# Collective instabilities in the Tevatron complex

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## *Talk outline*

1. Introduction
2. Head-tail instability in Tevatron
3. Suppression Head-tail instability in Tevatron by transverse damper
4. Resistive wall instability in recycler
5. Conclusions

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# 1. Introduction

- Tevatron complex has 6 rings. Wide range of instabilities
  - Only Debuncher has no problems with beam stability

| | Instability nature | | Damper | |
|---|---|---|---|---|
| | Longitudinal | Transverse | Longitudinal | Transverse |
| Booster | Multi-bunch, cavities | Space charge | Narrow band | - |
| Main injector | Multi-bunch | Multi-bunch | Bunch-by-bunch | Bunch-by-bunch |
| Accumulator | Stochastic cooling | Instability due to stored ions | - | Wide-band |
| Recycler | - | Head-tail, wall resistivity | - | Planned Wide-band |
| Tevatron | Bunch dancing | Head-tail, wall resistivity | Bunch-by-bunch | Bunch-by-bunch |

# 2. Head-tail instability in Tevatron[1]

- Transverse instability has been a problem for long time
  - It forces us to use large chromaticity
    - Bad effect on beam lifetime
- Significant progress achieved in understanding and correcting
  - Measurements of instability increment set the low boundary of $Z_\perp$ to ≈ 5 MΩ/m (100 MHz)
  - Main contribution came from 2 **laminated** Lambertson magnets which triple $Z_\perp$
    - The first unused Lambertson was removed in 2002
    - Another one (injection Lambertson) was shielded in 2003
  - Presently transverse impedance is dominated by the wall resistivity of main vacuum chamber (stainless steel)

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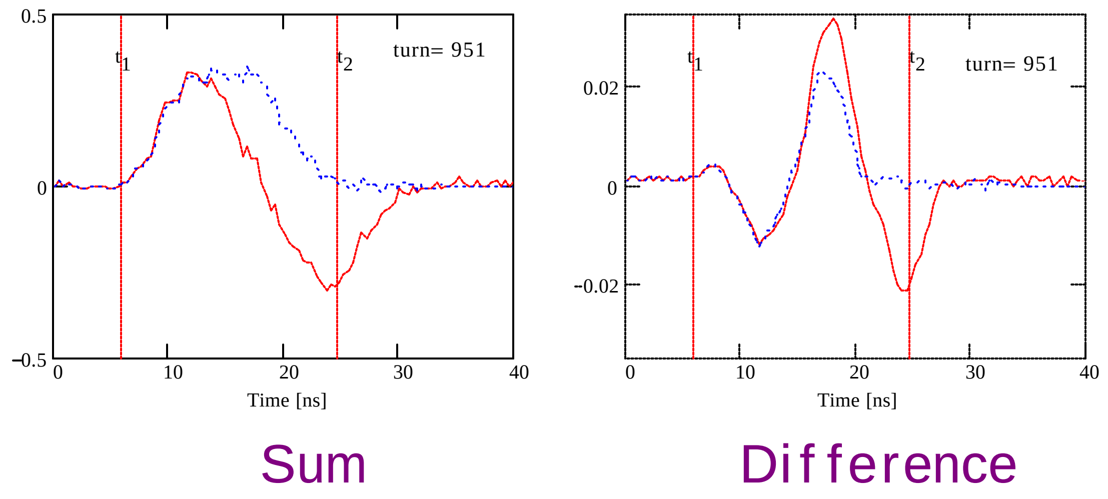
1 J. Annala, A. Burov, P. Ivanov, V. Lebedev, E. Lorman, V. Ranjbar, V. Scarpine, V. Shiltsev

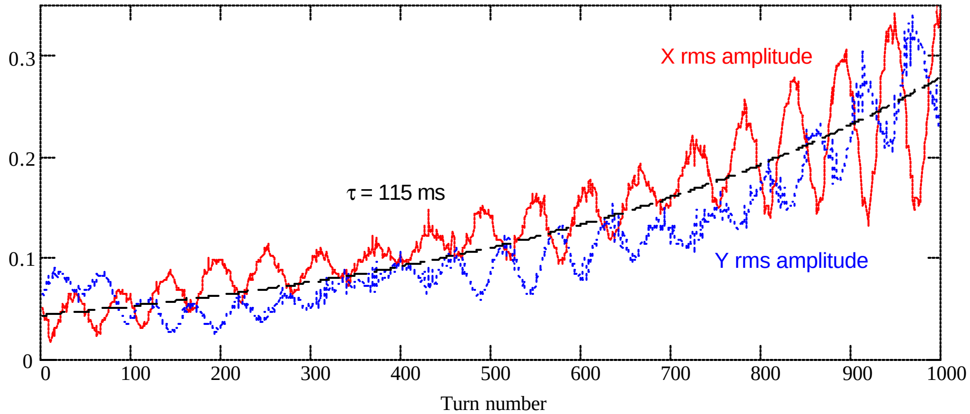
# Direct Instability Observations at injection (150 GeV)

- A fast segmented memory digital oscilloscope (0.4 ns sampling rate)
  - connected to the horiz. and vert. 1-meter long strip-line pickups
  - Single bunch (80 ns data taking), 2000 turns
- Data analysis
  - Both the sum and difference signals are deconvoluted.
    - Sum signal represents the particle distribution along the bunch
    - Difference signal represents the dipole moment along the bunch.

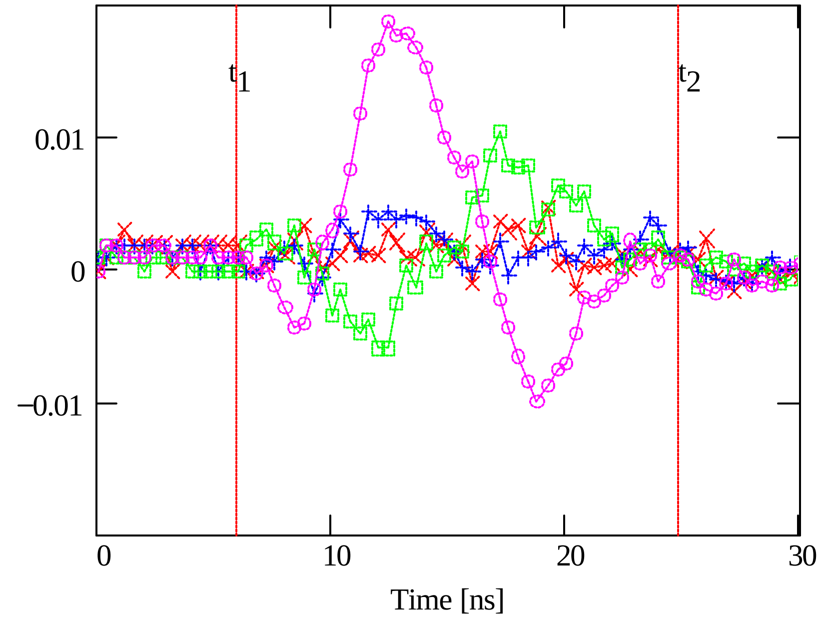



Signals of the vertical pickup and their deconvolution for turn 951. Measurements are performed before C0 lambertson magnet removal. Solid lines – unprocessed signals, dashed lines – deconvolution of the signals. The lattice chromaticities ($p dn/dp$) are: $n'_x$ = -1.75, $n'_y$ = -3.5; bunch population - $2.6{\cdot}10^{11}$. Vertical lines show boundaries of the RF bucket.

- Measurement are synchronized with inject.
- Chromaticities are set below zero so that the mode *l* = 0 would be unstable.




- Strong coupling between vertical and horizontal degrees of freedom results in the oscillations of the amplitudes with period about 57 turns.

