Luminosity Limitations for Electron-Ion Collider*

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Abstract. The major limitations on reaching the maximum luminosity for an electron ion collider are discussed in application to the ring-ring and linac-ring colliders. It is shown that with intensive electron cooling the luminosity of 10^{33} cm^{-2}s^{-1} is feasible for both schemes for the center-of-mass collider energy above approximately 15 GeV. Each scheme has its own pros and cons. The ring-ring collider is better supported by the current accelerator technology while the linac-ring collider suggests unique features for spin manipulations of the electron beam. The article addresses a general approach to a choice of collider scheme and parameters leaving details for other conference publications dedicated to particular aspects of the ring-ring and linac-ring colliders.

INTRODUCTION

Currently HERA at DESY and CEBAF at Jefferson Lab are two major players in the study of structure of the proton and light ions. HERA operates with polarized electrons and unpolarized protons, while CEBAF operates with polarized electrons and polarized protons or nuclei. HERA[1] is an electron-proton collider with very high center-of-mass energy (320 GeV) and moderate luminosity ($1.7\cdot10^{31}$ cm^{-2}s^{-1}), while CEBAF[2,3] is a fixed target machine with moderate energy and practically unlimited luminosity. The energy of the electron beam at CEBAF can presently be varied from 0.6 to 6 GeV and the machine is expected to be upgraded to 12 GeV[4] within about 5 years, boosting the center-of-mass energy of electron-proton collisions to 4.8 GeV. CEBAF luminosity for operation into the 4π CLAS detector of Hall B is mainly limited by the detector to about 10^{34} cm^{-2}s^{-1}. The luminosity is a few orders of magnitude higher for two other halls, which use spectrometers for particle detection.

For further progress in the study of nucleon spin structure a machine in the intermediate range of energies is required. The energy range of 15 to 50 GeV is currently considered to be interesting, with a request for the luminosity to be 10^{33} cm^{-2}s^{-1} or above. For physics of interest the effective luminosity of the collider is proportional to the square of each beam polarization, $L_{effective} = L_o p_e^{\,2} p_i^{\,2}$, and therefore achieving polarization above 70% for both electron and ion beams is of primary importance. Naive model of electron-quark collisions would require a ratio of

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the electron to proton energy to be about 1 to 6. We will not impose such a constraint but, as one will see later, an optimization of machine parameters yields close energy ratio.

Two general concepts for the collider have been suggested. The first is the classical ring-ring collider[5], where ion and electron beams are stored in independent storage rings of the same circumference. The second concept is the linac-ring collider[6], colliding protons in a storage ring against electrons from an energy recovery linac. Below we will try to analyze advantages and disadvantages of each scheme, as well as major factors limiting the collider luminosity. Reaching 10^{33} cm^{-2}s^{-1} luminosity with minimum cost will be our major optimization criterion. Because the machine luminosity generally grows with energy, we will also address the question at which energy each of the considered schemes can achieve the required luminosity. Although the parameter list and the luminosity optimization considered below are carried out for the electron-proton collider all results are also applicable to the electron-ion collider.

1. LUMINOSITY LIMITATIONS

There are three major limitations on the beam brightness. The first is the space charge effect in the ion beam. It causes nonlinear dependence of particle betatron tune on amplitude (Laslett tune shift) and, consequently, the loss of the particle motion stability in the case of large tune shift. This limits the phase density of the ion beam and is one of the major limitations at low ion energy. The second one is the beam-beam effect at the interaction point (IP) which limits the density of beams for both collider schemes. The third one is intrabeam scattering (IBS) in the ion beam. Although the intensity of IBS decreases with increasing energy, it is still an important limitation even at highest energy considered for the collider. In the estimates below we presume that the horizontal and vertical beam emittances, as well as the horizontal and vertical beta-functions at the IP, are equal for each of the beams (round beams). However, we do not necessarily consider the beam sizes for each of two beams to be equal. In this case the luminosity is determined by the following formula,

$$L = \frac{N_e N_i f_0}{2\pi\left(\sigma_i^{*2} + \sigma_e^{*2}\right)} \quad , \tag{1}$$

where N_e and N_i are the number of electrons and ions per bunch, σ_e^* and σ_i^* are the electron and ion beam sizes, and f_0 is the bunch frequency.

