μ_0=2π0.46, N=10^5

System with delayed response

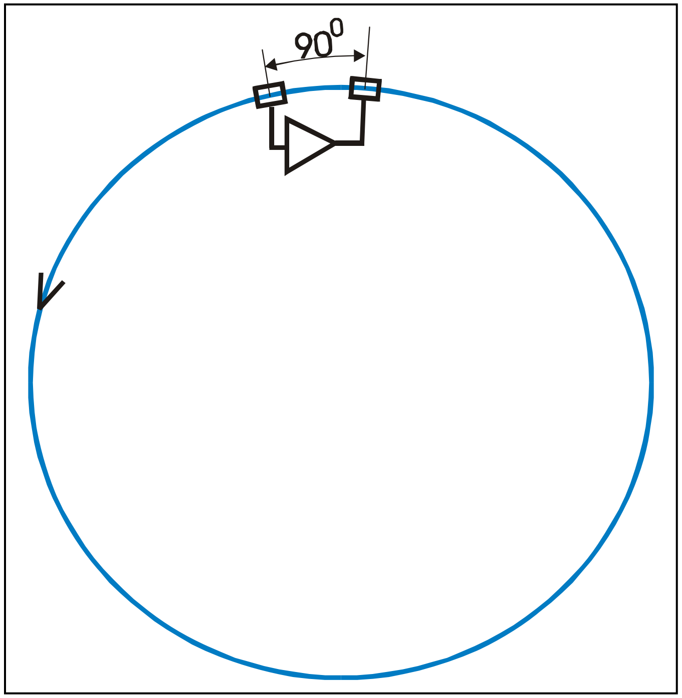
Single system with delayed response

$$\mathbf{V}_n = \mathbf{V}_0 \exp\left(i\omega \frac{T}{N} n\right)$$

$$\mathbf{V}_{n+N} = \begin{bmatrix} \cos(\mu_0) & \sin(\mu_0) \\ -\sin(\mu_0) & \cos(\mu_0) \end{bmatrix} \mathbf{V}_n - k(\omega) \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{V}_{n+N}$$

$$k(\omega) = \sum_{j=0}^{\infty} g\left(\frac{T}{N} j\right) \exp\left(-\frac{i\omega T}{N}(j + j_d)\right)$$

N – number of bunches
T – revolution period
$g(t)$ – response function
j_d – number of skipped bunches