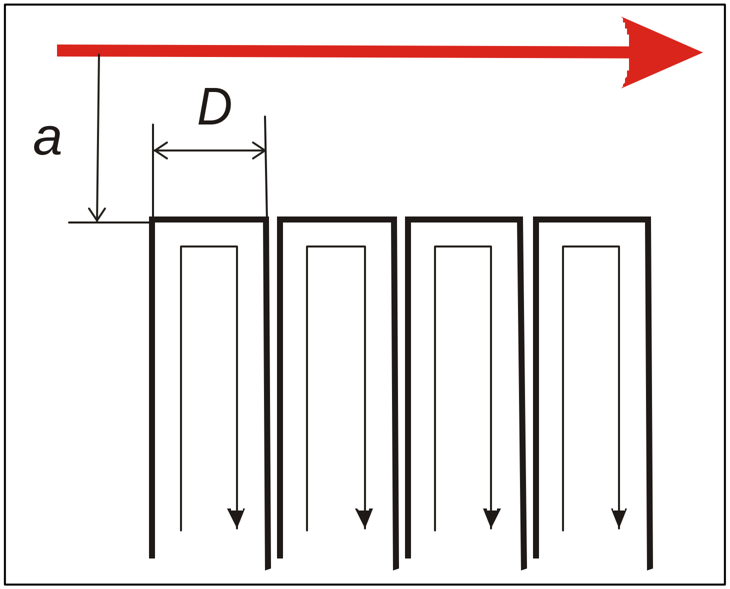



Dipole moments along the bunch for the same measurements as previous Figure. Curves are separated in time by 301 turns (about quarter of synchrotron oscillation).
´ - turn 0, + - turn 301,
- turn 602, - turn 903.

- In average the amplitudes exponentially grow with growth rate of 115±5 $s^{-1}$ (420 turns)

## Transverse impedance estimate

- Tevatron stainless steel vacuum chamber has a square cross section with $2h$ = 6 cm, $Z_\perp \sim 0.9$ MΩ/m at 100 MHz.
    - Comparison with numerical simulations for Gaussian beam yielded that the measured impedance value is about 5 times larger
- Two Lambertson magnets were identified as a major source of impedance. Its value can be estimated by integrating the resistance over the low frequency current passing through the laminas
- Impedance per unit length:




$$\Rightarrow \quad Z_\perp \approx \left[ (1-i) \frac{\gamma Z_o}{2\pi a^3} \cdot \frac{c}{\sqrt{2\pi\sigma\omega}} \right] \left[ \frac{2\sqrt{\mu a}}{d} \right]$$

where $Z_0 \approx 377$ Ω,

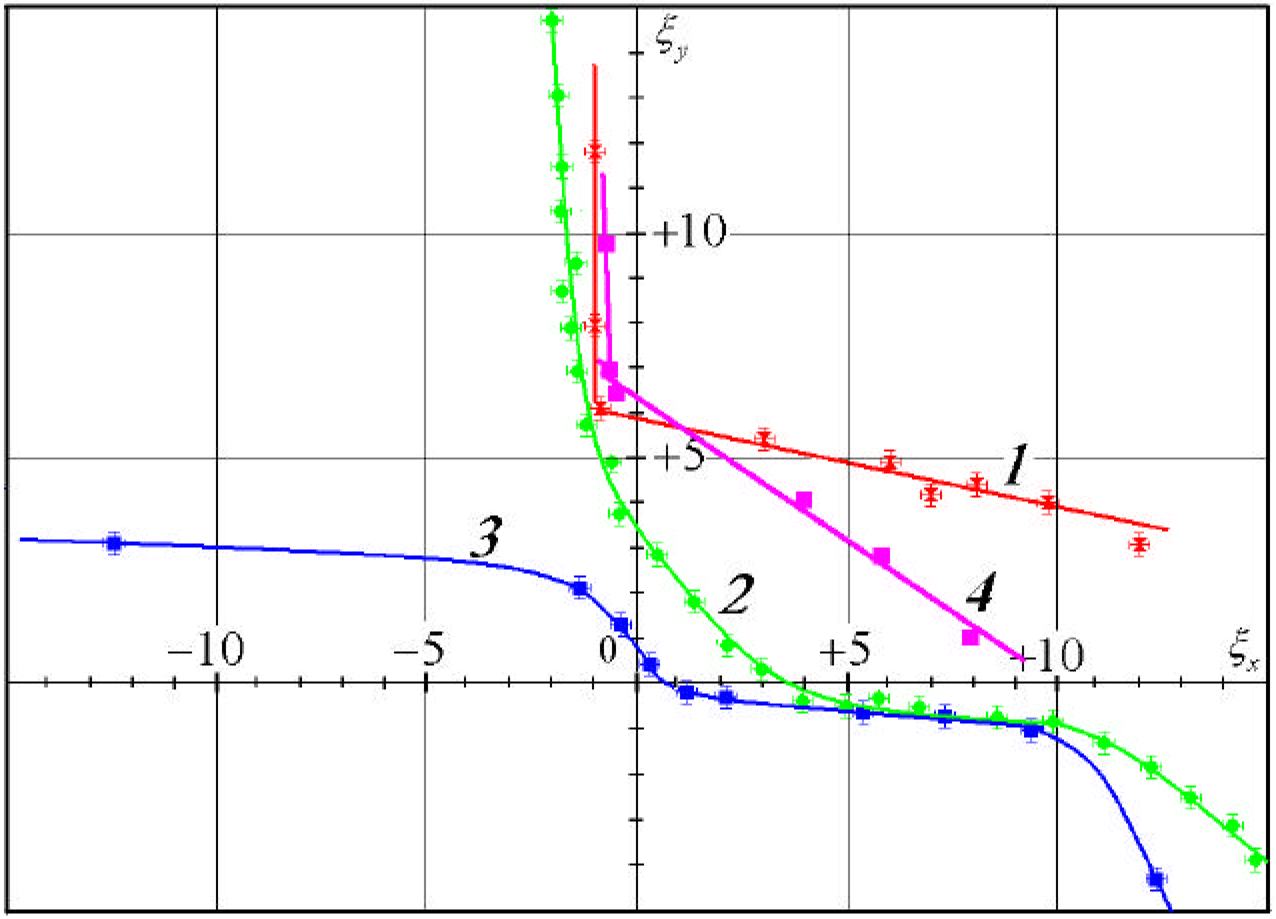
$d \approx 1$ mm is the lamination thickness,

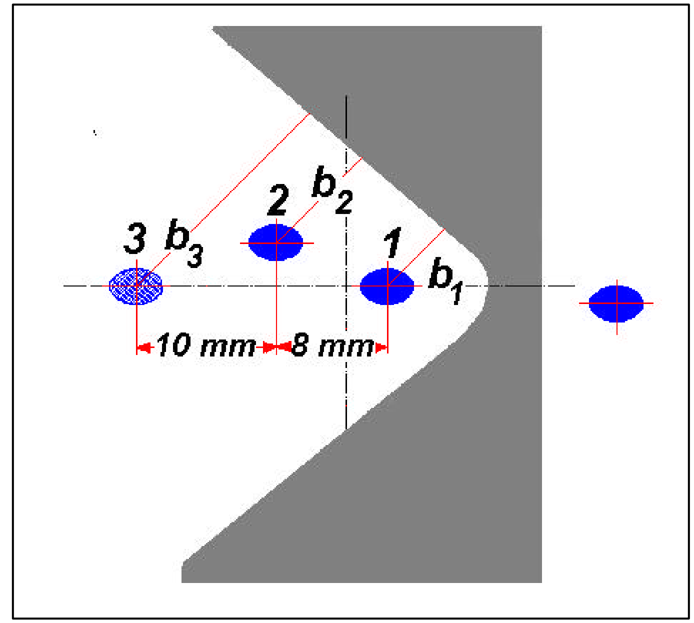
$\gamma \approx 0.5 - 1$ is a geometric form-factor.