1.1 Laslett Tune Shift Limit

We will start our consideration from the Laslett tune shift. Its value is determined by the following formula

$$\Delta\nu_i = \frac{Z^2 e^2 N_i R}{2\sqrt{2\pi}\, m_i c^2 \gamma_i^3 \beta_i^2 \sigma_{si} \varepsilon_i} \quad , \tag{2}$$

where Ze is the ion charge, $m_i = Am_p$ is the ion mass, R is the storage ring mean radius, γ_i and β_i are the ion relativistic factors, ε_i is the ion beam emittance, and σ_{si} is the rms longitudinal beam size. Substituting the ratio N_i/e_i from the above equation into the luminosity formula and choosing the beta-function at the IP to be equal to the bunch length, $b^* = s_{si}$, one obtains the following remarkably simple formula for the luminosity limit due to the Laslett tune shift:

$$L_{\Delta n} = \sqrt{\frac{2}{p}}\,\frac{h}{Z}\,\frac{I_e B_d}{e^2\left(1+s_e^{*2}/s_i^{*2}\right)}\,g_i^{\,2} b_i \Delta n_i \longrightarrow \left(\frac{h}{0.2}\right)\left(\frac{I_e}{1A}\right)\left(\frac{B_d}{4\,T}\right)\left(\frac{\Delta n_i}{0.05}\right)\left(\frac{E_i/A}{15\ GeV/nucleon}\right)^2 \frac{1.06\cdot 10^{33}\ cm^{-2}s^{-1}}{Z\left(1+s_e^{*2}/s_i^{*2}\right)} \quad . \tag{3}$$

Here B_d is the magnetic field of ring dipoles and h is the fraction of the machine circumference covered by the dipoles, I_e is the electron beam current, and we expressed the ring radius through the bending magnetic field, the beam momentum and the dipole occupation factor h.

The values of parameters in Eq. (3) with the exception of the electron beam current and ion beam energy are well determined and cannot be significantly increased. In particular, the electron cooling sets the maximum value of Laslett tune shift, Dn, to about 0.05 determined by the distance to the nearest non-linear resonance. Actually, the cooling cools the beam to the equilibrium between cooling and heating coming from non-linear resonances and intrabeam scattering. If machine parameters are chosen so that the last does not dominate, the beam is cooled as far as it is not heated by the resonance. In absence of cooling higher order resonances play more significant role and Dn is smaller. In this case the emittance is rather determined by the ion source emittance and the emittance growth in the course of acceleration. It is currently expected that $\Delta n_i = 0.01$ can be achieved without cooling.

The luminosity limit is proportional to the occupation factor and magnetic field of the dipoles. Therefore superconducting dipoles are strongly favored even in the case of comparatively small energy (~10-20 GeV), when conventional wisdom prefers a normal conducting synchrotron. The request for two IPs suggests that the ion ring should be a racetrack. Then, the interaction regions and other systems such as RF system, beam injection, electron cooling and Siberian snakes for suppression of depolarizing spin resonances should be located in the straight sections. The length of the straight sections does not depend much on the ring energy and expected to be slightly above 100 m. Using a strong magnetic field in the dipoles decreases the ring circumference and increases the luminosity limit. But at a proton energy of about 20 GeV the bending radius of SC dipoles is quite small (~10-20 m) which complicates the dipole design for very high magnetic field. The choice of superferric dipoles with about 4 T field is a reasonable compromise. Further increase of the magnetic field makes dipoles more complicated but does not bring significant gain in the machine circumference because it starts to be limited by the length of the straight sections. Taking all of the above into account we can estimate h = 0.2 for 4 T dipoles. For an energy of 20 GeV/nucleon, this yields a ring circumference of about 500 m.

The luminosity limit of Eq. (3) does not depend on the ion beam current and is proportional to the electron beam current. The recent commissioning of B-factories[7,8] suggests that an electron beam current of 2 A can be achieved in a storage ring with parameters required for the collider. That determines that to reach 10^{33} cm^{-2}s^{-1} luminosity one needs a proton energy above 15 GeV for the ring-ring scheme. For the linac-ring scheme the major limitation for electron beam current comes from the electron injector[6]. It sets the current maximum to about 0.2-0.3 A. In this case the energy of proton beam needs to be above about 30 GeV to achieve 10^{33} cm^{-2}s^{-1} luminosity, taking into account the factor of almost 2 which can be obtained with an electron beam size significantly smaller than the ion beam size, $\sigma_e^{*2} \ll \sigma_i^{*2}$. As one will see below such a choice is not limited by other constraints.

Note that in the absence of ion beam cooling $\Delta\nu$ is about five times smaller. For both schemes this requires an additional increase of ion energy by factor of $\sqrt{5}$ to achieve 10^{33} cm^{-2}s^{-1} luminosity. One can also see from Eq. (3) that the luminosity per nucleon LA is proportional to A/Z and therefore practically does not depend on Z for fully stripped ions. As will be seen below intrabeam scattering puts more severe request for the ion beam energy increase in the absence of cooling.

