Luminosity Modeling

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Talk outline

1. Parametric model of luminosity evolution
2. Luminosity evolution
   a. Current scenario
   b. Final Run II scenario
3. Beam-beam effects
4. Particle loss at injection

   Conclusions

1. Parametric Model of Luminosity Evolution

Takes into account the major beam heating and particle loss mechanisms

- **Phenomena taken into account**
  - ⇒ Interaction with residual gas
    - ♦ Emit. growth and particle loss due to E-M and nuclear scattering
  - ⇒ Particle interaction in IPs (proportional to the luminosity)
    - ♦ Emit. growth and particle loss due to E-M and nuclear scattering
  - ⇒ IBS
    - ♦ Energy spread and emittance growth due to multiple scattering
  - ⇒ Longitudinal dynamics
    - ♦ Nonlinearity and finite size of potential well
    - ♦ Bunch lengthening due to RF noise and IBS
    - ♦ Particle loss from the bucket due to single IBS (Touschek effect) and due to heating longitudinal degree of freedom (multiple IBS and RF noise)
    - ♦ Absence of tails after acceleration

- **<u>Phenomena presently ignored in the model</u>**

  ⇒ Beam-beam effects

  ⇒ Non-linearity of the lattice

  ⇒ Diffusion amplification by coherent effects
- Thus, it can be considered as **the best-case scenario**

  ⇒ It describes well our best present stores

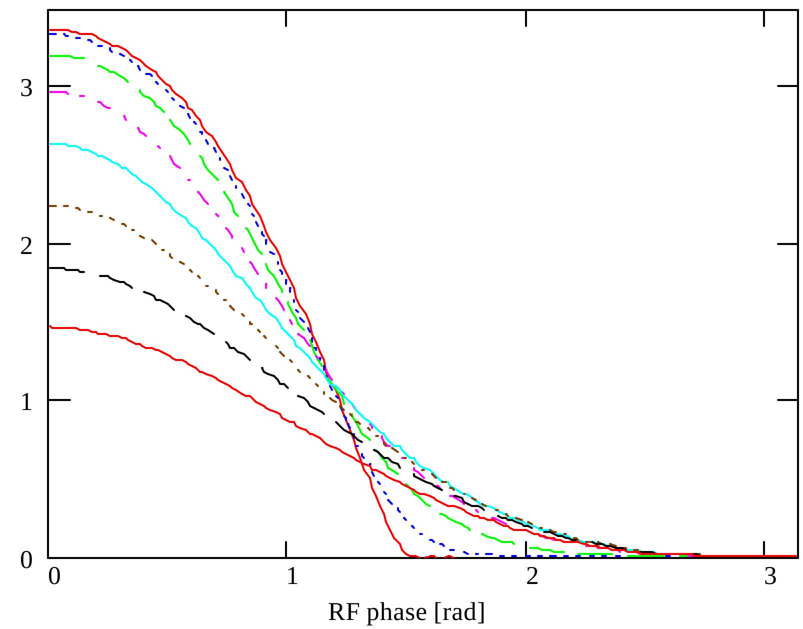
Beam Evolution in Longitudinal Degree of Freedom

- Longitudinal acceptance grows from 4 to 10 eV s during acceleration
  - Absence of tails after acceleration
- Interplay of single and multiple scattering
- The model based on a solution of integro-differential equation which describes both single and multiple IBS

$$\frac{\partial f}{\partial t} = \int_0^\infty W(I, I')\big(f(I', t) - f(I, t)\big)\mathrm{d}I'$$

Here the kernel is

$$\tilde{W}(E, E') = \frac{D w_0 w w'}{L_C (E - E')^2} \begin{cases} \dfrac{1}{2w} + \dfrac{I}{E' - E} \quad , \quad E' \geq E + dE \ , \\ \dfrac{1}{2w'} + \dfrac{I'}{E - E'} \quad , \quad E' \leq E - dE \ , \end{cases}$$



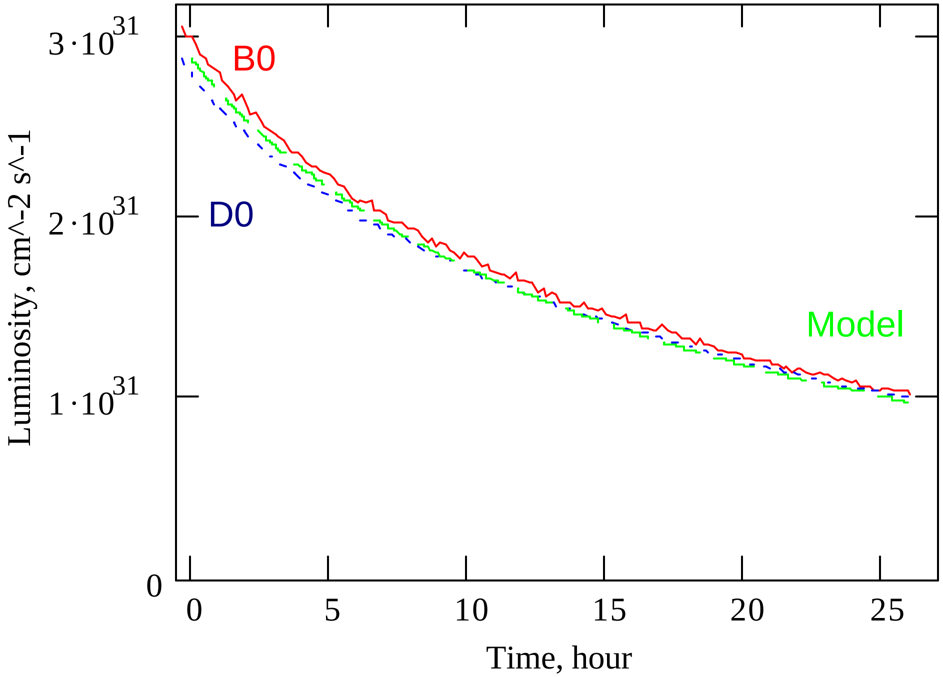
Numerical simulation of the longitudinal bunch profile evolution during store.

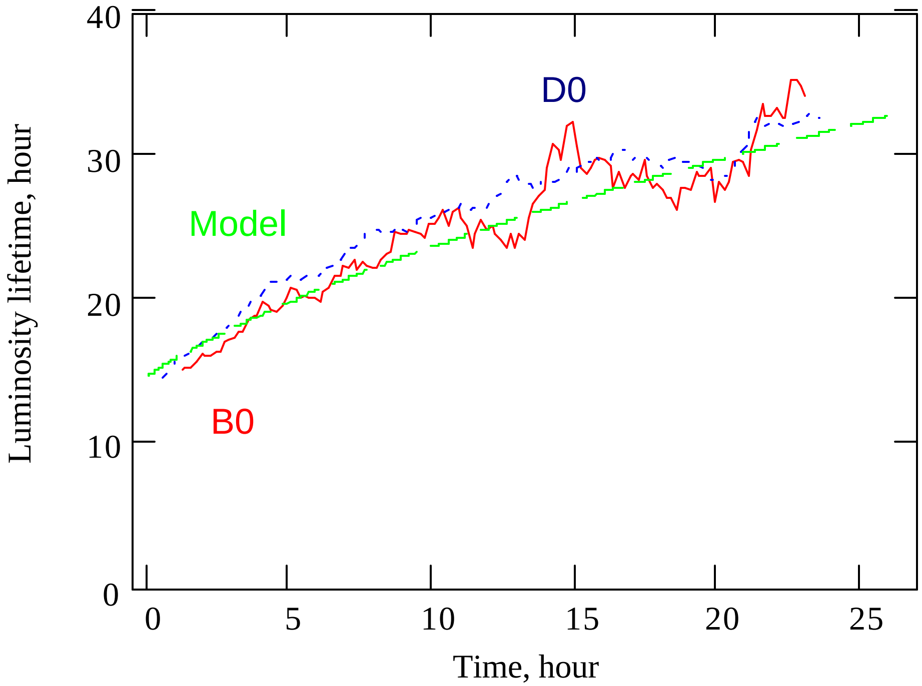
Parametric model of luminosity evolution

- Compromise between simplicity of the model and accuracy of the description
  - ➢ Finite accuracy of the measurements
- System of eight ordinary differential equations

$$\frac{d}{dt}\begin{bmatrix} e_{px} \\ e_{py} \\ s_{pp}^{2} \\ N_{p} \\ e_{ax} \\ e_{ay} \\ s_{pa}^{2} \\ N_{a} \end{bmatrix} = \begin{bmatrix} 2de_{px}/dt\big|_{BB} + de_{px}/dt\big|_{IBS} + de_{px}/dt\big|_{gas} \\ 2de_{py}/dt\big|_{BB} + de_{py}/dt\big|_{IBS} + de_{py}/dt\big|_{gas} \\ ds_{pp}^{2}/dt\big|_{total} \\ -N_{p}t_{scat}^{-1} - dN_{p}/dt\big|_{L} - 2Ls_{p\bar{p}}/n_{b} \\ 2de_{ax}/dt\big|_{BB} + de_{ax}/dt\big|_{IBS} + de_{ax}/dt\big|_{gas} \\ 2de_{ay}/dt\big|_{BB} + de_{ay}/dt\big|_{IBS} + de_{ay}/dt\big|_{gas} \\ ds_{pa}^{2}/dt\big|_{total} \\ -N_{a}t_{scat}^{-1} - dN_{a}/dt\big|_{L} - 2Ls_{p\bar{p}}/n_{b} \end{bmatrix}$$