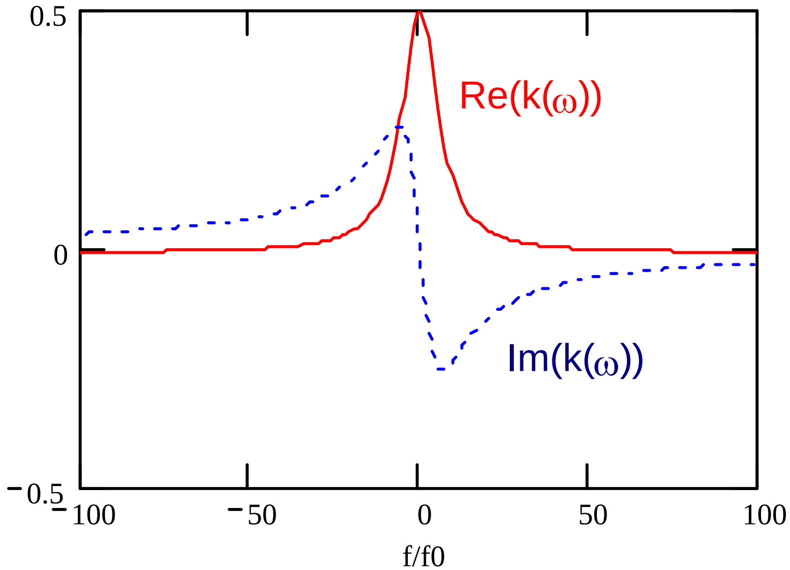


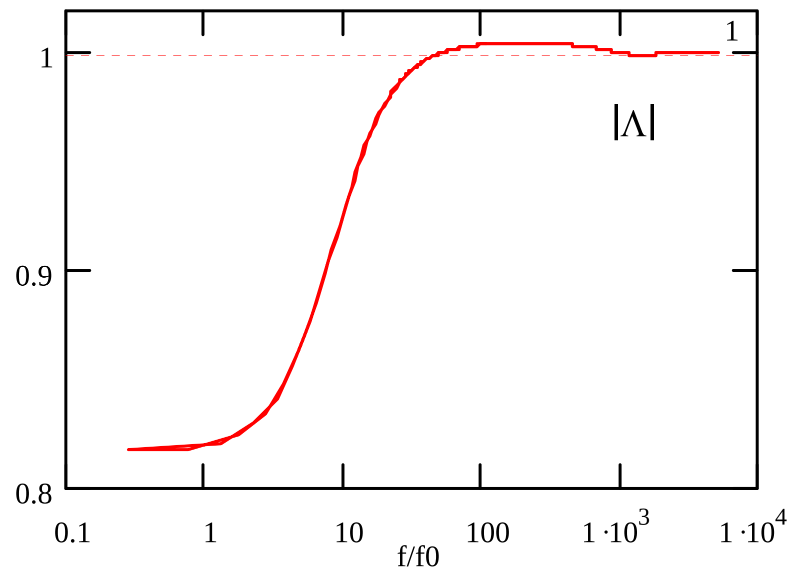
Perturbation theory solution

$$\Lambda_n \equiv \exp(i\omega T) = \frac{1 + k(\omega_n)/2}{1 + k(\omega_n)} \cos(\mu_0) \pm \sqrt{\left(\frac{1 + k(\omega_n)/2}{1 + k(\omega_n)}\right)^2 \cos^2(\mu_0) - \frac{1}{1 + k(\omega_n)}}$$

$$\omega_n T = \mu_0 + 2\pi n$$

Integrating type system $g(t) = g_0 \exp\left(-\frac{t}{\tau}\right)$, $k(\omega) = \dfrac{g_0}{1 - \exp\left(-\frac{T}{N}\left(\frac{1}{\tau} + i\omega\right)\right)}$





$g_0=2\cdot10^{-4}$, $\tau/T=.025$, $\mu_0=2\pi\cdot0.27$, N=10^5, j_d =50 – 50 skipped bunches (10^5/(4·500))

Optimization strategy

- Suppress resistive wall instability at low frequencies
- Achieve minimum increments at higher frequencies

Double system with delayed response

$$\mathbf{M} = \begin{bmatrix} \cos(\mu_0) & \sin(\mu_0) \\ -\sin(\mu_0) & \cos(\mu_0) \end{bmatrix} \quad , \quad \mathbf{M} = \mathbf{M}_1\mathbf{M}_2 \quad ,$$

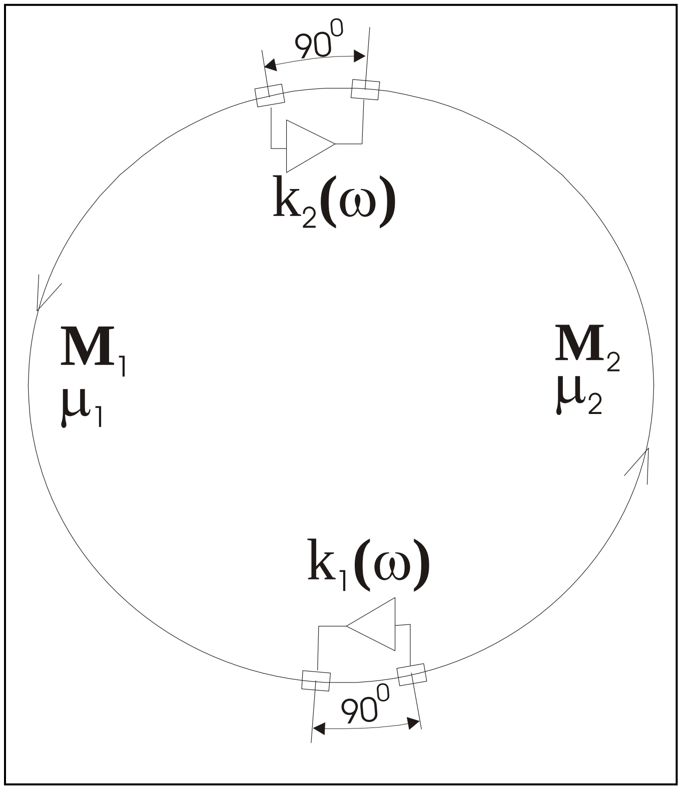
$l = 1,2$

$$k_l(\omega) = \sum_{j=0}^{\infty} g_l\left(\frac{T}{N}j\right)\exp\left(-\frac{i\omega T}{N}\left(j + j_d\right)\right)$$