- For $\mu$ = 200, a/d=10, the Lambertson aperture = half of the main vacuum chamber aperture, and total length of the magnet $L$ = 11.2 m

  $\Rightarrow$ each lambertson makes the same contribution as 6 km ring

## Stability region for the head-tail modes in the chromaticity space.




Measurements are performed with single proton bunch of $2.65 \cdot 10^{11}$ particles

4 – before C0 lambertson removal

2 – after C0 lambertson removal

1, 3 – orbit displaced in injection lambertson (after C0 removal)

## Growth rates of head-tail modes

- For air-bag (=hollow beam) distribution the single-bunch modes are described by a single head-tail wave number ***l*** with the transverse offset expressed as a sum over modes

$$x(\varphi) = \sum_l A_l \exp\left(il\varphi + i\chi\cos\varphi - i\omega_l t\right)$$

where $z = z_0 \cos\varphi$ is the longitudinal coordinate

$\chi = \nu' z_0 /(R\eta)$ is the head tail phase

- For coupled-bunch description with uniform bunch spacing the modes are described by two numbers: intra-bunch head-tail number ***l*** and multi-bunch number *m*. In this case,

$$x_n(\varphi) = \sum_{l,m} B_{lm} \exp\left(il\varphi + i\chi\cos\varphi + 2\pi nm/N - i\omega_{lm}t\right)$$

  - When bunches do not talk to each other, the eigen-frequencies do not depend of the multi-bunch mode number: $\omega_l = \omega_{lm}$

- For Tevatron the single-bunch modes are driven by the high-frequency impedance, $w \geq c / s_z$ or $f$ > 50 MHz, while the coupled-bunch modes are related to much lower frequency range of the impedance, $w_0 \leq w \leq N w_0$ . In general, the growth rates are

$$\Lambda_{lm} = \Lambda_l^s + \Lambda_{lm}^c$$

  - where for air-bag model the single- and coupled-bunch terms are

$$\Lambda_l^s = -\frac{N_b r_0}{2 p Z_0 g n_b} \int_{-\infty}^{\infty} dw \,\mathrm{Re}\, Z(w) J_l^2 (w z_0 / c - c)$$

$$\Lambda_{lm}^c = -\frac{N N_b r_0 w_0}{2 p Z_0 g n_b} J_l^2(c) \sum_{p=-\infty}^{\infty} \mathrm{Re}\, Z\big(w_0 (n_b + pN + m)\big)$$

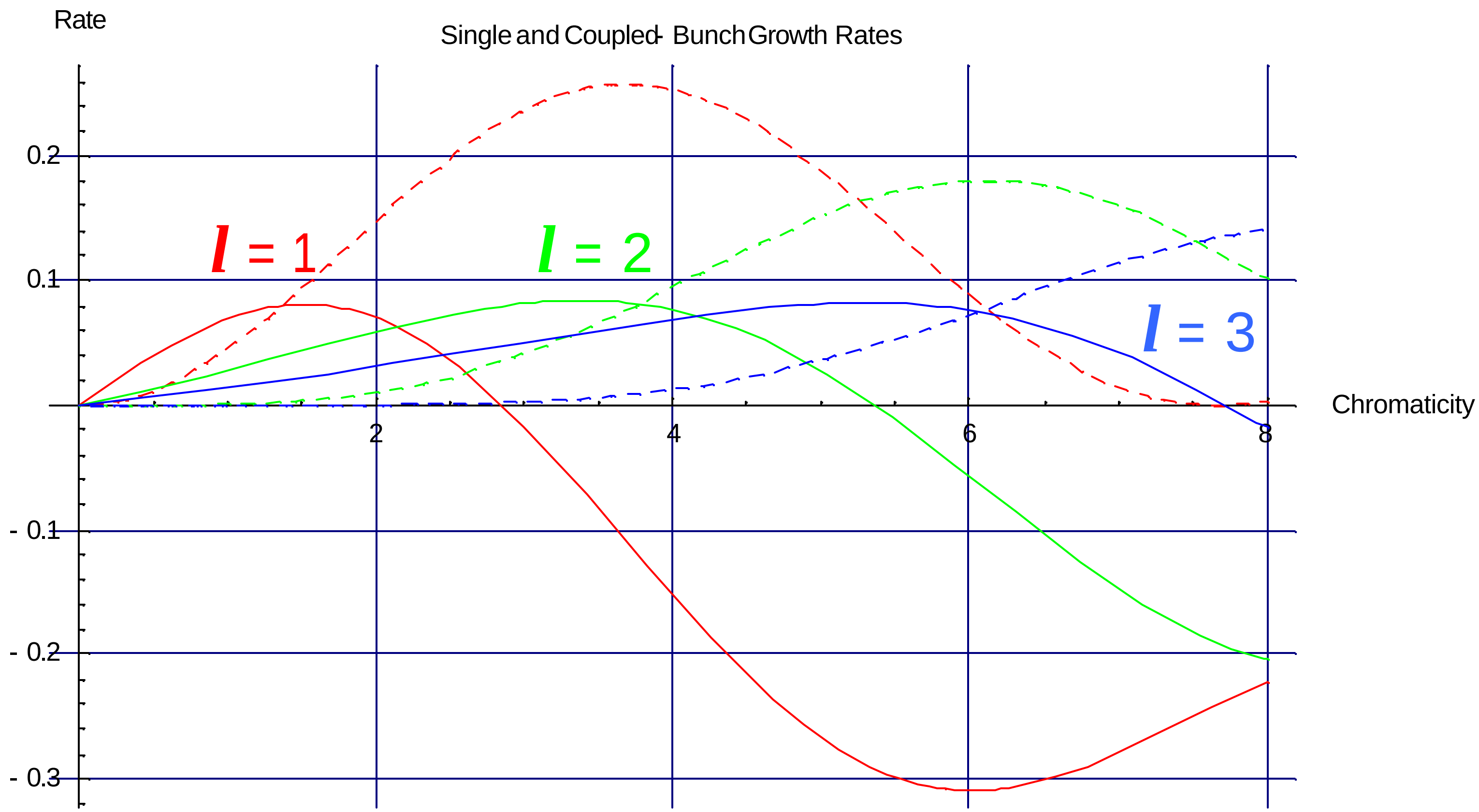
- The resistive wall impedance is slowly decreasing, $1/\sqrt{w}$. In the case of Tevatron that makes both contributions comparable. They are

$$\Lambda_l^s = \hat{\Lambda} \int_0^{\infty} \frac{dw}{\sqrt{w}} \left[ J_l^2 (w - c) - J_l^2 (w + c) \right]$$

$$\Lambda_{lm}^c \approx \hat{\Lambda} N J_l^2(c) \sqrt{\frac{z_0}{\big(N - [n_b] - m\big) R}} \quad , \quad N - [n_b] - m > 0$$

only one term making largest contribution in the sum for $\Lambda_{lm}^c$ is left

# Single and Coupled bunch Growth rates as functions of chromaticity




Single bunch (solid lines), $\Lambda_l^s$, and most unstable Coupled-Bunch (dashed), $\Lambda_{lm_0}^c$, growth rates for ***l=1*** (red), ***l=2*** (green) and ***l=3*** (blue)