2. Luminosity Evolution

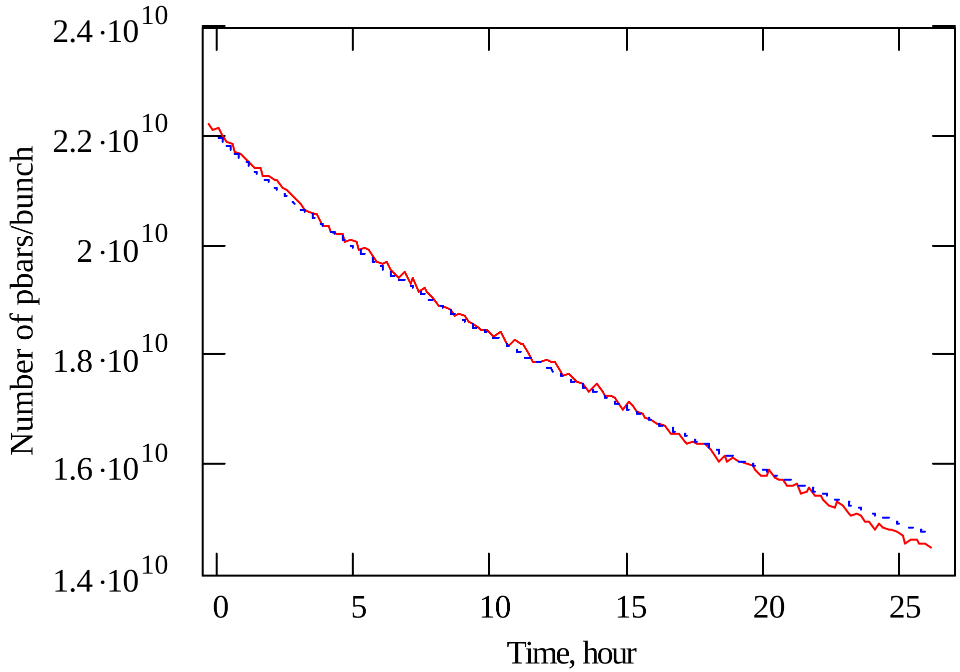




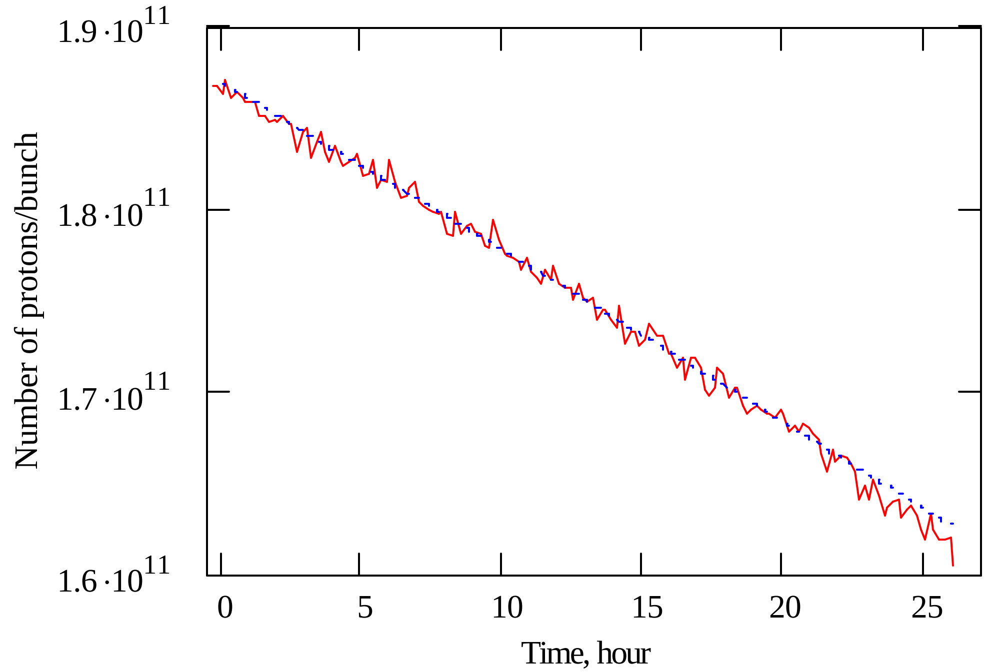
Good regular store !!! (Store 2138, Jan.5 2003)

- Three free parameters are used in the model
  - Residual gas pressure - P=1·10^{-9} Torr of N_2 equivalent
  - Spectral density of RF noise- $P_{ff}\left(f_s\right) \approx 5 \cdot 10^{-11} \quad \mathrm{rad}^2/\mathrm{Hz}$ (70μrad in Δf=100Hz)
  - X-Y coupling - k = 0.4 (strong coupling due to beam-beam effects)
- Their values are not very critical for the luminosity prediction but important for detailed comparison

Bunch population per bunch (Store 2138)

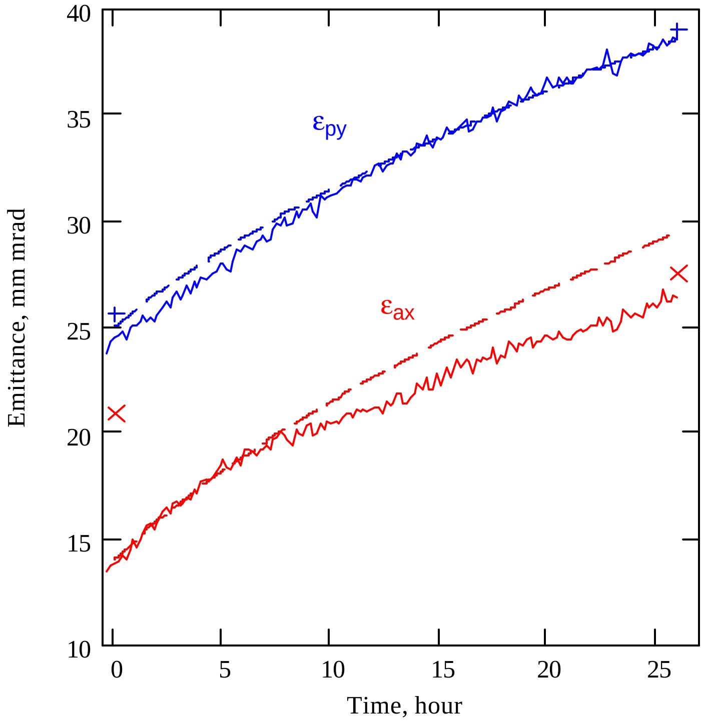


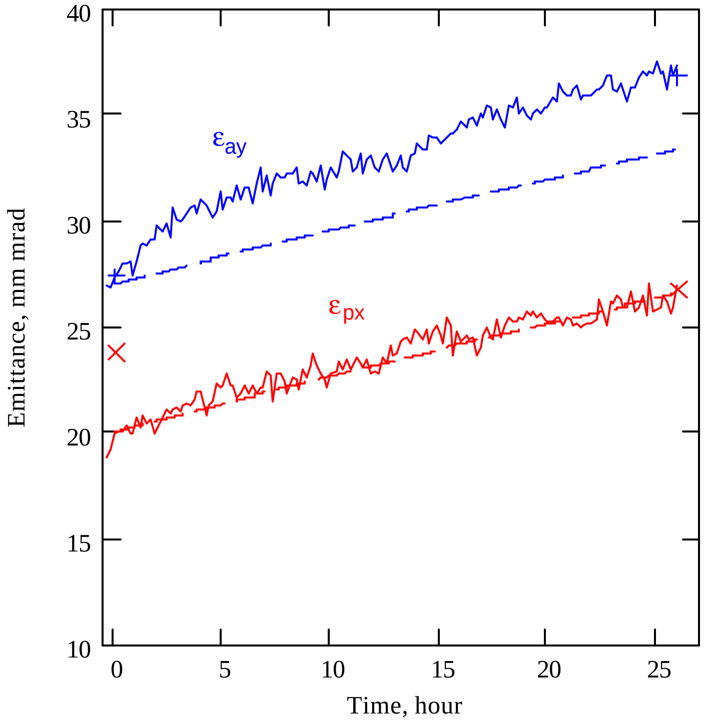
Pbars

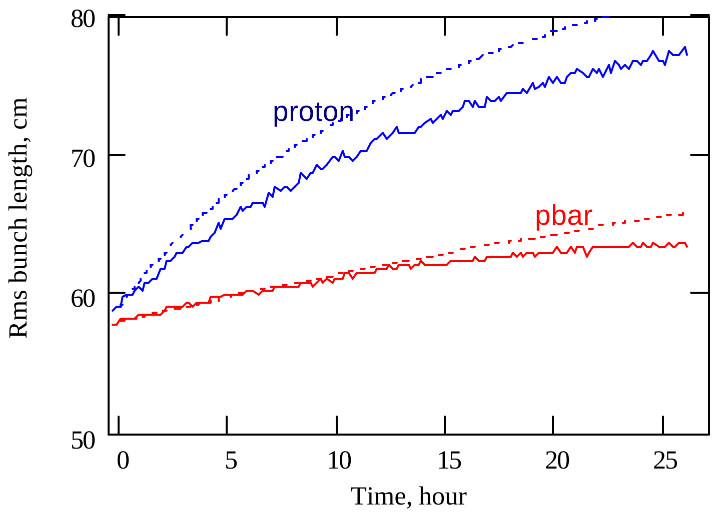


Protons

- **At the store beginning**
  - Pbar loss is large due to large initial luminosity
  - Proton loss is small due to short bunch length/absence of tails and, consequently, low longitudinal loss
- **Model describes well the particle loss during the store**

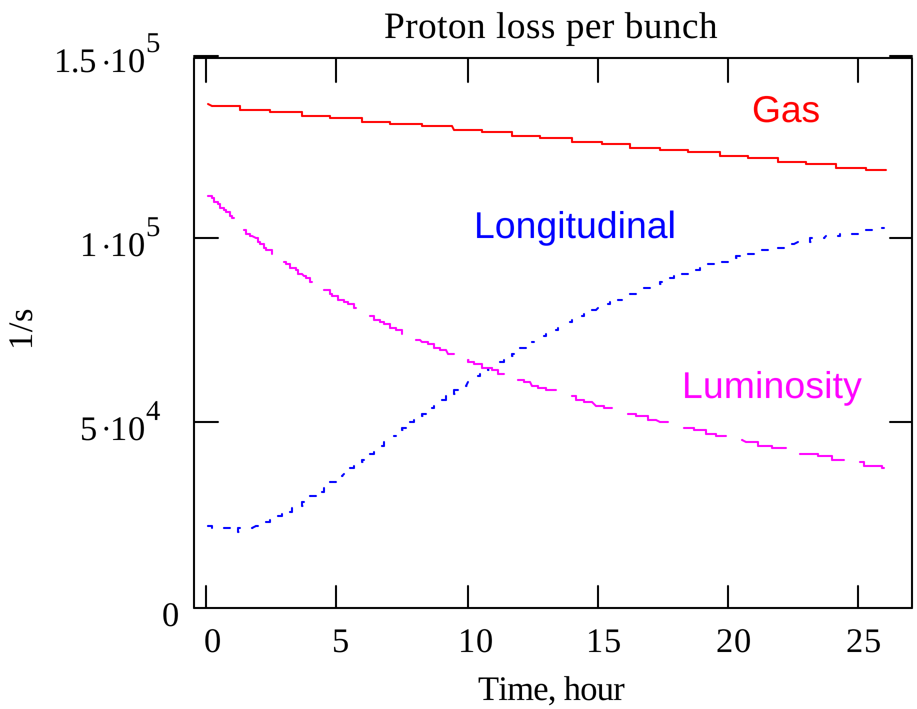


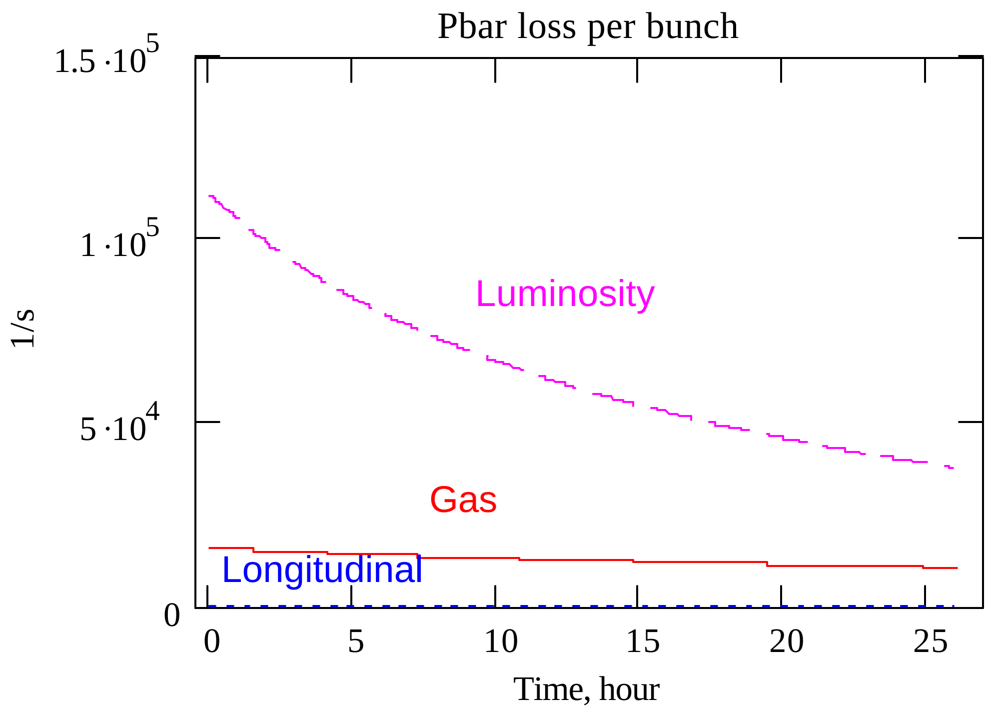




Emittances and bunch lengths on time for Store 2138

- **Beam-beam effects ?**
  - Vertical pbar emit. grows faster at the store beginning
  - Pbar bunch length does not grow at the second half of the store