$$\mathbf{k}_l = \begin{bmatrix} 1 & 0 \\ 0 & 1 + k_l(\omega) \end{bmatrix} \quad ,$$

$$\mathbf{M}\mathbf{V}_n = \left(\mathbf{k}_1 + \mathbf{M}_2\mathbf{k}_2\mathbf{M}_2^{-1}\mathbf{k}_1\right)\mathbf{V}_{n+N}$$

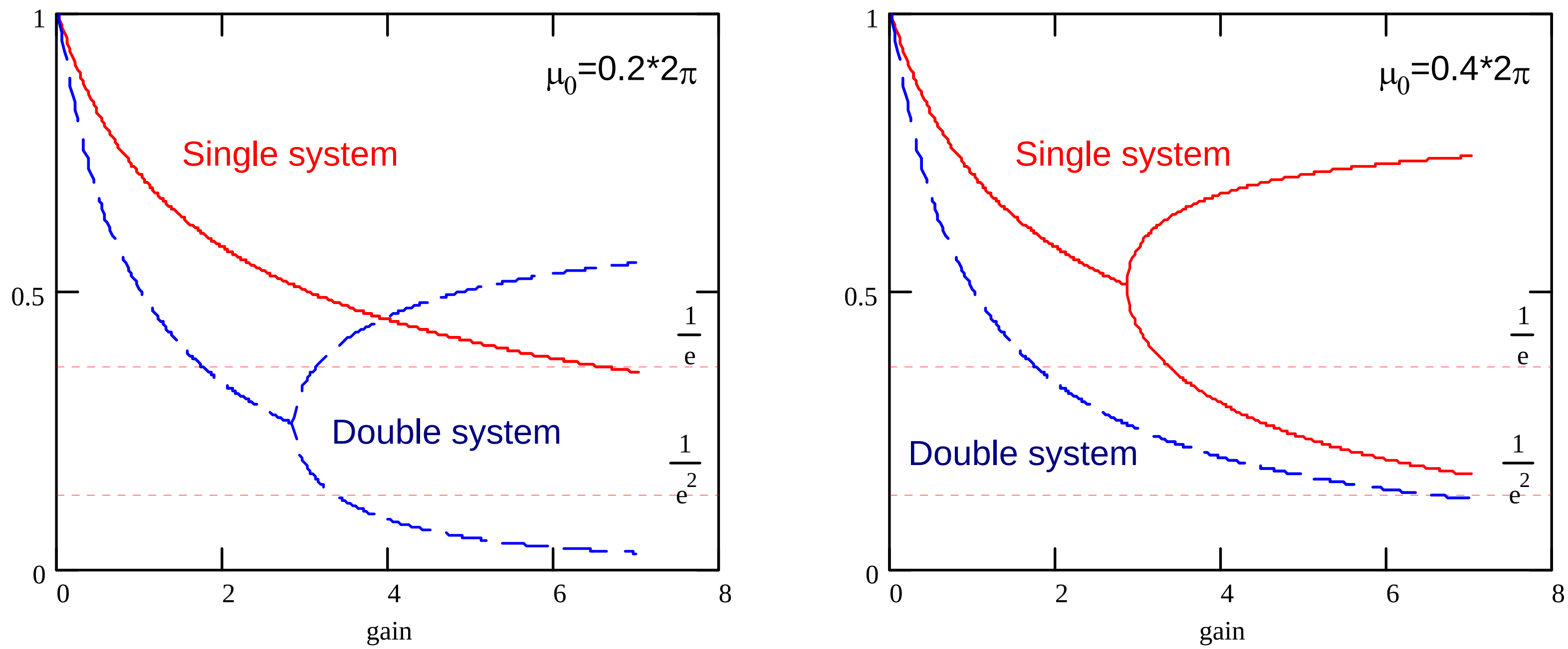
N – number of bunches
T – revolution period
$g(t)$ – response function
j_d – number of skipped bunches