## Other improvements in theoretical description

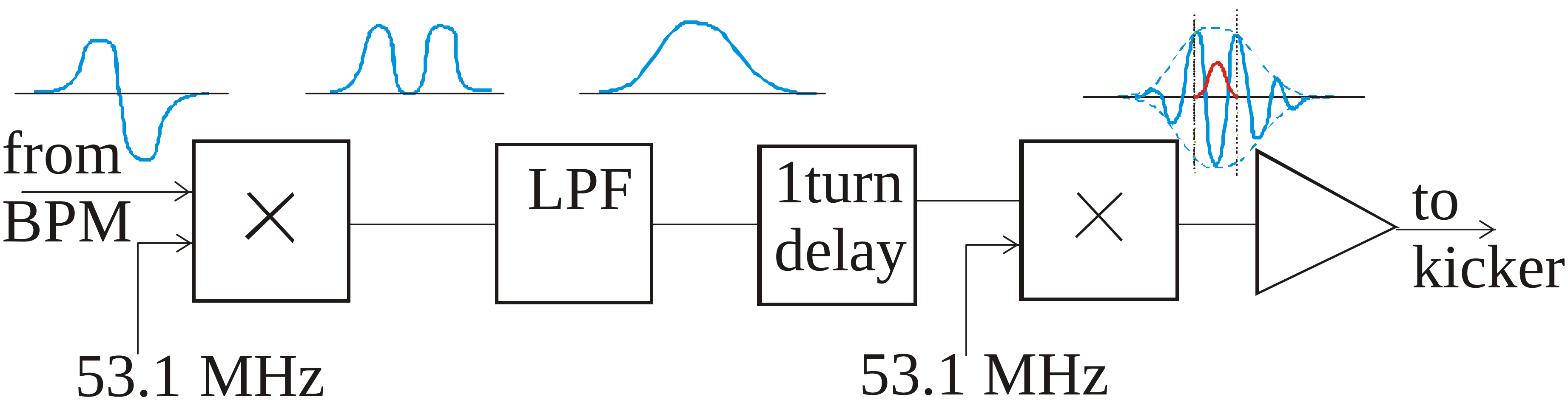
- To achieve better accuracy of the model two additional improvements of the theory have been taken into account
  - Numerical multi-particle simulations were carried out to get more accurate result for the instability growth for the gaussian distribution (instead of air-bag distribution)
    - Resistive wall impedance have been used
  - Coupling has been taken into account

$$l \propto \langle Z b \rangle \qquad \Rightarrow \qquad \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \propto \begin{bmatrix} \langle b_{1x} Z_x + b_{1y} Z_y \rangle_s \\ \langle b_{2x} Z_x + b_{2y} Z_y \rangle_s \end{bmatrix} \equiv \begin{bmatrix} \langle \mathbf{ß}_1 \mathbf{Z} \rangle_s \\ \langle \mathbf{ß}_2 \mathbf{Z} \rangle_s \end{bmatrix}$$

where Mais-Ripken beta-functions are used

# 3. Suppression Head-tail Instability in Tevatron by transverse damper

- Large chromaticity has been used to suppress the instability
  - Problems with beam-beam and dynamic aperture
- Transverse damper was designed to suppress bunch-by-bunch modes
  - Unexpectedly it also helped with the head-tail instability
  - It has moved the boundary chromaticity from ~6 to ~4 units
    - Thus, experiment verified that the damper damps the single bunch head-tail instability




Damper schematic

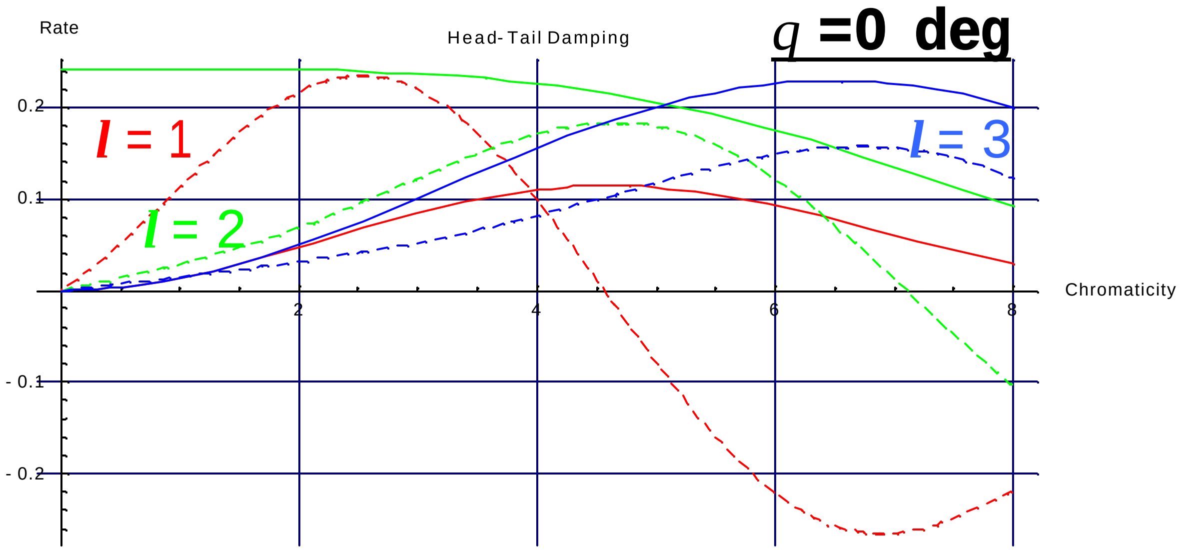
- For air-bag distribution, the damping rates are

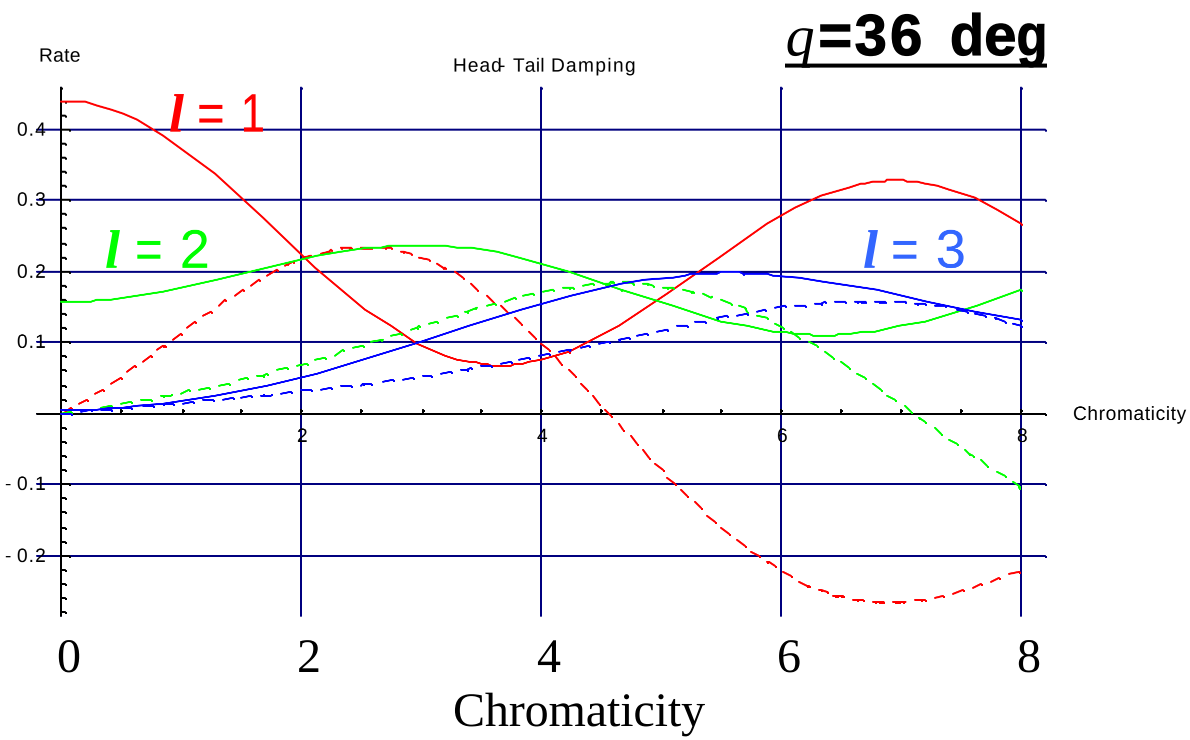
$$\Gamma_l = g\left|J_l(c+q)e^{iq} + J_l(c-q)e^{-iq}\right|^2$$