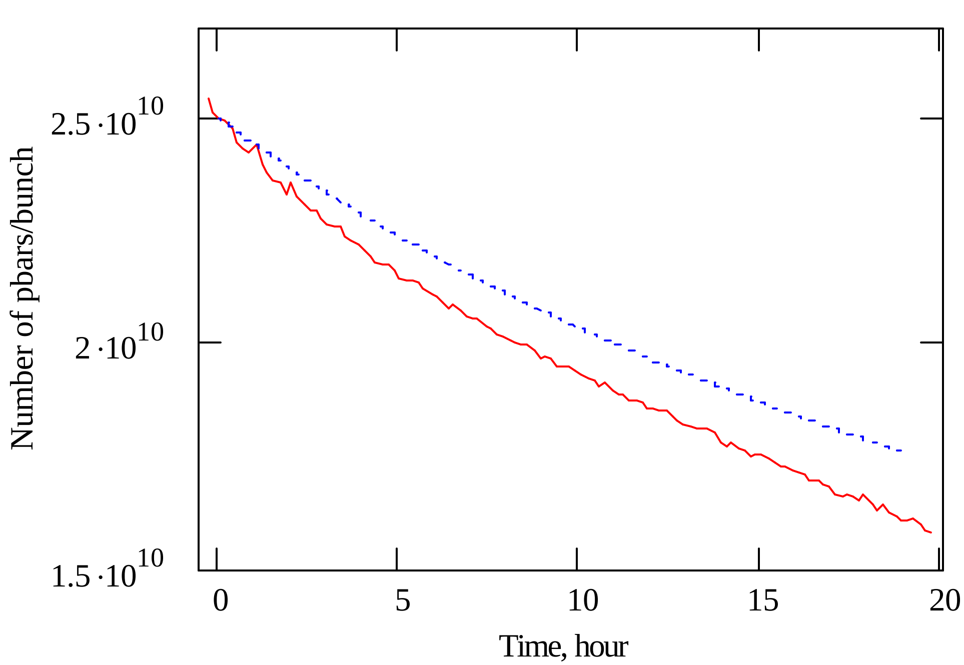


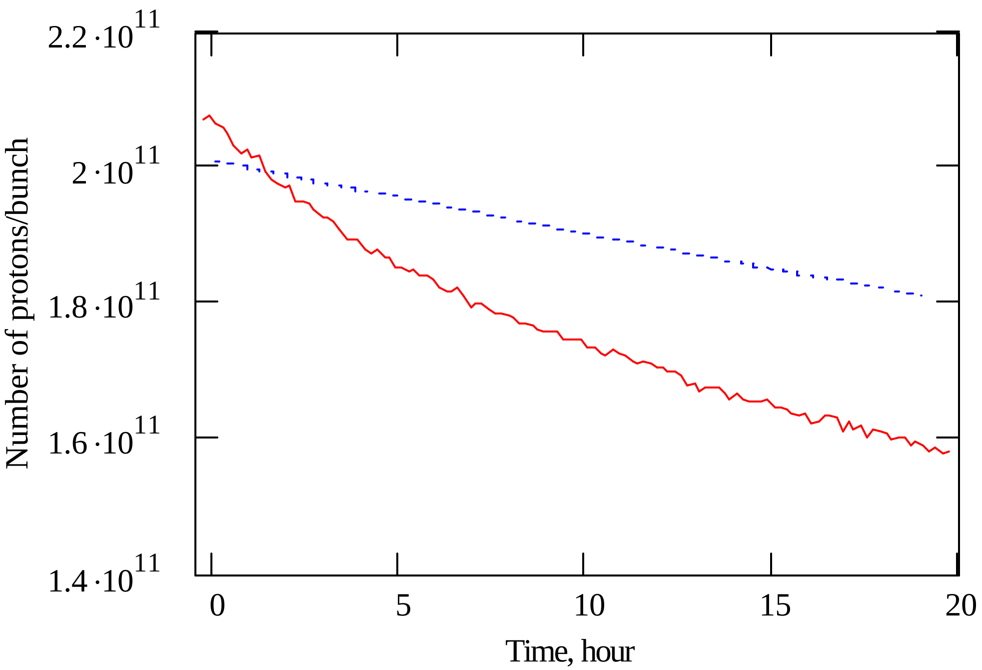


Particle loss computed for different loss mechanisms for Store 2138.

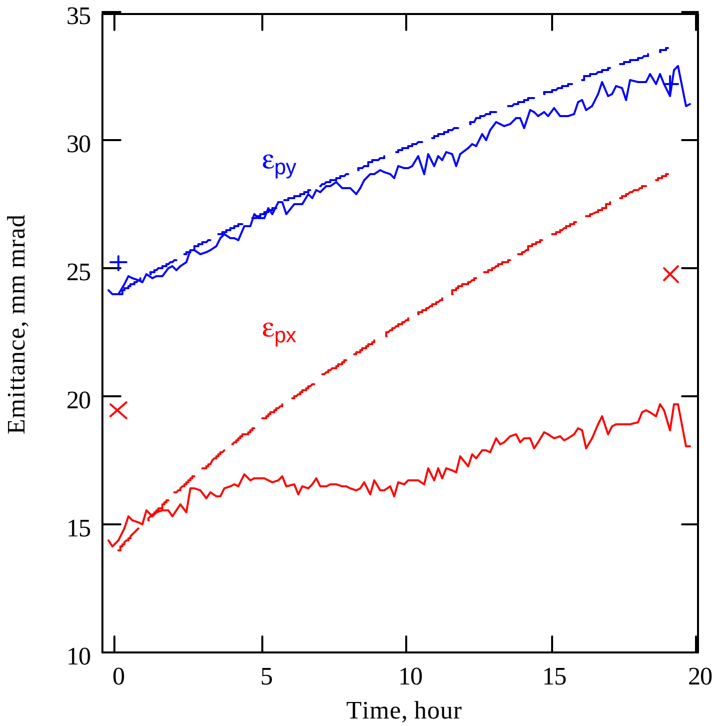
Comparison of Model Predictions to Store 2328 (Mar. 20 2003)

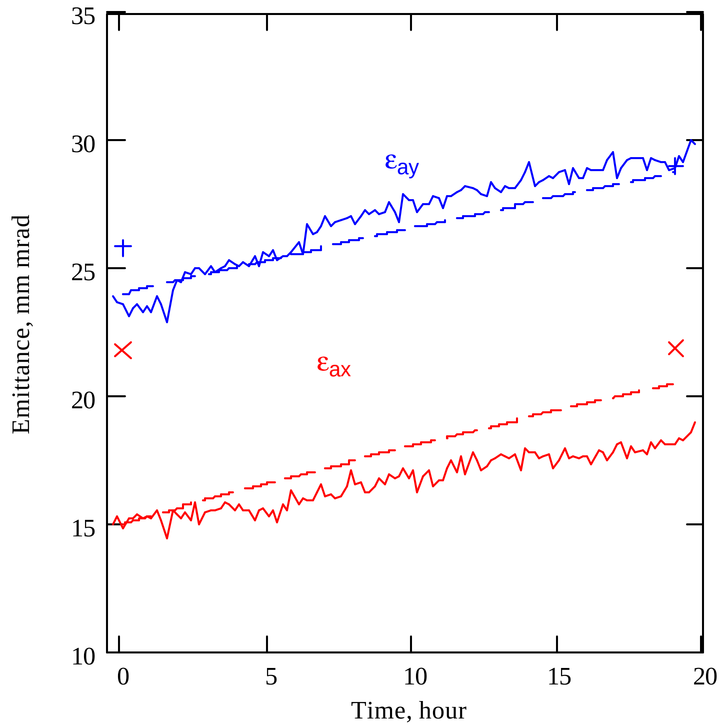
The store is strongly affected by the beam-beam effects !!!

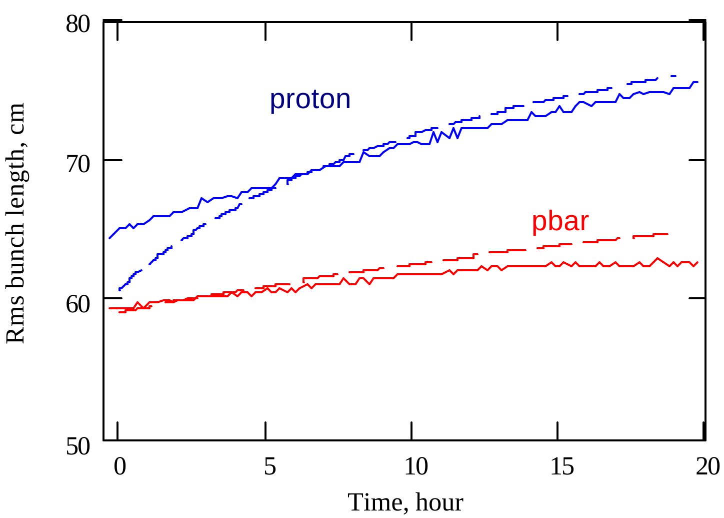




- At the store beginning both proton and pbar bunch intensities decay faster than the model predictions
  - Incorrect tune or too long bunch or both