1.2 Luminosity Limit due to Beam-beam Effects

Beam-beam effects in colliders are well known. They shift the betatron tunes of the beams and set the following two luminosity limits,

$$L_i = \frac{I_i E_i \xi_i}{e^3 Z \beta_i^*} \frac{2}{1+\sigma_i^{*2}/\sigma_e^{*2}} = \left(\frac{I_i}{1A}\right)\left(\frac{E_i}{15GeV}\right)\left(\frac{\xi_i}{0.02}\right)\left(\frac{7\,cm}{\beta_i^*}\right)\frac{2}{1+\sigma_i^{*2}/\sigma_e^{*2}} 1.85\cdot 10^{33}\,cm^{-2}s^{-1} \quad , \tag{4}$$

$$L_e = \frac{I_e E_e \xi_e}{e^3 Z \beta_e^*} \frac{2}{1+\sigma_e^{*2}/\sigma_i^{*2}} = \left(\frac{I_e}{1A}\right)\left(\frac{E_e}{5\,GeV}\right)\left(\frac{\xi_e}{0.035}\right)\left(\frac{7\,cm}{\beta_e^*}\right)\frac{2}{1+\sigma_e^{*2}/\sigma_i^{*2}} 1.08\cdot 10^{33}\,cm^{-2}s^{-1} \quad , \tag{5}$$

which corresponds to the tune shift limitations in the ion and electron beams. Here $E_i = Am_p c^2\gamma_i$ and $E_e = m_e c^2 \gamma_e$ are the energies of the ion and electron beams, and the parameters $\xi_i = e^2 Z N_e \beta_i^* / (4\pi E_i \sigma_e^{*2})$ and $\xi_e = e^2 Z N_i \beta_e^* / (4\pi E_e \sigma_i^{*2})$ correspond to the linear tune shifts in the ion and electron beams. There is not much freedom in choice of the parameters in the above equations. The achievable value of betatron tune shift for the ion beam depends on the beam cooling. We currently believe that with effective electron cooling (10 to 100 s damping time) ξ_i can be close to 0.02 per IP while without cooling it should be an order of magnitude lower. The beta-function at the IP is determined by the ion bunch length and is limited by the beam separation, the chromaticity of the final focus and its aperture. All these considerations limit β_i to the range of 6 to 10 cm for ion beam energy from 15 to 30 GeV. The luminosity limit described by Eq. (4) does not really depend on the collider scheme (both schemes have similar ion rings) and for the ion energy of 15-30 GeV sets the required value of the proton current to the range of 1-2 A.

In the case of the luminosity limit due to electron beam tune shift described by Eq. (5) the electron beam parameters depend on the collider scheme. Considerable experience acquired on the electron-positron colliders suggests that $\xi_e = 0.035$ can be achieved. For the linac-ring scheme with single IP when one can accept quite significant electron beam emittance growth after the collision the tune shift can be as large as one[6] and for practical machine parameters the beam-beam effects do not limit ξ_e. With two interaction points the tune shift will be limited by the emittance growth after the first collision and $\xi_e \le 0.2$.

For the linac-ring scheme there is another limitation on the product $\xi_e \xi_i$ which usually puts more severe limitations on possible collider parameters. In this case the interaction of electron beam with the ion beam transfers an electromagnetic excitation from the ion beam head to its tail and thus acts similar to the transverse impedance of the ion ring causing the ion beam kink instability above the threshold[9,10]. Taking into account an estimate of Ref. [6] and more accurate simulation results of Ref. [11] the threshold can be parametrized in the following form,

$$\xi_e \xi_p \le \frac{\nu_{si}}{5\pi} \frac{\beta_e^*}{\sigma_{si}} \quad , \tag{6}$$

where ν_{si} is the dimensionless synchrotron tune. Taking into account that the synchrotron tune for a high energy proton synchrotron hardly can be made more than 0.01 and ξ_p is desired to be about 0.005-0.01 that limits ξ_p to be less than about 0.1.

1.3 Intrabeam Scattering

Another important limitation on the beam brightness is determined by intrabeam scattering (IBS). For the above collider parameters the longitudinal energy spread in the beam frame is significantly smaller than the transverse one. That allows one to get comparatively simple formulas to describe IBS. In this case IBS transfers the energy from the transverse degrees of freedom to the longitudinal one and the growth rate can be approximated by the following formula[12],

$$\frac{d}{dt}\left(\theta_\parallel^{\,2}\right) \equiv \frac{d}{dt}\left(\frac{\overline{p_\parallel^{\,2}}}{p}\right) = \frac{(Ze)^4 N_i}{4\sqrt{2} A^2 m_p^{\,2} c^3 \gamma_i^{\,3} \beta_i^{\,3} \sigma_{si}} \left\langle \frac{\Xi_\parallel(\theta_x, \theta_y)}{\sqrt{\theta_x^{\,2} + \theta_y^{\,2}}} \frac{L_C}{\sigma_x \sigma_y} \right\rangle_s \quad . \tag{7}$$