$q = w_{RF} z_0 / c$ - the phase advance of modulation frequency, $w_{RF}$

- The modulation is assumed to be
  - as $\propto \sin(qz/z_0 + q)$ at the pickup
  - and $\propto \cos(qz/z_0 + q)$ at the kicker
- The phase shift $q$ is a parameter for optimization.
- This scheme makes all the head-tail modes damped simultaneously

# Dependence of damping rates on chromaticity




Solid lines - damper

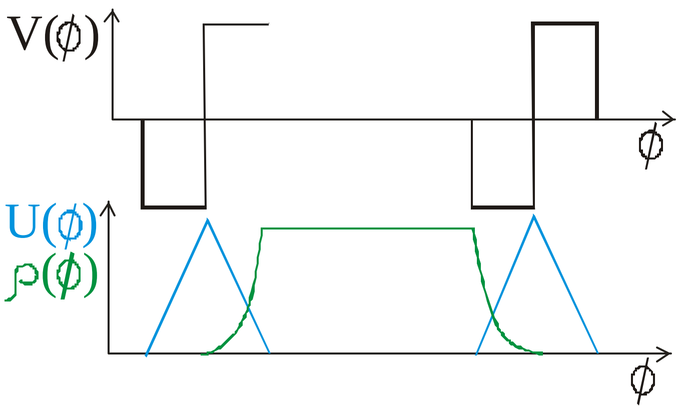
Dashed lines – instability

- For $q$=0, damping rates of odd modes (l=1,3…) vanish at low chromaticity, $n'$, as $n'^2$, while the head-tail rates go down linearly
  - The main stopper is the lowest-order odd mode, $l$ =1
- At optimal $q$=36° all the modes can be effectively damped for all chromaticities

# *4. Resistive wall instability in Recycler*

## Recycler parameters

- High current $\bar{p}$ accumulator with stochastic & electron cooling
- Kinetic energy — 8 GeV
- Circumference — 3328 m
- Tunes, $n_x / n_y$ — 25.58/24.42
- Number of particles — $(1.2 \to 6)\cdot 10^{12}$
- Number of bunches — 1 - 9
- Bunching factor — 0.2 - 0.8
- RF type — Barrier bucket



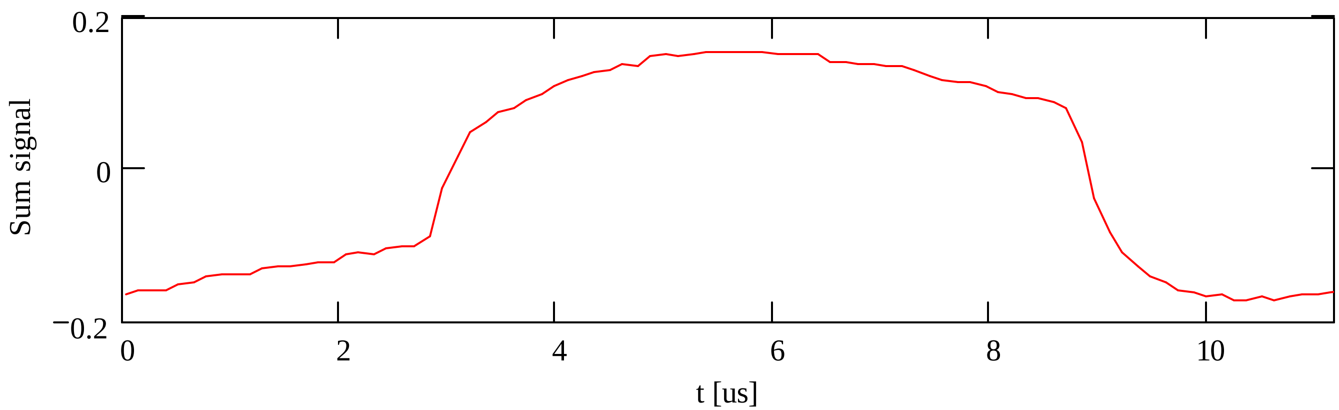

♦ Very first experimental observations showed that if machine chromaticity is reduced close to zero and the beam is sufficiently cold there appears a transverse instability

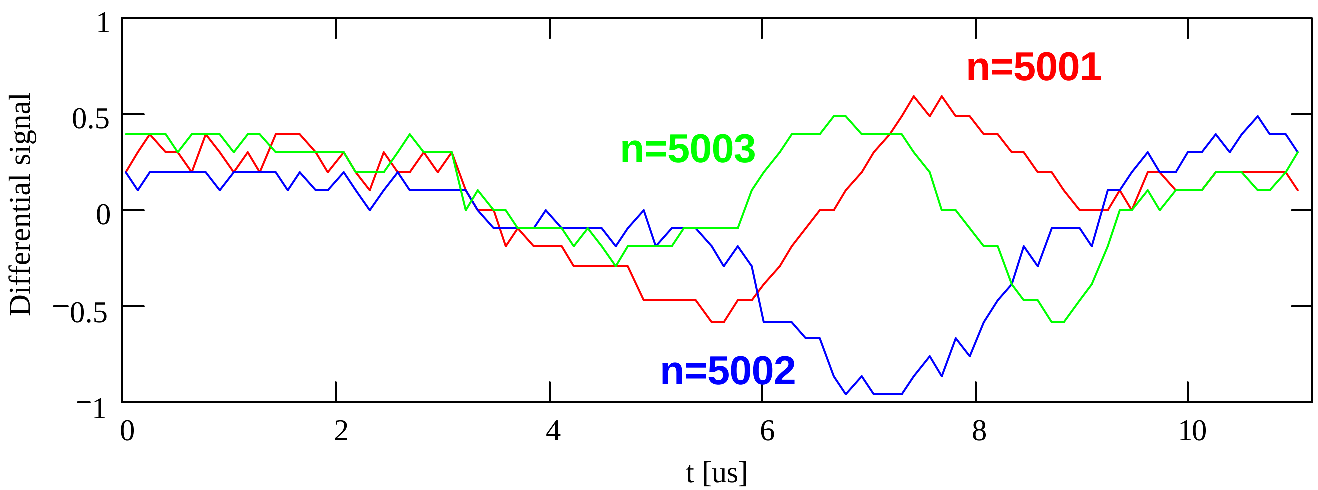
➢ It caused the beam loss but the emittance after the beam was stabilized was much smaller than it could be expected from the acceptance