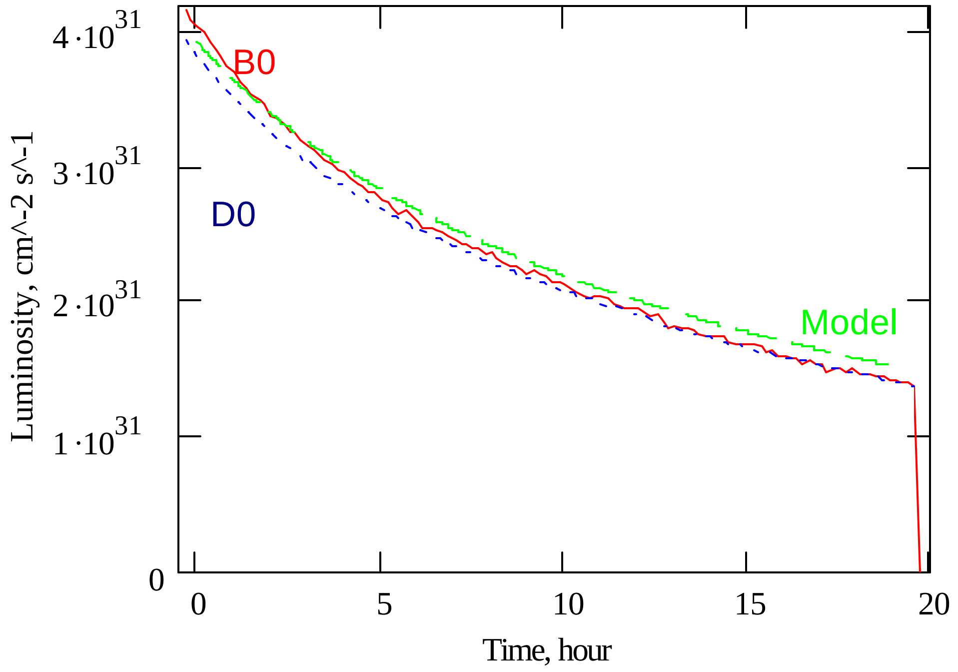


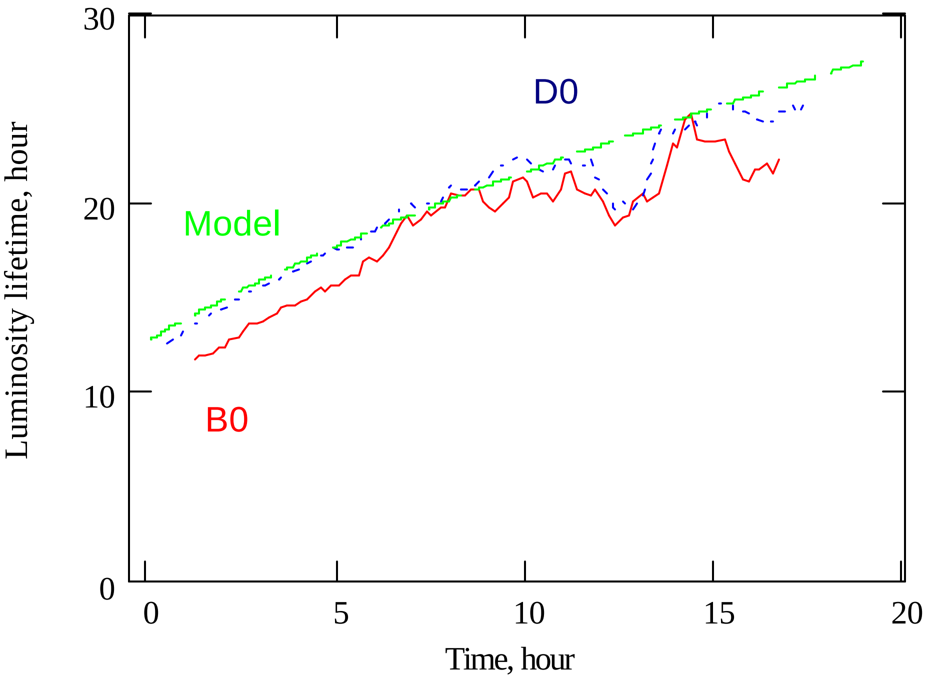




Emittances and bunch lengths on time for Store 2328

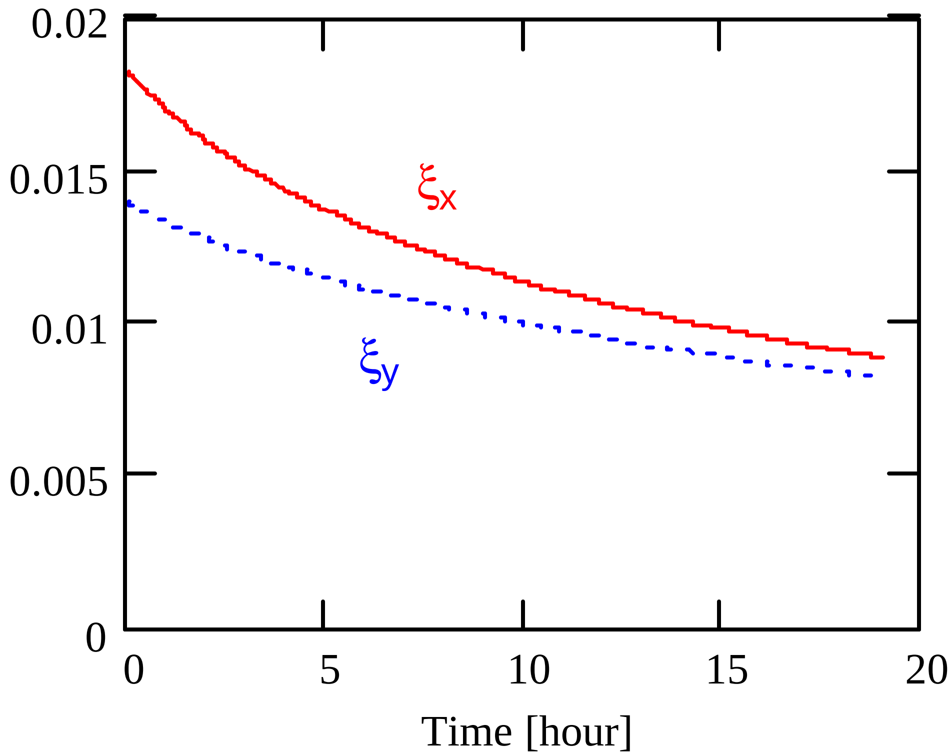
- **Beam-beam effects ?**
  - Proton bunch length and hor. emit. grow slower at the store beginning
    - Protons with large synchrotron amplitudes are lost due to beam-beam effects

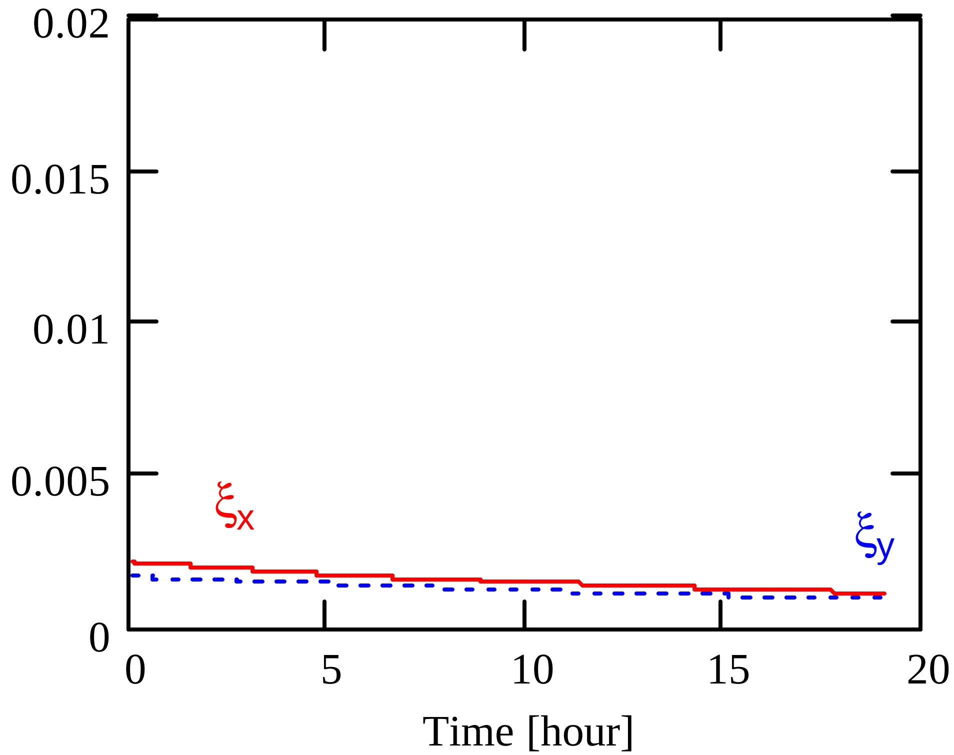




- Luminosity decays faster than the model predictions but the difference is small

Computed beam-beam linear tune shifts for Store 2328



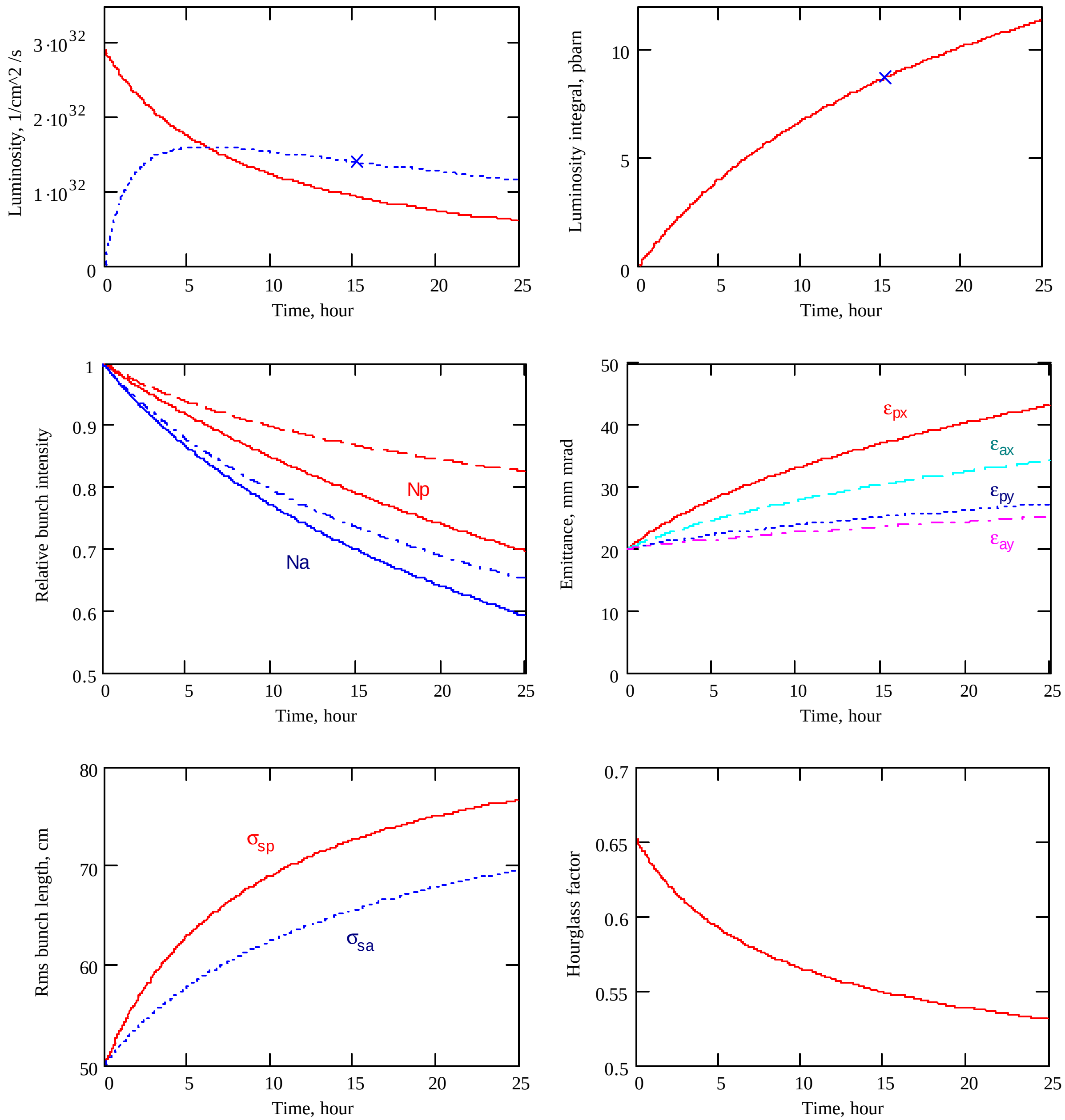


Pbars Protons

- We already close to the design linear tune shift of 0.02 for pbar beam (~80%)
  - and only about 20% for proton beam

Future Luminosity Scenario

- **Comparison of the model with present stores Luminosity yields**
  - The model makes good prediction of luminosity lifetime even in the case when there is strong influence of beam-beam effects
    - We do not know why we have “good” and “bad” stores
      - Incorrect tunes and too large longitudinal emittance are most probable reasons
      - New Shottky monitor will allow to track tunes and chromaticities for every bunch
  - Most, but not all, stores have abnormal proton and pbar loss at the store beginning
- **That assures us that this simplified model can be used for prediction of luminosity integral for the final parameters of Run II**
  - This is the best case scenario
  - Beam-beam effects and instabilities needs to be addressed separately
  - Balanced approach for both Tevatron and Antiproton source parameters