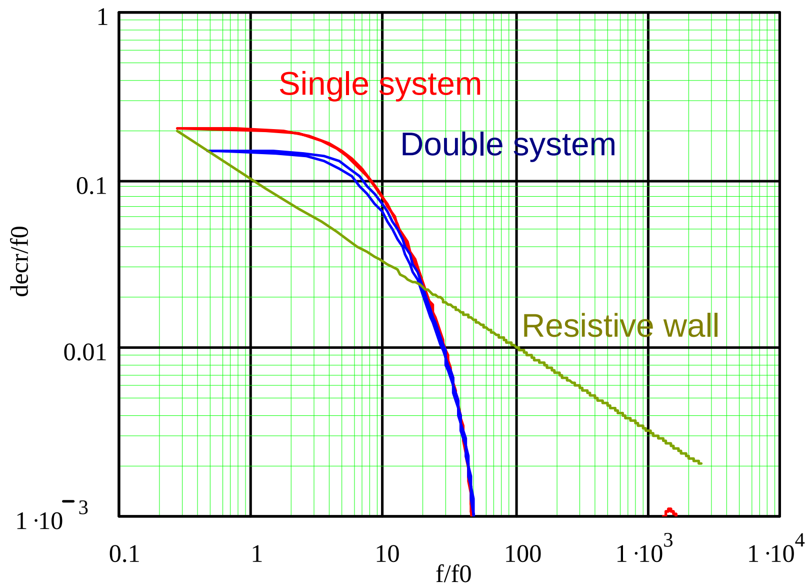
$$\Lambda_n^{\,2}\left[1 + k_1(\omega_n) + k_2(\omega_n) + k_1(\omega_n)k_2(\omega_n)\right] -$$

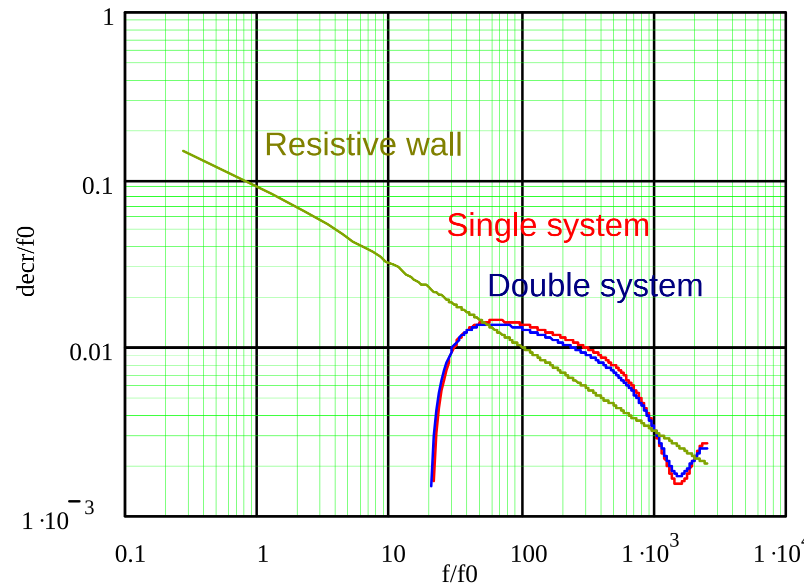
$$2\Lambda_n\left[\cos(\mu_0)\left(1 + \frac{k_1(\omega_n) + k_2(\omega_n)}{2}\right) + \cos(\mu_1)\cos(\mu_2)\frac{k_1(\omega_n)k_2(\omega_n)}{2}\right] + 1 = 0$$

Comparison of single (μ_0, g_0) and double systems ($\mu_1=\mu_2=\mu_0/2$, $g_1=g_2=g_0$)



Dependence of absolute values of eigen-numbers on system gain, g, for different betatron tunes.





Damping decrement   Residual instability increment

$N=10^5$, $j_d=50$; Single system: $g_0=2\cdot10^{-4}$, $\tau/T=2.5\cdot10^{-2}$, $\mu_0=2\pi\cdot0.27$,

Double system: $g_0=8\cdot10^{-8}$, $\tau/T=2\cdot10^{-2}$, $\mu_0=2\pi\cdot0.46$.