➢ The origin of the instability was unknown => detailed studies

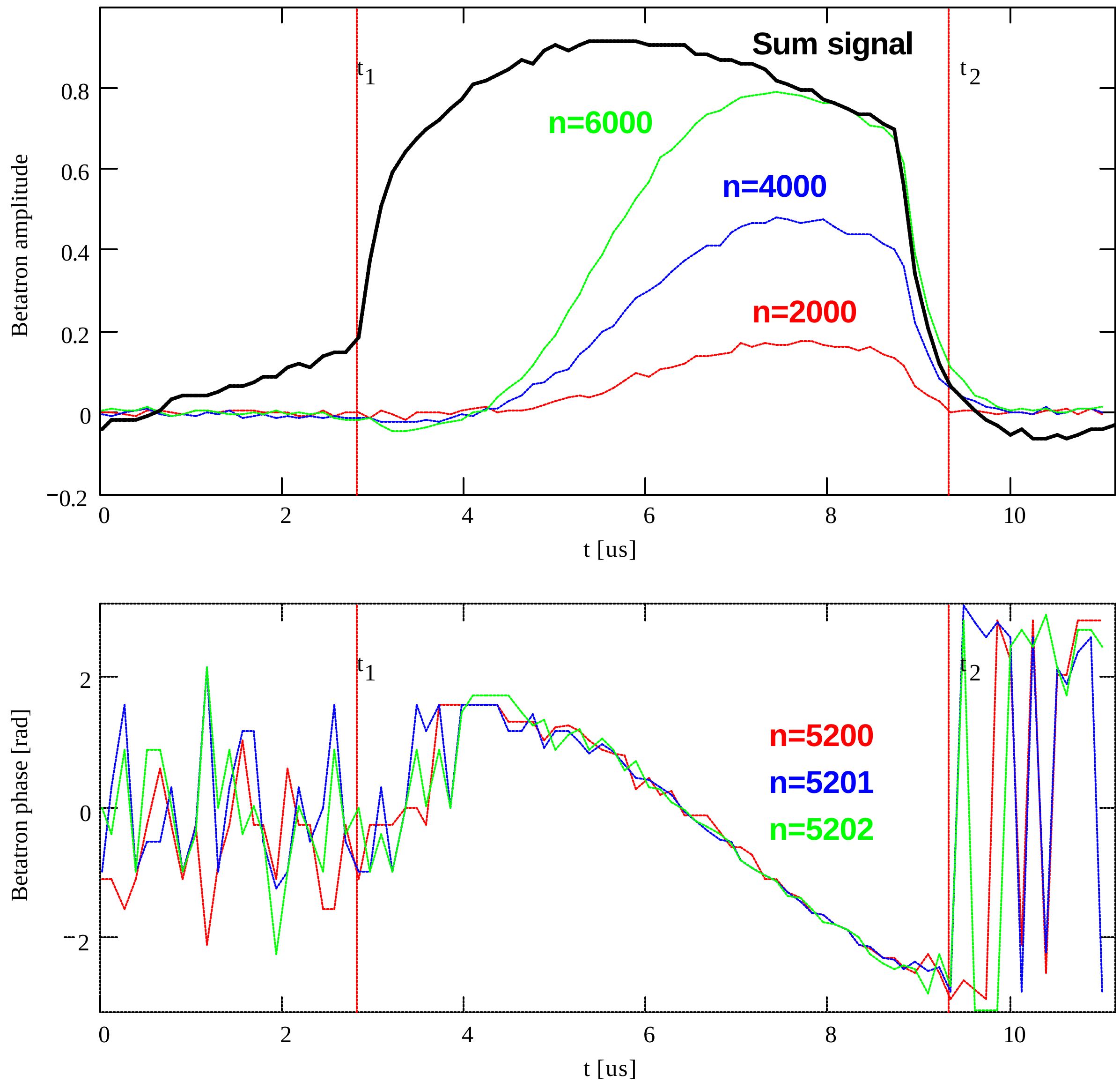
## Experimental results

- A fast digital oscilloscope connected to the sum and differential outputs of vertical pickups
  - Continuous record ~90,000 turns 128 ns sampling time



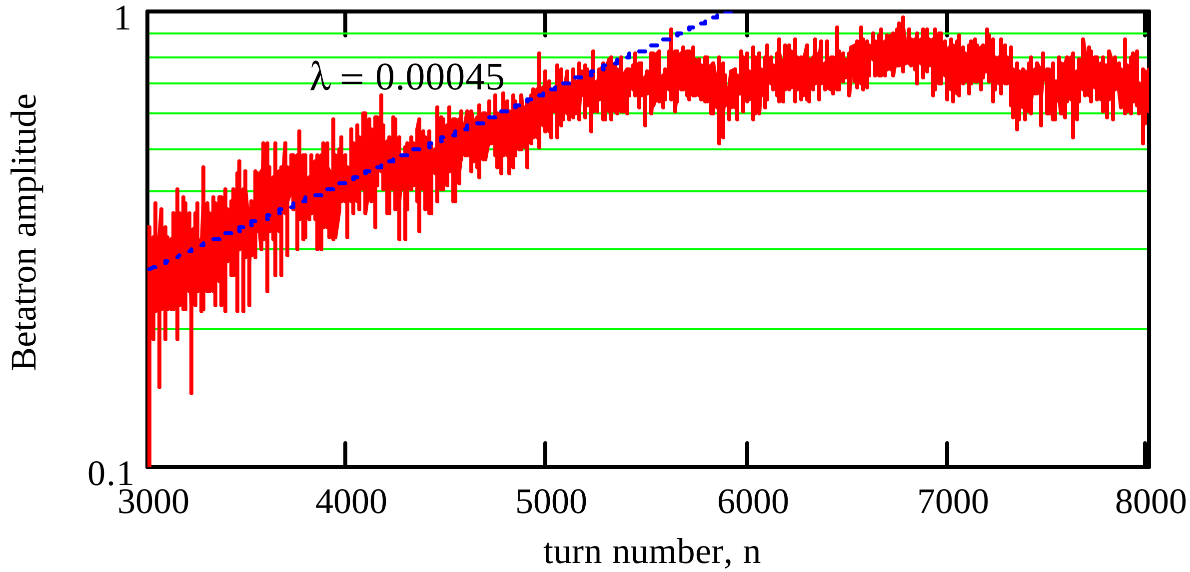



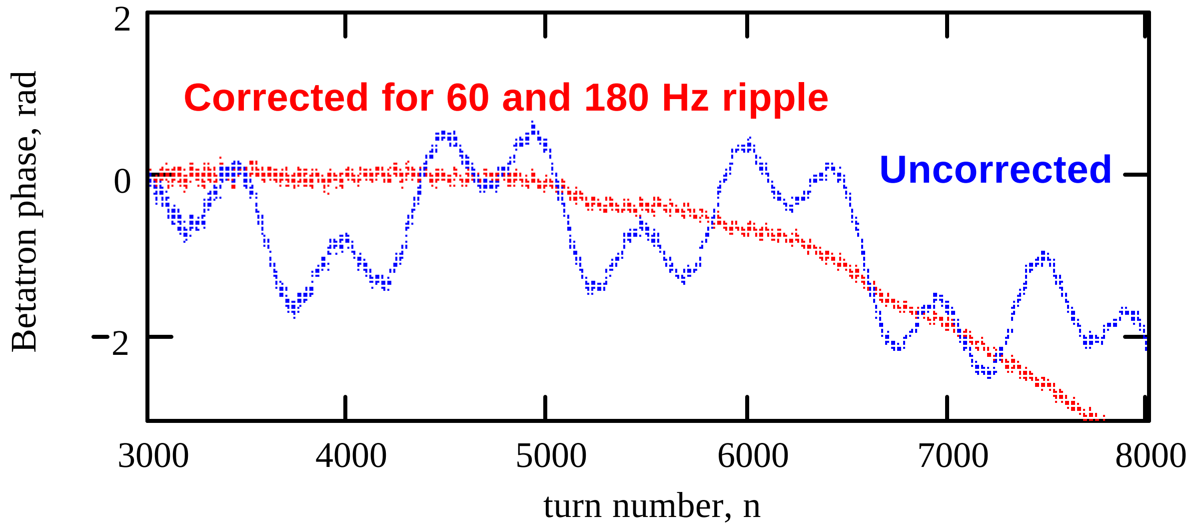

Raw BPM signals




Betatron amplitudes and betatron phases along the bunch for chosen turns

- Betatron amplitudes and phases along the bunch were computed from three consecutive turns
- There are very little motion in the bunch head
- The maximum amplitude is achieved at ~2/3 of bunch length