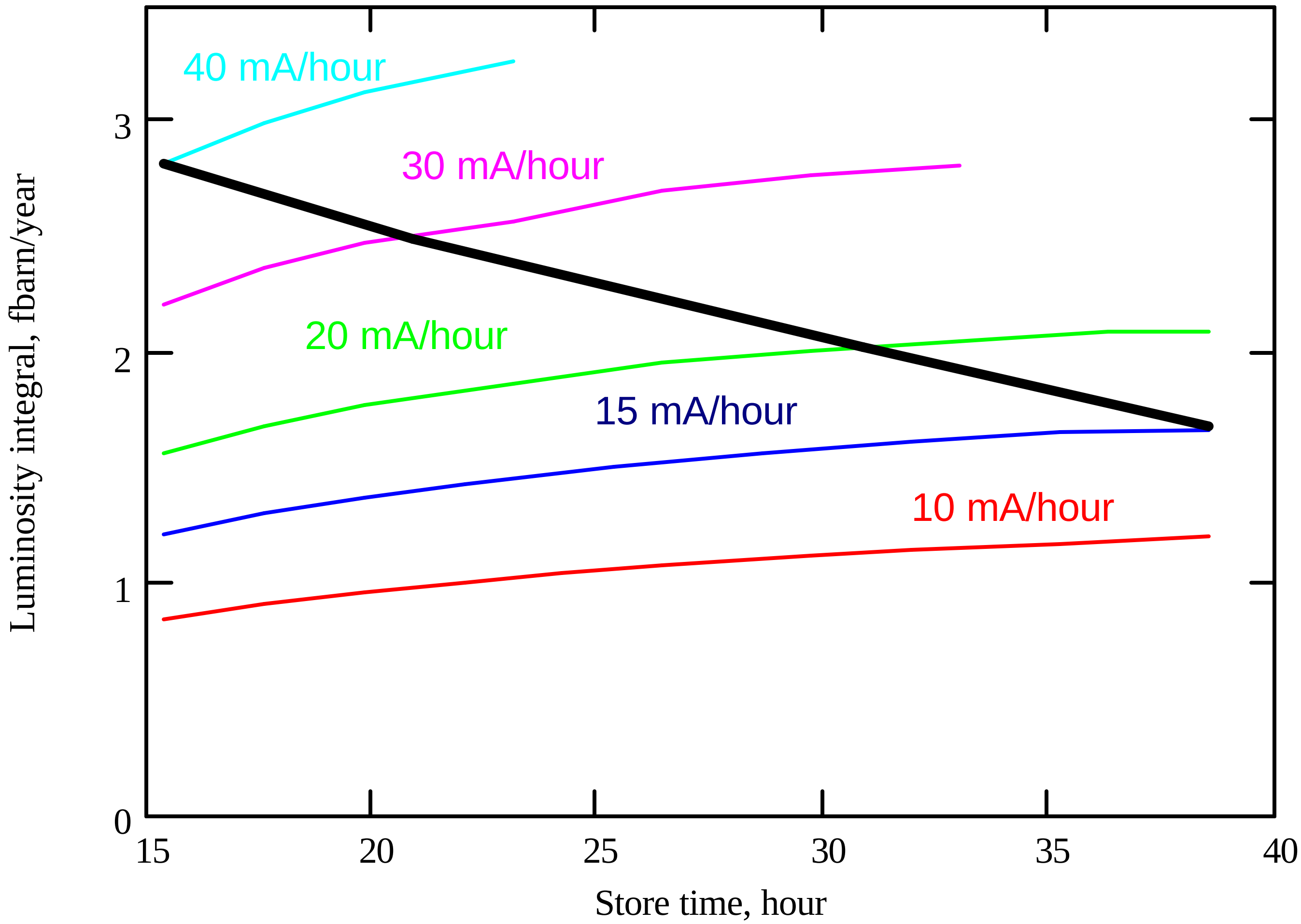


Break-up of the collider luminosity lifetime

	Lifetime [hour]
Prot.intens.	52
Pbar.intens.	29
Prot.H.emit.	9
Prot.V.emit.	32
Pbar.H.emit.	17
Pbar.V.emit.	56
Hourglass factor	32
Luminosity	7.2

Present and final Run II parameters of the collider

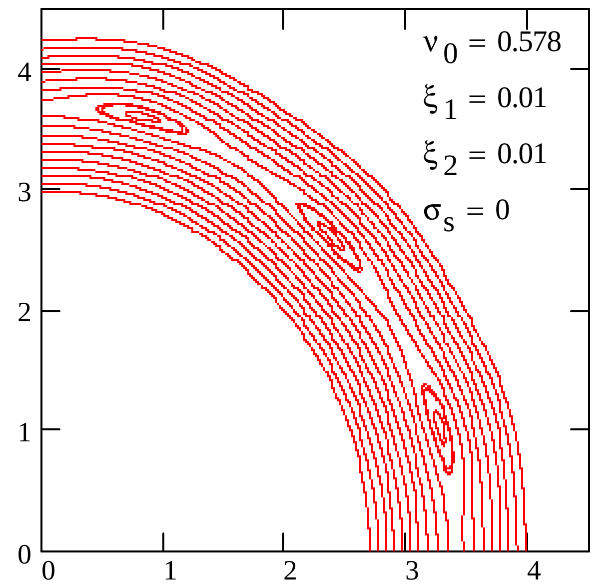
	Store 2328	Typical for April 2003	Final Run II
Number of protons per bunch, 10^{10}	20.7	20	27
Number of antiprotons per bunch, 10^{10}	2.54	2.2	13.5
Norm. 95% proton emitt, e_x / e_y, mm mrad	~14/24	~15/25	20/20
Norm. 95% pbar emitt, e_x / e_y, mm mrad	~15/24	~16/25	20/20
Proton bunch length, cm	65	62	50
Antiproton bunch length, cm	59	58	50
Initial luminosity, 10^{30} $cm^{-2}s^{-1}$	40.5	35	290
Initial luminosity lifetime, hour	11	12	7.1
Store duration, hour	19	20	15.2
Luminosity integral per store, pbarn	1.71	1.2	8.65
Shot setup time, hour	2	2	2
Luminosity integral per year, fbarn	-	0.225 ?	2.4
Transfer efficiency: stack to Tev at low-beta	60%	59%	80%
Average pbar production rate, 10^{10}/hour	-	11	40
Total antiproton stack size, 10^{10}	166	150	610
Antiprotons extracted from the stack, 10^{10}	154	140	610



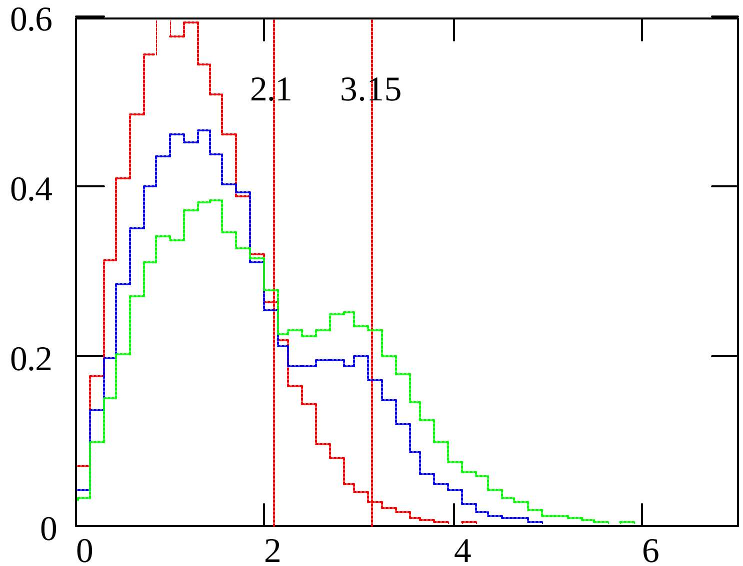
Dependence of luminosity integral per year on the store time for different antiproton production rates. Thick solid line shows where intensity of antiproton beam reaches $1.35 \cdot 10^{11}$ per bunch.

3. Beam-Beam effects

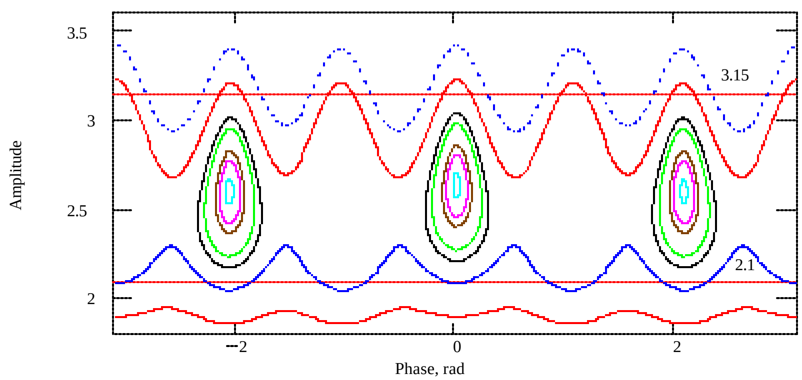
- 1 store ~ $4 \cdot 10^9$ turns – too much for any computer in visible future
- **Conclusion following from parametric model study**: for correctly tuned collider at present intensities the beam-beam effects and machine nonlinearity do not produce harmful effects on the beam dynamics while beams are in collisions
- We can not accept any significant worsening of the lifetime if we want to maximize the luminosity integral
- Theory should be build as perturbation theory to the diffusion model
- Diffusion amplification by resonances
  - Motion inside resonance island is fast comparing to the beam lifetime
    - 100-10,000 turns depending on ξ and the resonance order
  - Flattening distribution over resonance



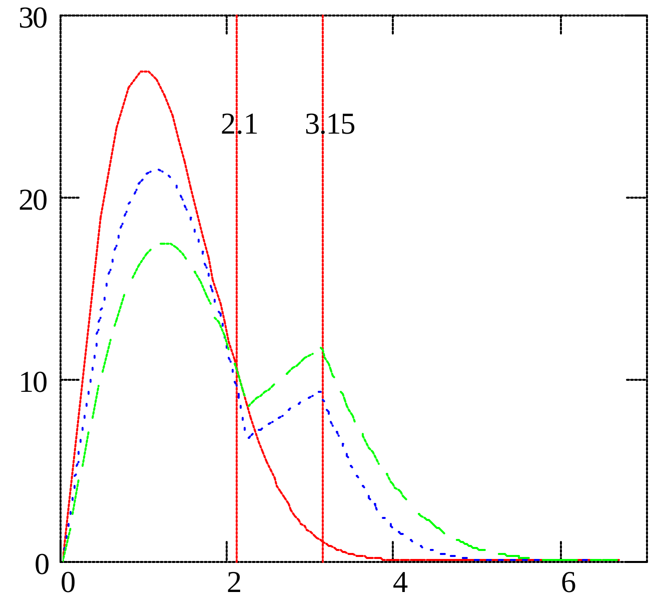
Phase trajectories in vicinity of 12-th order resonance ν_x=7/12; two Tevatron IPs but zero length of counter-rotating bunch, and zero synchrotron motion amplitude



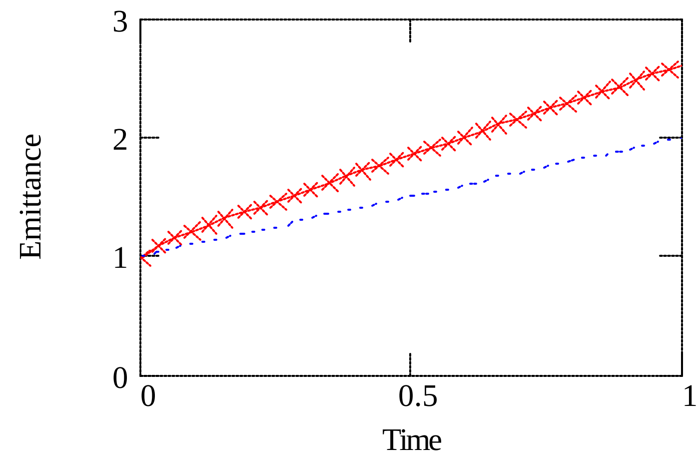
ν=0.325, ξ=0.02, $\Delta p/p$=0, $s_s \ll b^*$