Conclusions for a system with delayed response for VLHC

- System allows to reduce a requirement for the damping decrement of the bunch-bunch dumping system by an order of magnitude
- Installation of multiple systems does not allow to increase the total damping due to anti-damping at high frequency.
- Decreasing the number of skipped bunches one can improve performance but it increases noise effects due to smaller beta-functions

5. Emittance Growth Suppression

Emittance growth due to noises♦

$$\frac{d\varepsilon}{dt} = \pi f_0^{\,2}\xi^2 \sum_{n=-\frac{N}{2}+1}^{N/2} \left(\frac{3.3}{(2\lambda_n / f_0)^2 + \xi^2} + 67 \right) \langle \beta_2 S_{noise}(\omega_n) \rangle \quad , \quad \omega_n = \omega_0([\nu_\perp] + n) ,$$

Spectral density of transverse kicks, $\overline{\theta_\perp^{\,2}} = \int_{-\infty}^{\infty} S_{noise}(\omega_n) d\omega$, for the feedback system

is determined by spectral density of BPM noise $x_\omega^{\,2}$, system gain and beta-function at BPM, β_1 :

$$\langle \beta_2 S_{noise}(\omega_n) \rangle = \frac{x_\omega^{\,2}}{\beta_1} k^2(\omega_n) \quad .$$

β_1 – beta-function at BPM

β_2 – beta-function at kicker

♦ Emittance growth due to noise and its suppression with the feedback system in large hadron colliders, V. Lebedev, *et.al.* V.V.Parkhomchuk, V.D.Shiltsev, G.V.Stupakov; SSCL-Preprint-188, Dallas, March 1993 and Particle Accelerators, 1994, v.44, pp.147-164

Theoretical limit for the standard narrow band system

- Effective BPM noise of the feedback system is determined by the thermal noise of amplifiers is

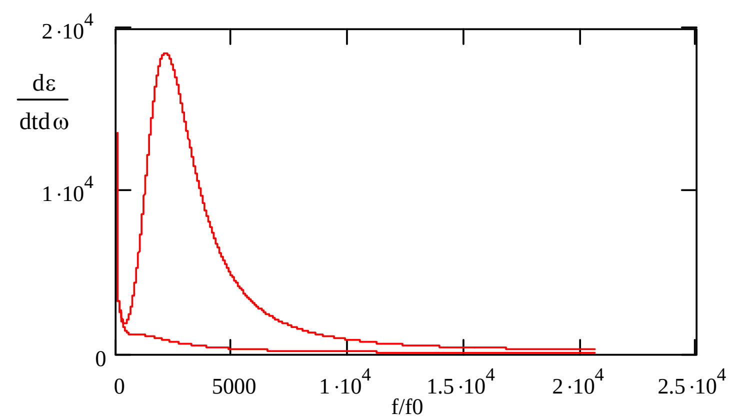
$$\overline{x_\omega^{\ 2}} = 2a^2 \frac{kT\rho}{\pi I_b^{\ 2} Z_{BPM}^{\ 2}}$$

Factor of 2 appeared because the system consists of two independent systems shifted by 90 deg in betatron phase

ρ = 50 Ω, T=300 K, I_b=0.19 A, Z_{BPM}=12 Ω $\Rightarrow$

$$\overline{x_\omega^{\ 2}} = 10^{-20}\ \text{cm}^2\,\text{s}, \qquad \sqrt{\overline{x^2}} = 0.02\ \mu\text{m}\ (\Delta f = 25\ \text{MHz})$$

$$\Rightarrow \boxed{\frac{\varepsilon_0}{d\varepsilon/dt} \approx 1.3\cdot 10^5\ \text{days}} \quad \text{- at the injection}$$



Relative contribution of different frequencies into emittance growth

- Required BPM accuracy

Emittance growth time > 100 hour $\Rightarrow$

bunch position measured with $\boxed{\sqrt{\overline{x^2}} = 3\ \mu\text{m}}$ (180 times of theoretical limit)

Conclusions

- Multi-bunch instabilities can be suppressed
- To suppress the resistive wall instability
  - ➢ Increase vacuum chamber thickness to 2 mm and vertical size to 2×12 mm will be extremely profitable
  - ➢ Two feedback systems required
    - ▪ Low frequency system with high gain, narrow band and delayed response to damp low frequencies
      - ♦ Unstable at high frequencies
    - ▪ High frequency system to restore stability at high frequencies
      - wide band, moderate gain and one turn delay
        - ♦ Unstable at low frequencies