Dependence of betatron amplitude and phase on turn number

- Betatron amplitude grows exponentially while it is smaller than the aperture
- Betatron phase has ripple at power line harmonics
  - Corresponding tune variations are: $\Delta\nu_{60}=4.6\cdot10^{-4}$, $\Delta\nu_{180}=3.2\cdot10^{-4}$
- Particle loss stabilizes instability and causes betatron phase slip

## Theoretical model

- Barrier bucket RF makes flat density along the bunch
- Bunch is so cold that the longitudinal particle displacement in the course of instability development is much smaller than the bunch length
- Tail-to-head feedback creating the instability is carried out through one turn delay

$$\frac{d^2x(s,t)}{dt^2}+\left(w_b{}^2-\frac{e^2N_b}{Mg}\tilde{D}(s)\right)x(s,t)=\frac{e^2N_b}{Mg}\tilde{W}(s)\,,$$

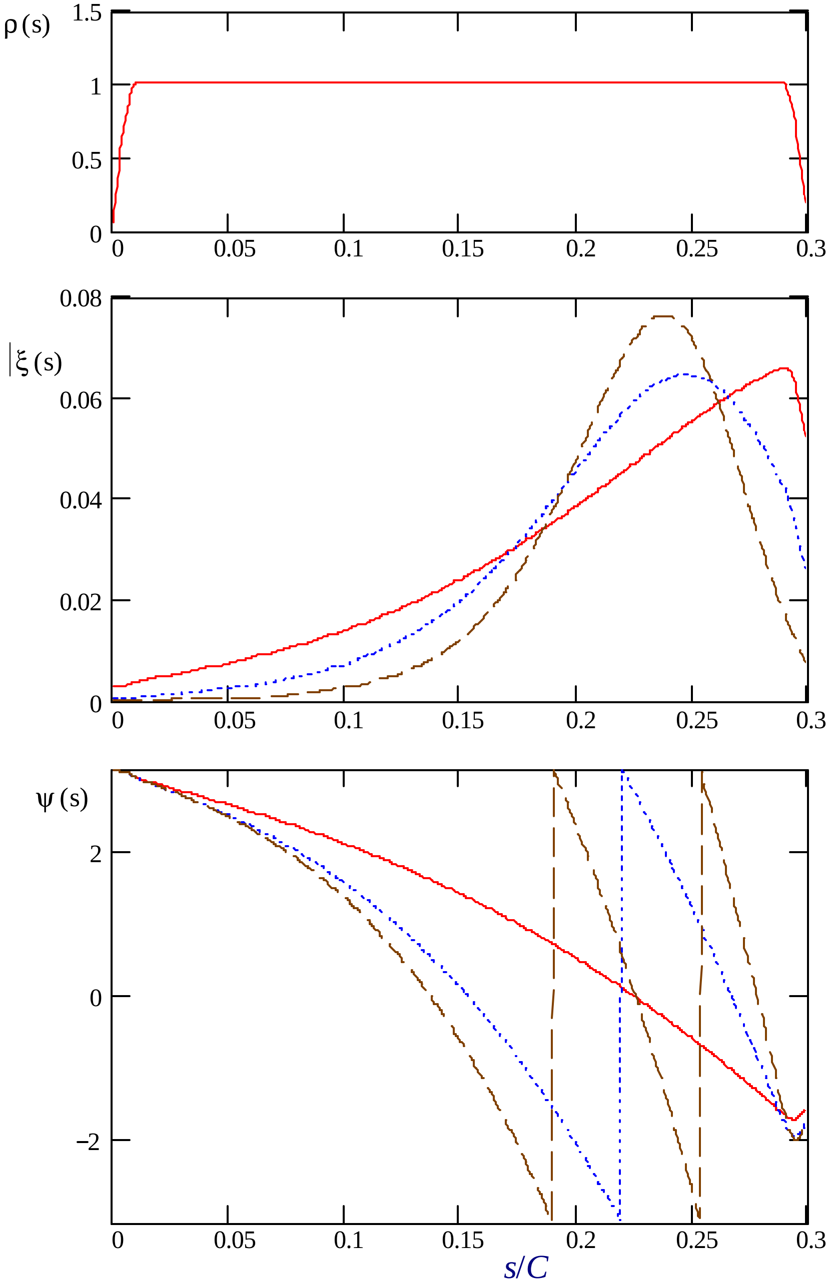
$$\tilde{D}(s)=\int_s^L D(s'-s)r(s')ds'+\sum_{n=1}^{\infty}\int_0^L D(s'+nC-s)r(s')ds'\,,$$

$$\tilde{W}(s)=\int_s^L W(s'-s)r(s')x(s',t-\frac{s'-s}{v_0})ds'+$$

$$\sum_{n=1}^{\infty}\int_0^L W(s'+nC-s)r(s')x(s',t-\frac{s'+nC-s}{v_0})ds'\,.$$

- Solution was carried out numerically for the resistive wall impedance
  - Approximation of flat vacuum chamber has been used

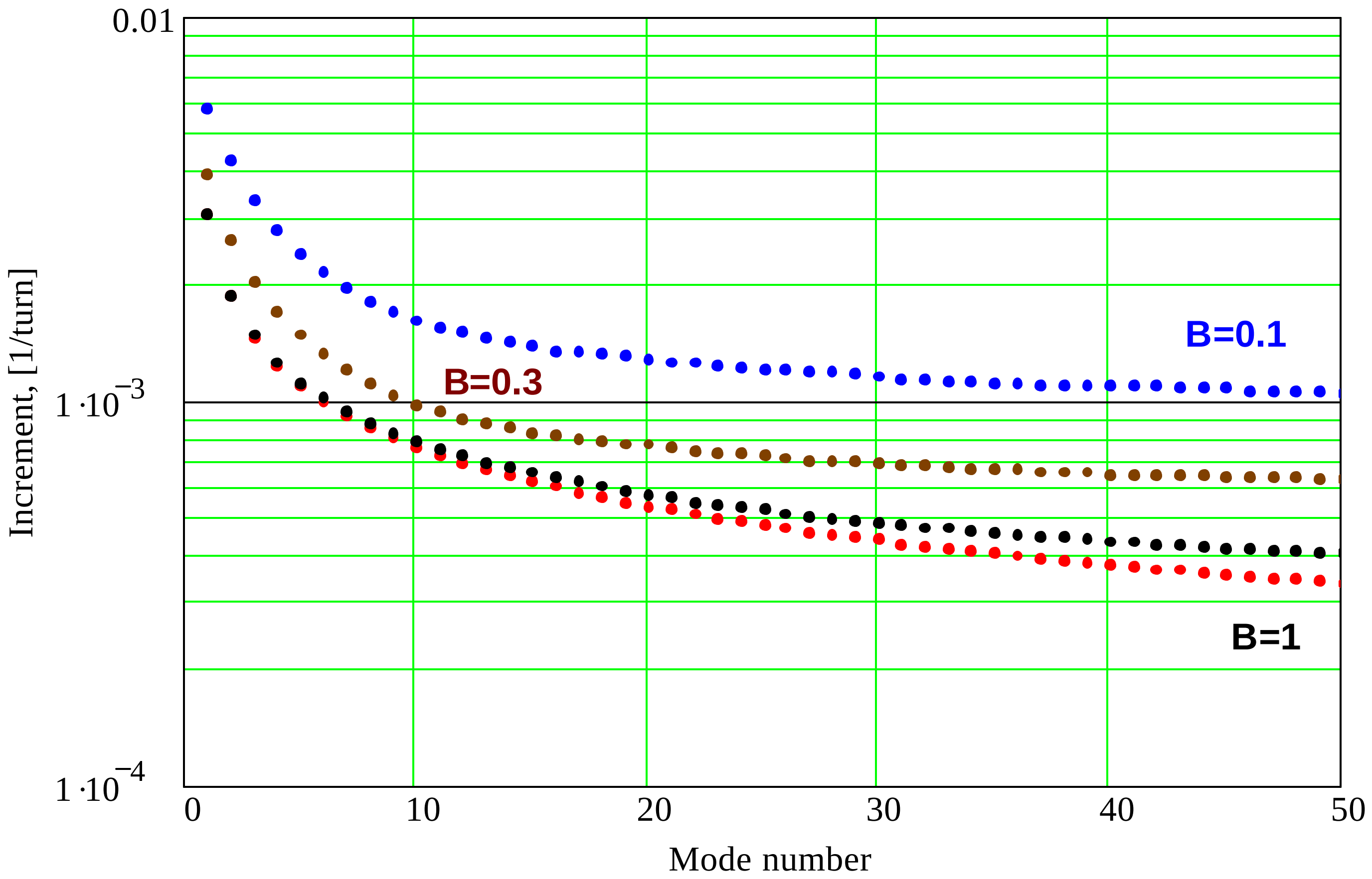
$$W_v(s)=2W_h(s)=\frac{pC}{6a^3}\sqrt{\frac{v_0}{s\ \ s}}\quad ,\qquad D_v(s)=-D_h(s)=\frac{pC}{12a^3}\sqrt{\frac{v_0}{s\ \ s}}$$