Here

$$\Xi_\parallel(x, y) \approx 1 + \frac{\sqrt{2}}{\pi} \ln\left(\frac{x^2 + y^2}{2xy}\right) - 0.055\left(\frac{x^2 - y^2}{x^2 + y^2}\right)^2, \tag{8}$$

averaging is performed along the beam orbit, $\sigma_x = \sqrt{\varepsilon_x \beta_y + D^2 \theta_\parallel^{\,2}}$, $\sigma_y = \sqrt{\varepsilon_y \beta_y}$, $\theta_x = \sqrt{\varepsilon_x / \beta_x}$ and $\theta_y = \sqrt{\varepsilon_y / \beta_y}$ are the beam sizes and angular spreads along the ring, and

$$L_C = \ln\left(\sqrt{\sqrt{\frac{\pi}{2}} \frac{\sigma_x \sigma_y \sigma_{si} \gamma_i}{N_i} \left(\frac{A m_p c^2 \gamma_i^{\,2} \beta_i^{\,2} \left(\theta_x^{\,2} + \theta_y^{\,2}\right)}{2e^2}\right)^3}\right) \tag{9}$$

is the Coulomb logarithm.

In the smooth focusing approximation, for equal horizontal and vertical emittances, $e_x = e_y$, equal betatron tunes, $n_x = n_y$, and small contribution of energy spread into the beam size, $D(\Delta p/p) << s_x$, the momentum spread growth rate can be written in the following form

$$\Lambda_{\|} \equiv \frac{1}{q_{\|}^{2}} \frac{d}{dt}\left(q_{\|}^{2}\right) = \frac{Z^{4}}{8A^{2}} \frac{e^{4} N_i L_C}{m_p^{2} c^{3} g_i^{3} b_i^{3} s_{si} e_x^{3/2} q_{\|}^{2}} \sqrt{\frac{n_x}{R}} \quad . \tag{10}$$

The heating of the longitudinal degree of freedom, consequently, causes cooling for both transverse degrees of freedom; but there is another mechanism, which additionally heats the horizontal degree of freedom. At regions with non-zero dispersion, changes in longitudinal momentum change the particles reference orbits, which additionally excites the horizontal betatron motion,

$$\frac{de_x}{dt} = \left\langle A_x \frac{dq_{\|}^{2}}{dt} \right\rangle_s \quad , \tag{11}$$

where

$$A_x = \frac{D^2 + \left(D' b_x + a_x D\right)^2}{b_x} \quad . \tag{12}$$

Finally, one can write for the emittance growth rates

$$\begin{bmatrix} \Lambda_x \\ \Lambda_y \end{bmatrix} \equiv \frac{1}{e_{x,y}} \frac{de_{x,y}}{dt} = \frac{(Ze)^4 N_i}{8\sqrt{2} A^2 m_p^{2} c^3 g_i^{3} b_i^{3} s_{si} e_{x,y}}$$
$$\left\langle \frac{1}{\sqrt{q_x^{2} + q_y^{2}}} \frac{L_C}{s_x s_y} \begin{bmatrix} 2A_x \Xi_{\|}(q_x, q_y) - \frac{b_x}{g_i^{2}} \Xi_{\perp}(q_x, q_y) \\ -\frac{b_y}{g_i^{2}} \Xi_{\perp}(q_y, q_x) \end{bmatrix} \right\rangle_s \quad . \tag{13}$$

where

$$\Xi_{\perp}(x, y) \approx 1 + \frac{2\sqrt{2}}{p} \ln\left(\frac{\sqrt{3x^2 + y^2}}{2y^2} x \right) + \frac{0.5429 \ln(y/x)}{\sqrt{1 + \ln^2 (y/x)}} \quad . \tag{14}$$

The energy conservation requires $\Xi_{\perp}(x, y) + \Xi_{\perp}(y, x) = 2\Xi_{\|}(x, y)$ which for considered approximation is fulfilled with accuracy better than 1%.

In the smooth focusing approximation for equal emittances and betatron tunes we obtain for the horizontal emittance growth rate,

$$\Lambda_x = \frac{Z^4}{16A^2} \frac{e^4 N_i L_C}{m_p^{2} c^3 g_i^{3} b_i^{3} s_{si} e_x^{5/2}} \sqrt{\frac{R}{n_x}} \left(\frac{2}{n_x^{2}} - \frac{1}{g_i^{2}} \right) \quad . \tag{15}$$

Note that for considered collider the length of the straight sections is close to the length of arcs and therefore Eq. (15) underestimates this growth rate by about factor of two.

1.3 Choice of Basic Parameters

The limitations considered above allow one to choose the basic machine parameters for both collider schemes. As a major goal we will consider achieving the luminosity of 10^{33} cm^{-2}s^{-1} for the electron-proton collider with the minimum machine energy and, consequently, the minimal cost. In this case, the Laslett tune shift (see Eq. (3)) and the electron beam current limitations set the minimum energy of the proton ring. They are 15 GeV for the ring-ring collider and 30 GeV for the linac-ring collider.