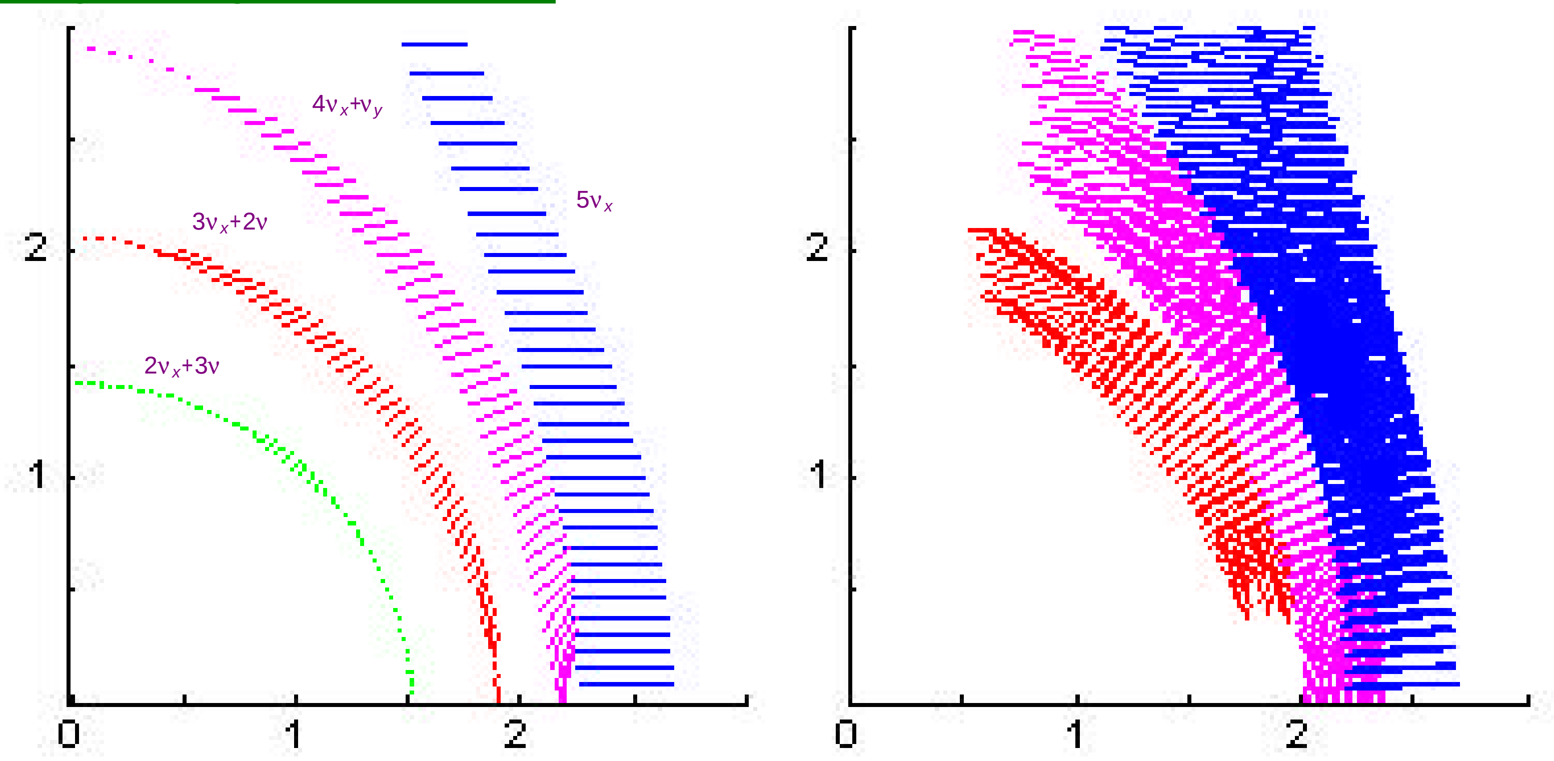
Tracking of 20,000 particles for 5,000 turns in vicinity of 6-th order resonance, n_x=2/6



Particle distribution obtained by diffusion equation solving



Long Range collisions



$d_p = 0$ $d_p = 1.25 \cdot 10^{-4}$

Swing of the normalized transverse amplitudes on the 5th order resonances and their synchrotron satellites at synchrotron amplitude $d_p = 0$ (left) and $d_p = 1.25 \cdot 10^{-4}$ (right), lattice chromaticity is zero, $n_x = 20.585$, $n_y = 20.575$.

Courtesy of Yu. Alexahin

How to perform tracking

- The number of turns is determined by minimum acceptable noise
  - Resonances with width above ~ 0.05σ need to be taken into account (5 such resonances change emittance growth time by ~25%)
  - Displacement due to diffusion during one revolution in the resonance needs to be much smaller than the resonance width
    - For 12-th order resonances one revolution is about few thousand turns
    - For displacement of about 0.1 of resonance width we obtain the minimum number of turns

$$\Delta a \equiv \Delta\sqrt{N_{res}} << 0.05s \xrightarrow{\Delta \approx 3\cdot 10^{-4}\ s} N = \frac{\Delta e}{\Delta^2} \approx \frac{0.1}{\left(3\cdot 10^{-4}\right)^2} \approx \mathbf{10^6\ turns}$$

- Number of particles is determined by
  - The statistic accuracy of emittance calculations

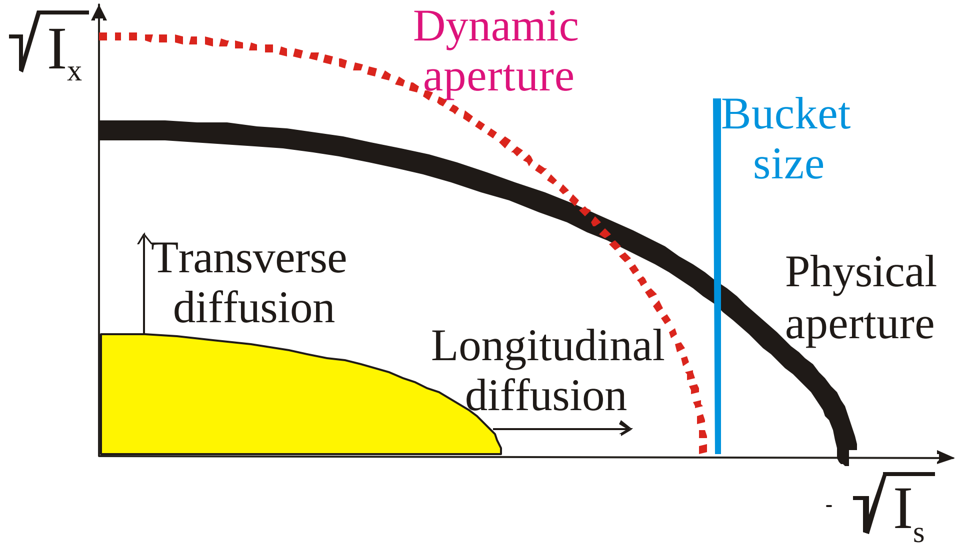
    1% accuracy of emittance calculations requires about 10,000 particles
  - Coverage of the phase space by particles for 3-D phase space

    10,000 particles correspond to an average particle distance (in 3 dimensional action phase space) of about 0.05σ

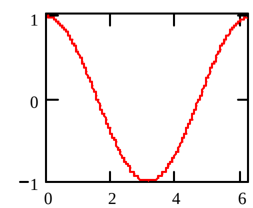
4. Particle Loss at Injection

- **Experimental observations**
  - Proton lifetime at proton helix (1– 4 hour) is much worse than at central orbit (~10 hour)
  - Lifetime is affected by the machine chromaticity
    - Smaller chromaticity improves the lifetime but its reduction is limited by head-tail instability
  - Strong dependence of the lifetime on bunch length
  - Intensity lifetime is much worse than the emittance lifetimes
    - Proton intensity decays as $N\left(1-\sqrt{t/t}\right)$
  - Additionally to mentioned above, the **pbar lifetime** is strongly affected by beam-beam effects

- **Basic mechanisms and reasons of the proton loss**

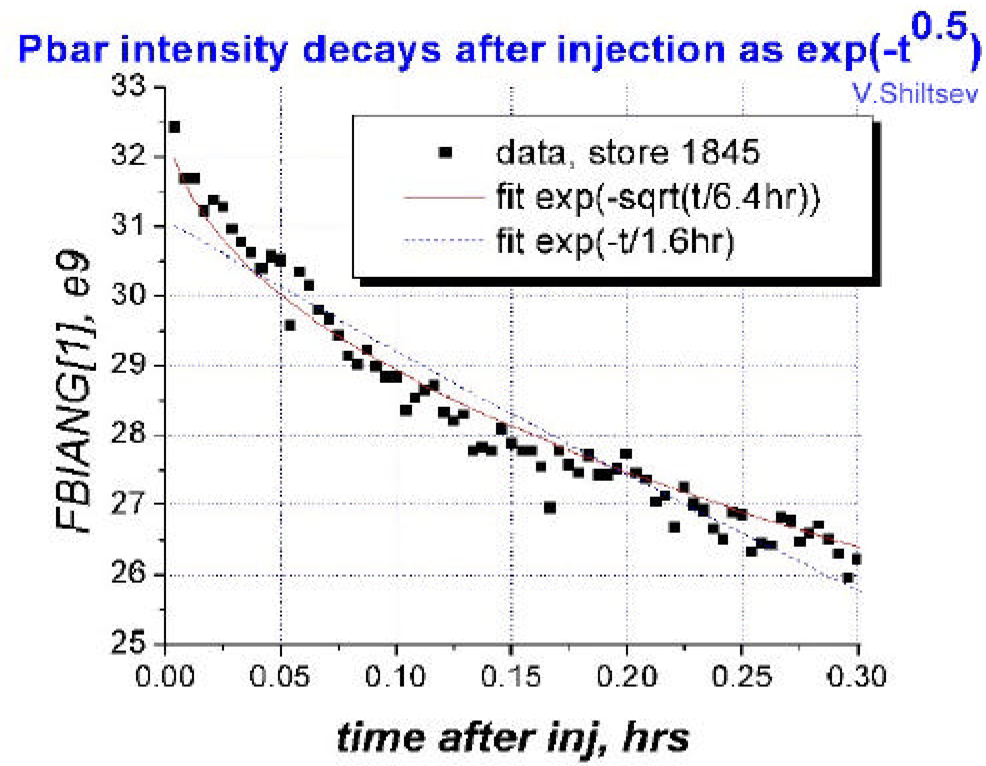


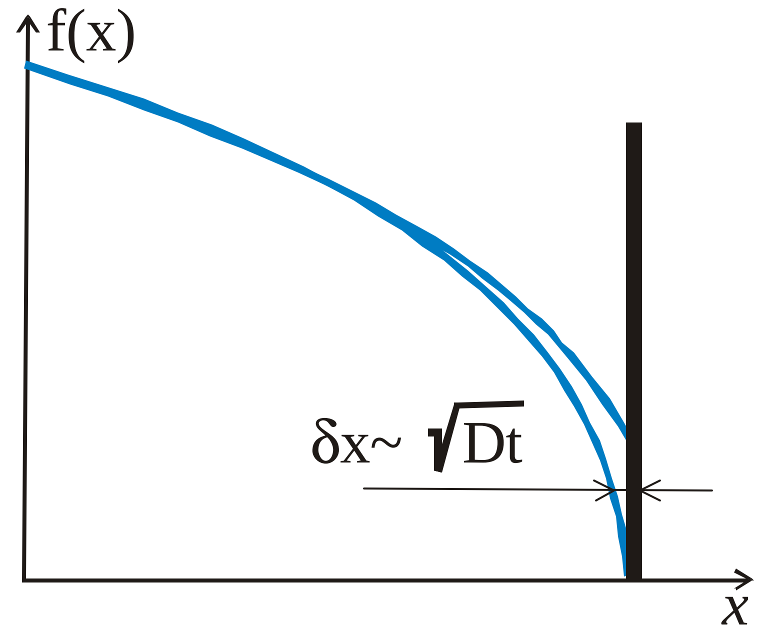
- Effects of longitudinal diffusion due to IBS and RF noise are amplified by
  - Overfilled bucket at injection
  - Shallowing the potential well near separatrix
  - Instability of motion at large synchrotron amplitudes



- Effects of transverse diffusion create loss due to
  - aperture limitations
  - reduced dynamic aperture for particles with large synchrotron apertures
- Major transverse diffusion mechanisms are
  - the residual gas scattering ($de_x/dt|_{Gas} \approx de_y/dt|_{Gas} \approx 1.1$ mm mrad/hour)
  - IBS (for protons $de_x/dt|_{IBS} + de_y/dt|_{IBS} \approx 1.2$ mm mrad/hour)