The distance between bunches is determined by the beam separation after collision and was chosen to be 3 m for both schemes. The limitation coming from the beam-beam effects determines the proton beam current and the energy of the electron beam. Main parameters of the considered colliders are shown in Table 1.

Although the energies of electron beams for the linac-ring and the ring-ring colliders are equal they are set by different limitations. For the ring-ring collider the energy is determined by the beam-beam tune shift of electrons in the field of protons. For the linac-ring collider the energy is determined by the kink instability threshold; and parameters are chosen so that the beam is at the instability threshold of Eq. (6). The threshold of the kink instability is proportional to the synchrotron tune and therefore achieving highest possible tune is desirable. Expressing the tune through the bunch length and the energy spread we obtain the following formula:

$$\nu_s = \frac{\gamma_i}{1-\gamma_i^2}\frac{q\sigma_{si}}{2\pi R}\frac{eV_0}{Am_p c^2\theta_{\|}} \quad , \qquad (16)$$

TABLE 1. Parameters of Ring-ring and Linac-ring Colliders with Luminosity of 10^{33} cm^{-2}s^{-1}				
	Ring-ring		Linac-ring	
	Electrons	Protons	Electrons	Protons
Center of mass energy, GeV	13.8		19.3	
Kinetic beam energy, GeV	3	15	3	30
Circumference, m	420	420	-	650
Betatron tunes	≈11	≈12	-	≈16
Critical kinetic energy, GeV	-	7.2	-	9.7
Beam current, A	1.7	1.8	0.27	2.7
Beta-functions at IP, cm	7	7	7	7
Rms normalized beam emittance, mm·mrad	780	2.26	130	1.4
Rms beam size at IP, μm	96	96	40	55
Laslett tune shift	-	0.05	-	0.05
Beam-beam tune shift per IP	0.032	0.006	0.15	0.0029
Synchrotron tune	-	0.0075	-	0.007
Rms momentum spread	$6\cdot10^{-4}$	$8\cdot10^{-4}$	$<5\cdot10^{-3}$	$8\cdot10^{-4}$
Rms bunch length, cm	≤ 3	7	≤ 0.3	7
Longitudinal intrabeam scattering lifetime, $\Lambda_{\|}^{-1}$, s	-	110	-	80
Transverse intrabeam scattering lifetime, Λ_x^{-1}, s	-	370	-	150

where V_0 is the total voltage of the ion ring RF system, and q is the ring's harmonic number. Further decreasing of the momentum spread, $\theta_{\|}$, is limited by the intrabeam scattering and the single bunch longitudinal instability. Therefore the only free parameter we have is the RF voltage. Its increase requires increasing the momentum compaction factor and, consequently, decreasing the betatron tune, which leads to a fast growth of the transverse IBS growth rate. As result a compromised value $n_x \approx 16$ has been chosen.

Note that the threshold of the kink instability was computed in the linear theory, which overestimates the instability threshold. We currently work on a more detailed non-linear simulation of the instability and we believe that this more accurate theory should increase the instability threshold by a factor of two. Consequently, the electron beam energy could be decreased to about 1.5 GeV, but in the case of two IPs this additional decrease of the electron beam energy is limited by deterioration of the electron beam emittance after the first collision.

If fully striped ions are used in the machine optimized for protons the Laslett tune shift is going to be the major limitation for luminosity. Actually, in this case $A/Z \approx 2$, and the energy per nucleon is about two times lower. Substituting this into Eq. (3) yields that the luminosity per nucleon, LA, is also two times smaller than for the proton case. Usually one would like to keep the same bunch length and the beam size at the IP and, consequently, the same beam emittance. Then, Eq. (2) yields that the number of ions should be $4Z$ times less than the number of protons and the ion beam current should be 4 times less than the proton beam current. That determines that the tune shift in the electron beam is one forth of the tune shift for the electron-proton case, and the tune shift of the ion beam is the same as for the proton beam. The IBS increments grow by approximately Z/2 times and, as one will see in the next section, they can be compensated by increased strength of the electron cooling.

2. ELECTRON COOLING

The intrabeam scattering in the ion beam is so strong that it is impossible to reach the required luminosity without strong cooling of the ions. Electron cooling is the only cooling method, which works for the ion density discussed above. For the collider parameters the velocity spread in the ion beam is sufficiently large and cooling can be considered non-magnetized. Then the cooling force in the beam frame is[13]:

$$\mathbf{F}'(\mathbf{v}') = \frac{4\pi\, n_e' Z^2 e^4}{m_e} \int \frac{\mathbf{v}' - \mathbf{v}_e'}{\left|\mathbf{v}' - \mathbf{v}_e'\right|^3} f_e(\mathbf{v}_e') d\mathbf{v}_e'^3 \quad , \tag{17}$$

where $f_e(\mathbf{v}_e')$ is the electron distribution function. The longitudinal velocity spread for the electron beam accelerated in an electrostatic accelerator is usually much smaller than its transverse energy spread. In our case of very high energy electron cooling, it is expected that the electron beam will be accelerated by a low frequency linear accelerator with energy recovery, and therefore the longitudinal velocity spread is expected to be significantly higher. For the estimate we assume that the longitudinal and transverse energy spreads of electrons are equal. Then, the friction force can be approximated by the following formula:

$$\mathbf{F}'(\mathbf{v}') \approx \frac{4\pi\, n_e' Z^2 e^4 L_{Ce}}{m_e} \frac{\mathbf{v}'}{\left(\mathbf{v}'^2 + 2.4\overline{\mathbf{v}_e'^2}\right)^{3/2}} \quad , \tag{18}$$

where $\overline{\mathbf{v}_e'^2} = \overline{\mathbf{v}_{ex}'^2} = \overline{\mathbf{v}_{ey}'^2} = \overline{\mathbf{v}_{ez}'^2}$, and L_{Ce} is the Coulomb logarithm.

To find the damping decrement one needs to perform averaging of the force over betatron motion. For the sake of this estimate we consider that the ion longitudinal velocity spread is much smaller than the transverse one, $\overline{\mathrm{v}_x'^2}, \overline{\mathrm{v}_y'^2} \gg \overline{\mathrm{v}_z'^2}$, and we put that $\mathrm{v}_z' = \beta_i c\theta_{\|}$, $\mathrm{v}_x' = \sqrt{2}\beta_i\gamma_i c\theta_x \cos\varphi_x$ and $\mathrm{v}_y' = \sqrt{2}\beta_i\gamma_i c\theta_y \cos\varphi_y$, where a factor of $\sqrt{2}$ takes into account that the amplitude is $\sqrt{2}$ times larger than the rms value. After performing averaging and returning back to the lab frame we obtain the following estimate for the damping decrements:

$$\tau_{\|}^{-1} \approx \frac{16\pi n_e Z^2 e^4 L_{Ce}}{3m_e m_i \gamma_i^5 \beta_i^3 c^3} \frac{\eta_c}{\left(2\theta_x^2 + 0.66\left(\theta_{\|}^2/\gamma_i^2\right) + 1.6\theta_e^2\right)\sqrt{\left(\theta_{\|}^2/\gamma_i^2\right) + 2.4\theta_e^2}} \quad ,$$
$$\tau_{\perp}^{-1} \approx \frac{16\pi n_e Z^2 e^4 L_{Ce}}{3m_e m_i \gamma_i^5 \beta_i^3 c^3} \frac{\eta_c}{\left(2\theta_x^2 + 1.2\left(\theta_{\|}^2/\gamma_i^2\right) + 2.8\theta_e^2\right)^{3/2}} \quad . \tag{19}$$

Here the decrements are defined as $\tau_{\perp}^{-1} \equiv 1/\varepsilon \cdot d\varepsilon/dt$ and $\tau_{\|}^{-1} \equiv 1/\overline{\Delta p^2} \cdot d\overline{\Delta p^2}/dt$,θ_e is the angular spread in the electron beam, η_c is a fraction of the ring orbit used for cooling, and the following two approximate equations have been used to perform averaging:

$$\int_0^{\pi}\!\!\int_0^{\pi} \frac{dxdy}{\pi^2} \frac{1}{\left(\cos^2 x + \cos^2 y + \Delta^2\right)^{3/2}} \approx \frac{2}{3} \frac{1}{\Delta\left(1 + \frac{2}{3}\Delta^2\right)} \quad ,$$
$$\int_0^{\pi}\!\!\int_0^{\pi} \frac{dxdy}{\pi^2} \frac{\cos^2 x}{\left(\cos^2 x + \cos^2 y + \Delta^2\right)^{3/2}} \approx \frac{1}{1.55\Delta\left(1 + \left(\frac{2}{1.55}\right)^{2/3}\Delta^2\right)^{3/2}} \quad , \tag{20}$$