Measured dependence of proton beam intensity on time at injection





Conclusions

1. During last year we made significant progress in understanding of
   - Beam heating mechanisms and luminosity lifetime
   - Beam-beam effects
   - Transverse and longitudinal instabilities
   - Sources of background in particle physics detectors
2. We are still far away to understand all details of the beam-beam effects and it brings an uncertainty for the final Run II parameters
   - But the goal $L \sim 3{\cdot}10^{32}$ $cm^{-2}s^{-1}$ and $\int Ldt \sim 2.4$ fb^{-1}/year looks achievable

3. The following major actions are planned to achieve this goal
   a. To mitigate beam-beam effects
      - Optimization of helical orbits for all stages
      - Increasing of HV separator strength and installation of new separators
      - On-line tune measurements and the tune feedback
      - Bunch length reduction
   b. To mitigate head-tail instability
      - Shielding of F0 lambertson magnet
   c. Further improving of pbar transfer efficiency
      - Coalescing improvements and long. Emittance reduction
      - Reducing the emittance growth at transfers
   d. Improvements in machine tunability and reproducibility
      - Instrumentation and software
      - Tevatron alignment and correction of skew-quad in dipoles (smart bolts)
      - Tevatron optics model

Interaction with Residual Gas

Beam lifetime

$$\tau_{scat}^{-1} = \frac{2\pi c r_p^2}{\gamma^2 \beta^3}\left(\sum_i n_i Z_i (Z_i+1)\right)\left(\frac{\overline{\beta_x}}{\varepsilon_{mx}} + \frac{\overline{\beta_y}}{\varepsilon_{my}}\right) + \sum_i n_i \sigma_i c\beta$$

where: $\overline{\beta_{x,y}} = \frac{1}{C}\int \beta_{x,y} ds \approx 70\ \mathrm{m}$

ε_{mx}, ε_{my} – acceptances are chosen to be $6^2 \cdot 20$ mm mrad

- Average vacuum is adjusted to match the beam lifetime and the emittance growth rate for small intensity beam, $P=1\cdot10^{-9}$ Torr of N_2 equivalent
  - Coulomb scattering (~6000 hour)
  - Nuclear absorption **(~400 hour)**
  - Total gas scattering lifetime **(~380 hour)**
  - Gas composition used in the simulations

Gas	H_2	CO	N_2	C_2H_2	CH_4	CO_2	Ar
Pressure [nTorr]	1.05	0.18	0.09	0.075	0.015	0.09	0.15

Emittance growth time due to gas scattering

$$\frac{d\varepsilon_{x,y}}{dt} = \frac{2\pi c r_p^2}{\gamma^2 \beta^3}\left(\sum_i n_i Z_i (Z_i+1) L_{C_i}\right)\overline{\beta_{x,y}} \quad \Rightarrow \quad \frac{d\varepsilon_x}{dt} \approx \frac{d\varepsilon_y}{dt} \approx 0.2\ \text{mm mrad/hour}$$

- Beam based measurements of vacuum were carried out in July 2003. Analysis will follow.

Scattering in IP

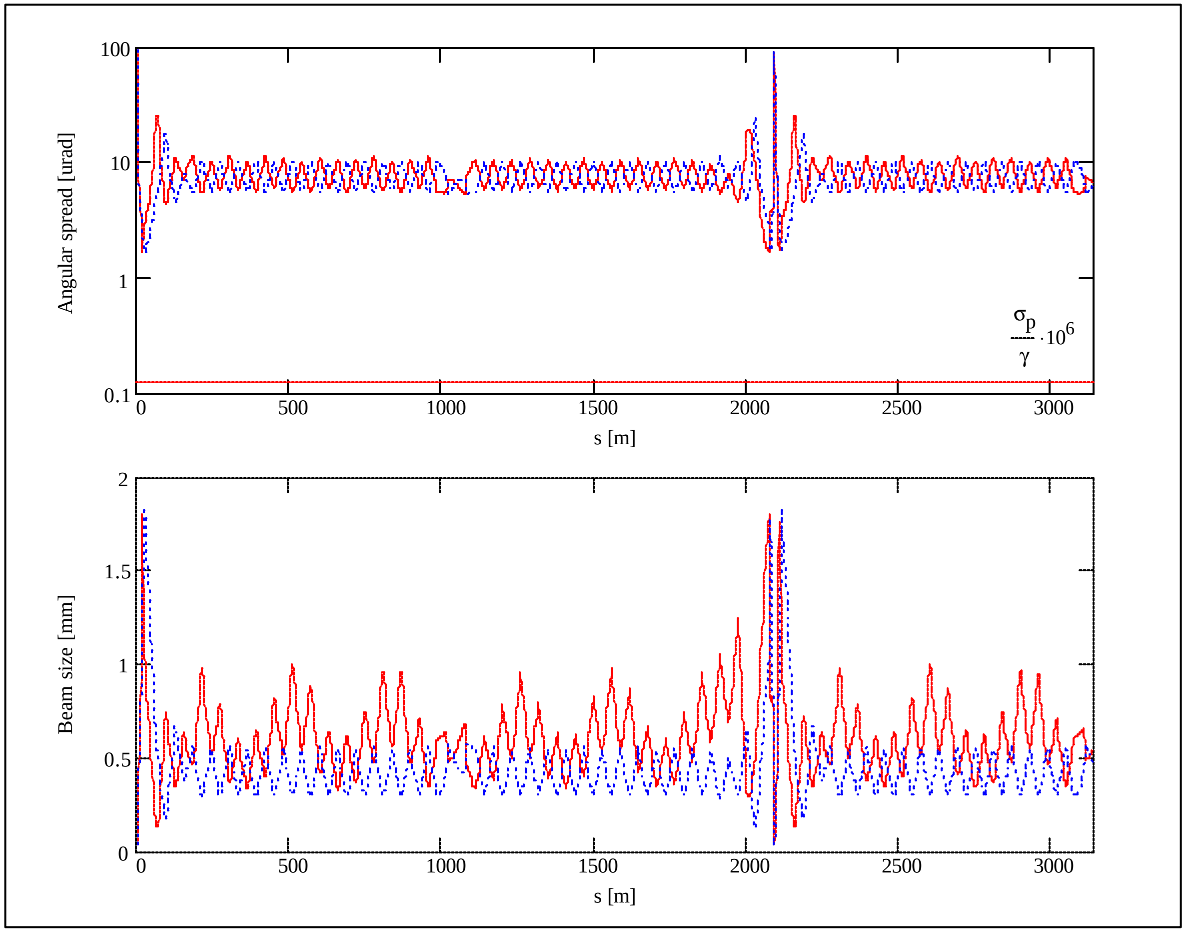
- Nuclear interaction
  - Main mechanism for loss of antiprotons
  - $p-\overline{p}$ cross-section ~ 69 mbarn
    - Inelastic – 60 mbarn
    - Elastic – 15 mbarn
      - 40% scatters within the beam (3σ)
- Electromagnetic scattering
  - Emittance growth

$$\frac{de_{x,y}}{dt} = \frac{4 r_p{}^2 N L_{bb} f_0}{g^2 b^3 \sqrt{\left(e_{px} + e_{py}\right)\left(e_{ax} + e_{ay}\right)}}$$

  - dε /dt ≈ 0.01 mm mrad and is negligible in comparison with gas scattering

Intrabeam Scattering

- Pancake distribution function allows one to use simple IBS formulas
- Integration over Tevatron lattice was carried out and results were compared to the smooth lattice approximation
  - Comparison yielded coincidence within 10%
- Therefore the smooth lattice approximation has been used for IBS to simplify the model
- The following corrections has been taken into account
  - Bunch length correction due to non-linearity of longitudinal focusing
  - Average dispersion and dispersion invariant, A_x, were calculated using lattice functions



Intrabeam Scattering (Continue)

$$\frac{d}{dt}\left(q_{\parallel}{}^{2}\right) \equiv \frac{d}{dt}\left(\frac{\overline{p_{\parallel}{}^{2}}}{p}\right) = \frac{1}{4\sqrt{2}}\frac{e^{4}N_{i}L_{C}\Xi_{\parallel}\left(q_{x},q_{y}\right)}{m_{p}{}^{2}c^{3}g_{i}{}^{3}b_{i}{}^{3}s_{x}s_{y}s_{s}\sqrt{q_{x}{}^{2}+q_{y}{}^{2}}} \quad ,$$

$$\frac{de_{x}}{dt} = (1-k)\left\langle A_{x}\frac{dq_{\parallel}{}^{2}}{dt}\right\rangle_{s} \quad ,$$

$$\frac{de_{y}}{dt} = k\left\langle A_{x}\frac{dq_{\parallel}{}^{2}}{dt}\right\rangle_{s}$$

where

$$\Xi_{\parallel}(x,y) \approx 1+\frac{\sqrt{2}}{p}\ln\left(\frac{x^{2}+y^{2}}{2xy}\right)-0.055\left(\frac{x^{2}-y^{2}}{x^{2}+y^{2}}\right)^{2},$$

$$s_{x} = \sqrt{e_{x}b_{y}+D^{2}q_{\parallel}{}^{2}},\ s_{y} = \sqrt{e_{y}b_{y}},\ q_{x} = \sqrt{e_{x}/b_{x}} \text{ and } q_{y} = \sqrt{e_{y}/b_{y}}$$

$$A_{x} = \left.\left\langle\frac{D^{2}+(D'b_{x}+a_{x}D)^{2}}{b_{x}}\right\rangle\right|_{s}$$

k – coupling coefficient (measurements yield that presently *k* ~ 0.4)

A_x = 19.7 cm, b_x=b_y = 48.5 m, *D* = 2.84 m

Beam Evolution in Longitudinal Degree of Freedom

- Diffusion mechanisms
  - IBS
    - Multiple and single scattering
  - RF noise
    - Phase noise
    - Amplitude noise
- Diffusion differently depends on action for all three mechanisms

- The first iteration of the model solved diffusion equation in a sinusoidal potential well under constant diffusion, $D(I) = D$,

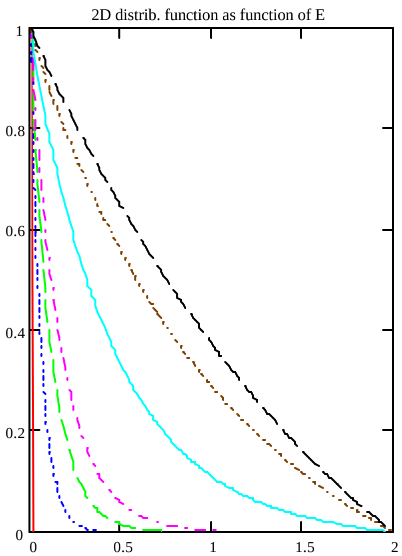
$$\frac{\partial f}{\partial t} = D\frac{\partial}{\partial I}\left(\frac{I}{dE/dI}\frac{\partial f}{\partial I}\right)$$

where

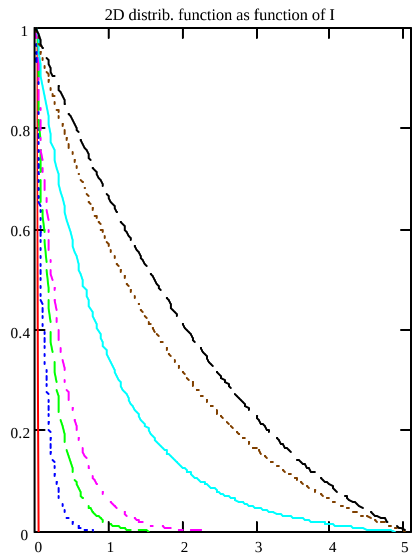
$$E = \frac{p^2}{2} + \Omega_s{}^2(1-\cos f) \qquad , \qquad I = \frac{1}{2p}\oint p\,df$$