Assuming that electron beam size is three times larger than the rms ion beam size in the cooling section, $r_{eb} = 3\sqrt{\varepsilon_i \beta_c}$ we can finally rewrite Eq. (19) in the following form

$$\tau_{\|}^{-1} \approx \frac{8Z^2 e^3 I_0 L_{Ce}}{27 m_e m_i \gamma_i^5 \beta_i^4 c^4 \varepsilon_i^2} \frac{\eta_c}{\left(1 + \left(0.33\frac{\theta_{\|}^2}{\gamma_i^2} + 0.8\theta_e^2\right)\frac{\beta_c}{\varepsilon_i}\right)\sqrt{\left(\theta_{\|}^2/\gamma_i^2\right) + 2.4\theta_e^2}} \quad ,$$
$$\tau_{\perp}^{-1} \approx \frac{4\sqrt{2} Z^2 e^3 I_0 L_{Ce}}{27 m_e m_i \gamma_i^5 \beta_i^4 c^4 \varepsilon_i^{5/2}} \frac{\eta_c\sqrt{\beta_c}}{\left(1 + \left(0.6\frac{\theta_{\|}^2}{\gamma_i^2} + 1.4\theta_e^2\right)\frac{\beta_c}{\varepsilon_i}\right)^{3/2}} \quad . \tag{21}$$

which will be used for the cooling time estimates considered below. Here $I_0 = \pi r_e^2 n_e \beta_i c$ is the electron beam current.

Table 2 presents parameters of the required cooling devices for the ring-ring and the linac-ring colliders considered in the previous section. The following issues have determined the choice of the parameters. The cooling decrements are proportional to the cooling length and one would like to choose this length as long as possible. To keep the ion beam inside the electron beam along the entire cooling length, the ion beam beta-functions should be larger, or equal to the cooling length. However an increase of beta-functions increases electron beam size and it decreases the electron density, which compensates the growth of the decrements due to the decrease of the angular spread. As one can see from Eq. (21) the effect of beta-function increase on damping decrements depends on the actual values of parameters. For the considered here parameters, an increase of beta-functions decreases the decrements. Also note that an increase of beta-functions raises the requirements on the angular spread of electrons due to smaller angular spread in the ion beam. Therefore, we choose the beta-functions to be equal to the cooling length for both cases. The cathode temperature and size set the minimum for the angular spread in the electron beam. We believe that the electron beam acceleration and transport needs to be done (and can be done) with a sufficiently small additional emittance growth. Then, the transverse temperature of electrons is equal to the cathode temperature and the longitudinal temperature was chosen to be equal to the transverse one. A smaller longitudinal temperature does not bring any significant gain, while the larger one affects both decrements.

TABLE 2. Electron Cooling Parameters for Ring-ring and Linac-ring Colliders

	Ring-ring	**Linac-ring**
Kinetic energy of electrons, MeV	8.2	16.3
Peak electron beam current, A	3	10
Mean electron beam current in energy recovery linac and 40 cm bunch, A	0.4	1.3
Electron beam radius, cm	0.6	0.34
Electron density in the beam frame, cm^{-3}	$3.2\cdot 10^{7}$	$1.7\cdot 10^{8}$
Beta-function of proton beam at the center of cooling section, m	30	30
Rms proton beam size at the center of cooling section, cm	0.2	0.11
Cooling section length, m	30	30
Rms angular spread in the proton beam,	$6.7\cdot 10^{-5}$	$3.7\cdot 10^{-5}$
Rms transverse velocity spread of ions in the beam frame, cm/s	$3.4\cdot 10^{7}$	$3.7\cdot 10^{7}$
Rms longitudinal velocity spread of ions in the beam frame, cm/s	$2.4\cdot 10^{7}$	$2.1\cdot 10^{7}$
Effective electron temperature in the beam frame, eV	0.2	0.2
Rms velocity spread of electrons in the beam frame, cm/s	$1.9\cdot 10^{7}$	$1.9\cdot 10^{7}$
Relative rms energy spread of electrons	$6.2\cdot 10^{-4}$	$6.2\cdot 10^{-4}$
Relative rms angular spread of electrons	$3.7\cdot 10^{-5}$	$1.9\cdot 10^{-5}$
Longitudinal emittance damping time, s	110	70
Transverse emittance damping time, s	240	150

As one can see from Table 2, the electron velocity spread is already comparable to the ion velocity spread and its further increase would cause fast decrease of the damping decrements. There are two major issues limiting the angular spread for electrons: uncontrolled transverse dipole fields and the space charge effects. The first issue is important only inside the cooling section where achieving the beam angle variation below 10-20 mrad is not going to be a challenging problem. This issue is worse for the linac-ring case where the angular perturbations have to be 2 times lower. The space charge effects are a major concern at low energy and solution of this problem will be another challenge. If an energy recovery linac is chosen for the acceleration of electrons, there is another effect, which can significantly affect the transverse velocities. This is a time dependent focusing of the accelerating cavities, which focuses differently particle in the head and in the tail of the bunch.

If for the ring-ring case the electrostatic acceleration to 8 MeV is still feasible, this is not a viable option for the linac-ring scheme with 16 MeV, where the energy recovery linac is the only choice. In that case to maintain effective cooling for all particles in the ion bunch the electron bunch should have a bunch length of at least 5σ, corresponding to about 40 cm. In this case achieving $6\cdot 10^{-4}$ energy spread will pose a challenge. One of possible solutions can be an addition of higher harmonics to correct the accelerating profile. In particular adding the third and the fifth harmonics allows one to reach the accelerating gradient uniformity within $\pm 10^{-4}$ band for 45 cm bunch and 3 m bunch spacing.

The parameters of the electron coolers required for both projects are well beyond current state-of-the-art technology for the beam current, and the angular and momentum spread in the electron beam. The required electron beam current for the linac-ring scheme is three times higher and the angular spread is two times lower making this choice significantly more difficult, if possible at all. Also note that the electron beam parameters were calculated for the most optimistic case. In reality, we may have difficulties to achieve the desired velocity spread in the electron beam. That will require higher electron beam current making the project more complicated.

3. NO COOLING SCENARIO

Taking into account that the electron cooling is still a significant pending problem we also consider a collider with no cooling at the top energy. However some cooling can be used at low energy to shape the beam. In this case the IBS is the major obstacle in achieving high brightness in the ion beam; usually the longitudinal IBS is more limiting. Combining Eqs. (1) and (10) and taking into account that the bunch length and the beta-function at the IP are equal one obtains the following luminosity limitation:

$$L_{IBS} = \frac{4A^2}{\pi Z^4} \frac{m_p^{\,2} c^3 \gamma_i^{\,3} \beta_i^{\,3} \theta_\parallel^{\,2} I_e \Lambda_\parallel}{e^5 L_C \left(1 + \sigma_e^{*2} / \sigma_p^{*2}\right)} \sqrt{\frac{\varepsilon_x \nu_x}{R}} \quad . \tag{22}$$

For high energy collider without cooling the emittance is determined by the injector, and the normalized emittance, $\varepsilon_{xn}=\varepsilon_x\gamma_i$, is conserved. The ratio R/γ_i is determined by the magnetic field of dipoles. Thus, we can write the following scaling for the luminosity limit: $L_{IBS} \propto \gamma_i^{\,2} \theta_\parallel^{\,2} I_e \Lambda_\parallel \sqrt{\varepsilon_{xn} \nu_x}$. As one can see, the only effective free parameter to compensate a longer IBS time is the ion beam energy. Table 3 depicts the tentative parameters of the ring-ring collider with 10^{33} $cm^{-2}s^{-1}$ luminosity and minimum center of mass energy.

TABLE 3. Tentative Parameters of Ring-ring Collider with Luminosity of 10^{33} $cm^{-2}s^{-1}$ and without Cooling

	Electrons	Protons
Center of mass energy, GeV	42.6	
Kinetic beam energy, GeV	3	150
Circumference, m	1600	1600
Beam current, A	1.7	0.8
Beta-functions at IP, cm	5	10
Rms normalized beam emittance, mm·mrad	110	6.6
Rms beam size at IP, μm	30	64
Laslett tune shift	-	$2 \cdot 10^{-4}$
Beam-beam tune shift per IP	0.023	0.002
Rms momentum spread	$6 \cdot 10^{-4}$	$8 \cdot 10^{-4}$
Rms bunch length, cm	≤ 3	10
Longitudinal intrabeam scattering lifetime, $\Lambda_{\parallel}^{-1}$, hour	-	17
Transverse intrabeam scattering lifetime, Λ_{x}^{-1}, hour	-	23

In the case of the linac-ring collider the ion ring energy should be increased by another factor two or three. As one can see, although achieving the desired luminosity is still feasible with high-energy ion ring, its energy becomes disproportionally high.

4. DISCUSSION

Achieving the luminosity of 10^{33} $cm^{-2}s^{-1}$ for the electron-proton collider is a challenging problem. Two possible collider schemes have been proposed. They are the ring-ring and the linac-ring colliders. Both of them require electron cooling of the proton beam at the collision energy. The electron beam parameters for the cooler are far beyond of the current electron cooling technology, and the development of cooling is one of the highest priorities of the project R&D. In the comparison of two collider schemes considered above, the electron beam current for the linac-ring collider has been limited to 200-300 mA. Even with this optimistic choice supported by estimates, but staying far away from the current technology level, the luminosity of 10^{33} $cm^{-2}s^{-1}$ requires two times higher energy for the ion ring and significantly more complicated electron cooling. Although, at the present time the linac-ring collider does not look competitive to the ring-ring if a new machine with ultimate luminosity is built, the linac-ring collider can be a good choice for already existing high energy synchrotrons. In the case of RHIC collider it does not look reasonable to build a full circumference electron ring, but in a ring of smaller circumference the beam-beam effects are much worse. A linac with energy recovery may be a much better choice.

If a collider optimized for the electron-proton collisions is used for the electron-ion collisions with fully stripped ions the ultimate luminosity for the electron-ion mode is expected to be about half of luminosity for the proton-electron case with approximately the same requirements for the electron cooling.

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