  - Equation is solved numerically for initial distribution f(I) = δ(I)
  - The boundary condition $f(I) = 0$ at the RF bucket boundary is used

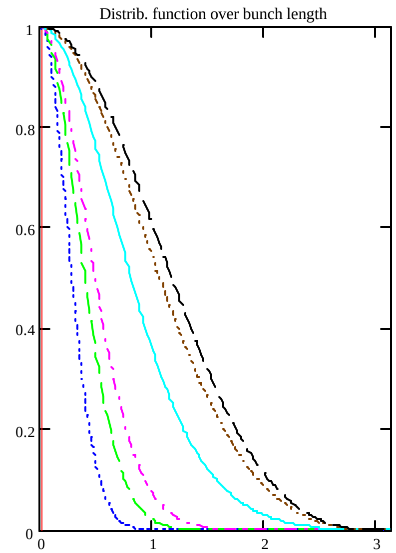
Beam Evolution in Longitudinal Degree of Freedom (continue)



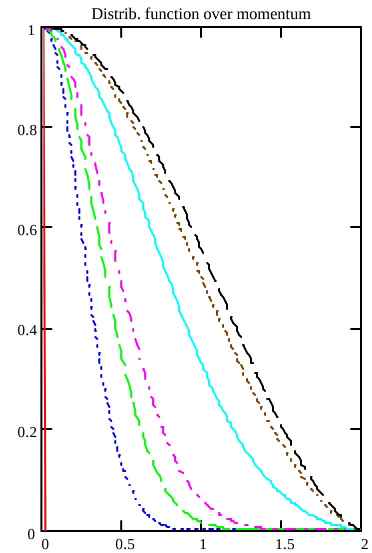
$f(I(E))$



$f(I)$



$f_f(f)$

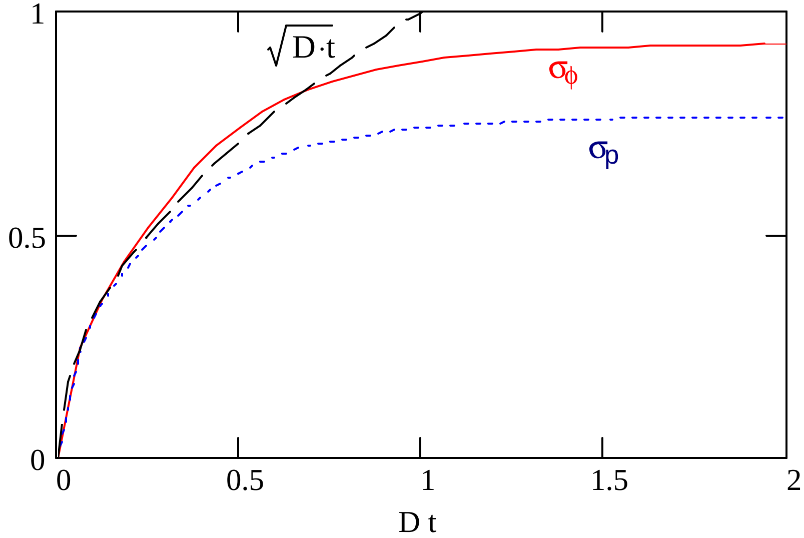


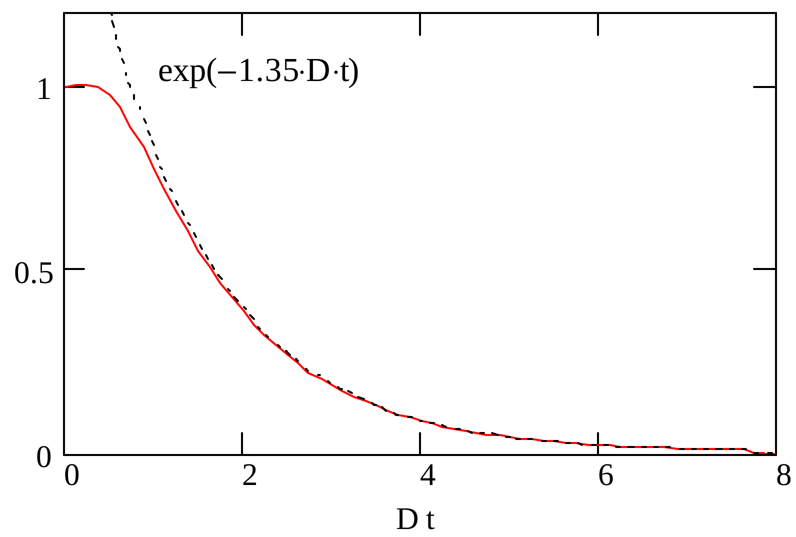
$f_p\,(p)$

Distribution functions as functions of the beam energy, action, longitudinal coordinate and the particle momentum deviation

where: $\int_0^{I_{\max}} f(I)dI = 1$, $f_f(f) = \int_{-p_{\max}(f)}^{p_{\max}(f)} f(I(f,p))dp$, $f_p(p) = \int_{-f_{\max}(p)}^{f_{\max}(p)} f(I(f,p))df$.

Beam Evolution in Longitudinal Degree of Freedom (continue)





- Asymptotic behavior
  - Shape of distribution function does not depend on time
  - Exponential decay of beam intensity

Beam Evolution in Longitudinal Degree of Freedom (continue)

To find compromise between completeness and simplicity of the model the following approximate relations were deduced from the numerical solution:

$$s_s \approx \Gamma_s s_{\Delta p/p}\left(1+\frac{1}{4}\left(\frac{2s_{\Delta p/p}}{\Delta P/P|_{sep}}\right)^2+\frac{1}{6}\left(\frac{2s_{\Delta p/p}}{\Delta P/P|_{sep}}\right)^3\right)$$

$$\frac{1}{N}\frac{dN}{dt} \approx \frac{2.425\,(2ps_s)^7}{l_{RF}{}^7+1.65\,(2ps_s)^7}\left(\left(\frac{2p\Gamma_s}{l_{RF}}\right)^2\left.\frac{d\left(s_{\Delta p/p}{}^2\right)}{dt}\right|_{IBS}+\left.\frac{d\left(s_f{}^2\right)}{dt}\right|_{RF}\right)$$

$$\left.\frac{d\left(s_{\Delta p/p}{}^2\right)}{dt}\right|_{total} \approx \left(1-\left(\frac{2s_{\Delta p/p}}{0.765\Delta P/P|_{sep}}\right)^5\right)\left(\left.\frac{d\left(s_{\Delta p/p}{}^2\right)}{dt}\right|_{IBS}+\left(\frac{l_{RF}}{2p\Gamma_s}\right)^2\left.\frac{d\left(s_f{}^2\right)}{dt}\right|_{RF}\right)$$

where $\Gamma_s=\left(a_M-1/g^2\right)q\,l_{RF}/(2pn_s)$ is the parameter of longitudinal focusing

Beam Evolution in Longitudinal Degree of Freedom (continue)

The bunch lengthening due to RF phase noise

- At small amplitude the bunch lengthening due to RF phase and amplitude noise is determined by its spectral density at synchrotron frequency,

$$\left.\frac{d\left(s_f^{\,2}\right)}{dt}\right|_{RF} = p\,\Omega_s^{\,2}\left(P_f(\Omega_s) + \frac{1}{2}s_f^{\,2}P_A(2\Omega_s)\right) \quad ,$$

where the spectral density of RF phase noise is normalized as

$$\overline{df_{RF}^{\;2}} = \int_{-\infty}^{\infty} P_f(w)\,dw \quad , \quad \overline{\frac{dA_{RF}^{\;2}}{A_{RF}^{\;2}}} = \int_{-\infty}^{\infty} P_A(w)\,dw$$

- Spectral density and bunch lengthening measurement are in decent agreement, and they yield that

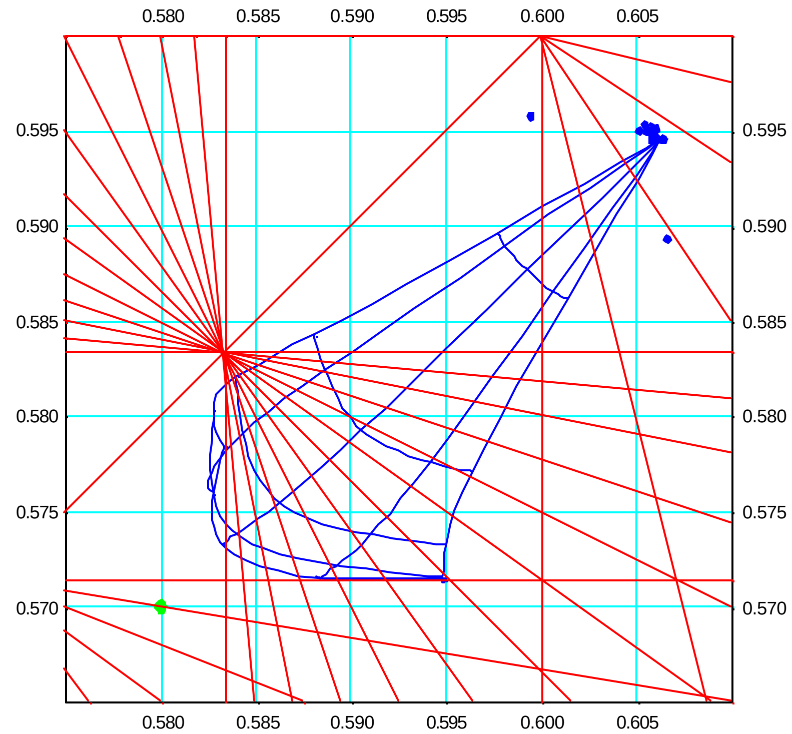
$$P_{ff}(\Omega_s/2p) = 4pP_f(\Omega_s) \approx 5\cdot 10^{-11} \quad \mathrm{rad}^2/\mathrm{Hz}$$

$$\left.\frac{d\left(s_f^{\,2}\right)}{dt}\right|_{RF} \approx 2200 \quad \mathrm{mrad}^2/\mathrm{hour}$$

- Beam-beam effects are important at all stages
  - Injection
  - Acceleration
  - Squeeze
  - Collision
- Two types of the beam-beam effects
  - **Head-on**
    - Run IB proton bunch population of ~$2.7\cdot10^{11}$ proton/bunch was set by the head-on collisions
    - We aim to achieve the same number of protons per bunch
      - Linear beam-beam tune shift $\xi \approx 0.01$ for each of two interaction points
  - **Long range**
    - Much stronger than for Run IB
    - Additional tune spread within one bunch
      - $\Delta\nu \approx 5\cdot10^{-3}$
    - Tune spread between bunches ($N_p=2.7\times10^{11}$)
      - At injection: $\Delta\nu_x \approx 5\cdot10^{-3}$, $\Delta\nu_y \approx 2.5\cdot10^{-3}$
      - At flat top: $\Delta\nu_x \approx \Delta\nu_y \approx 8\cdot10^{-3}$

Beam-beam effects on Tevatron

- Tunes are between 5-th and 7-th, and on 12-th order resonance
  - 5-th and 7-th order are excited by long large and lattice nonlinearity
  - 12-th order are excited by head-on
- Long range interactions make different tune shifts for different bunches
  - It can and must be mitigated
- Distance between 5-th and 7-th order resonances is 0.0285
  - Pbars from Protons
    - Head-on – 2·0.01=0.02
    - Long range within a bunch – 0.005
    - Bunch to bunch difference – 0.007
  - Protons experience only half of this due to smaller pbar intensity



Footprint of pbar bunch #6 in the tune diagram with ν_x=0.580, ν_y=0.580 (green dot) and nominal beam parameters. Dots show small amplitude tunes for other bunches. Footprint lines go in 2σ and 22.5 deg in the space of actions (angle or $(a_x^2 + a_y^2)^{1/2}$ =const on a line).