Amplitudes and phases along the bunch for first 3 unstable modes

- Detuning (quadrupole) wake makes tunes dependent on position along the bunch
  - It is responsible for the fact that maximum amplitude is achieved at 2/3 of bunch length
  - Without detuning wake the maximum amplitude would be achieved at the bunch tail
- Good agreement between simulation results and the experimental measurements for both the instability growth rate and mode structure of the lowest mode

- 600 macro-particles have been used in numerical simulations
- Good agreement with analytical model for B=1 (continuous beam)
  - Divergence for large mode numbers is related with insufficient number of particles per oscillation length

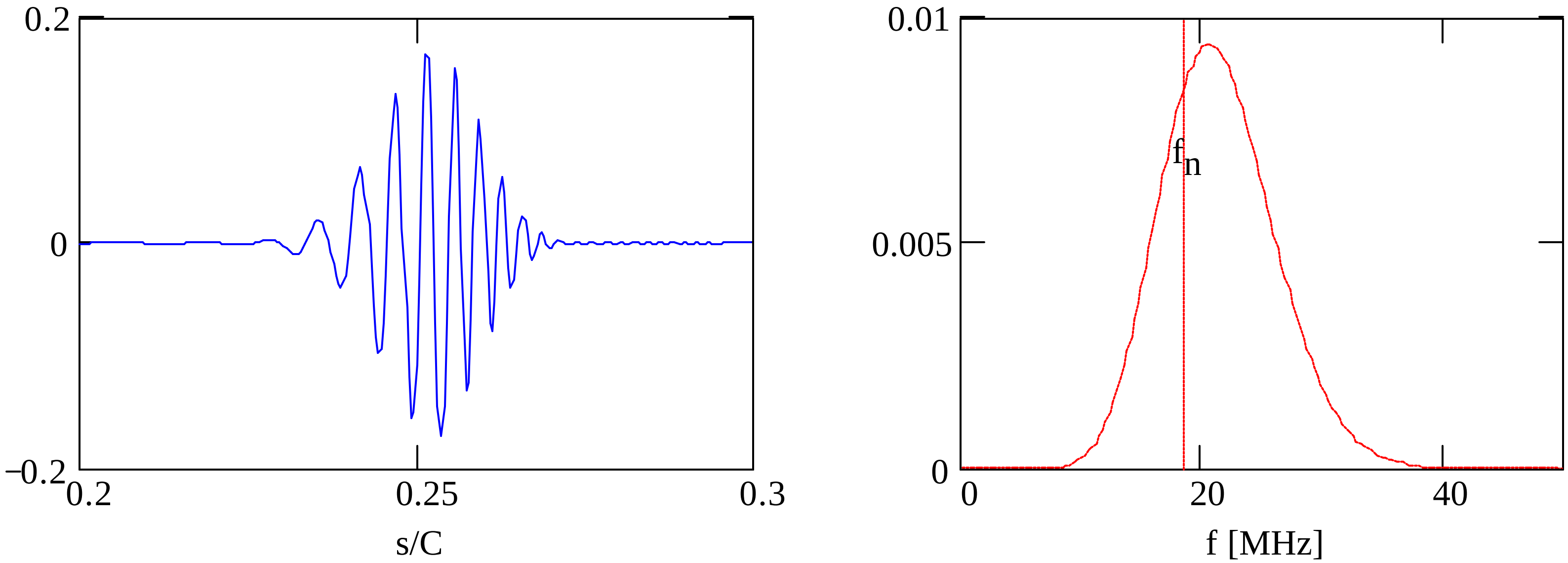



- For the same number of particles the increment grows with bunching

$\propto 1/B^{1/4}$

2 particle model

Instability growth rates of unstable modes for different bunching factors, $B$; $N_b = 6\cdot10^{12}$.




Structure and spectrum of mode $m$ = 101, $B$ = 0.3.

- Maximum amplitude is achieved at 0.25/0.3 ~ 0.83 of bunch length
- Oscillation frequency grows from head to tail
- Maximum of spectral density is achieved at

$$f_n \approx \frac{2.5n}{B} f_0$$

- For given mode number the bunching moves both the mode frequency and the mode growth rate to higher values
  - More rigid requirements for the feedback system to control instability

## Requirements for instability damping

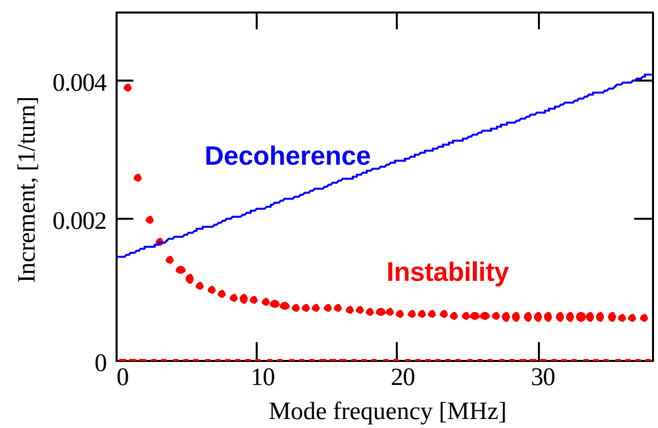
- Tune spread is a basic mechanism for beam stabilization

$$\Delta n_n \approx (h\,n - n')\frac{\Delta p}{p}$$

- Space charge (incoherent) tune shift is expected to be very large $\Delta\nu_{sc} \approx 0.03 - 0.1$
  - It will suppress Landau damping due to tune spread up to very high frequencies

$$\Delta n_n \approx \Delta n_{sc} / F_{sc} \quad , \quad F_{sc} \approx 3-6$$

  - As result, the required frequency band of the instability damper goes to well above 100 MHz



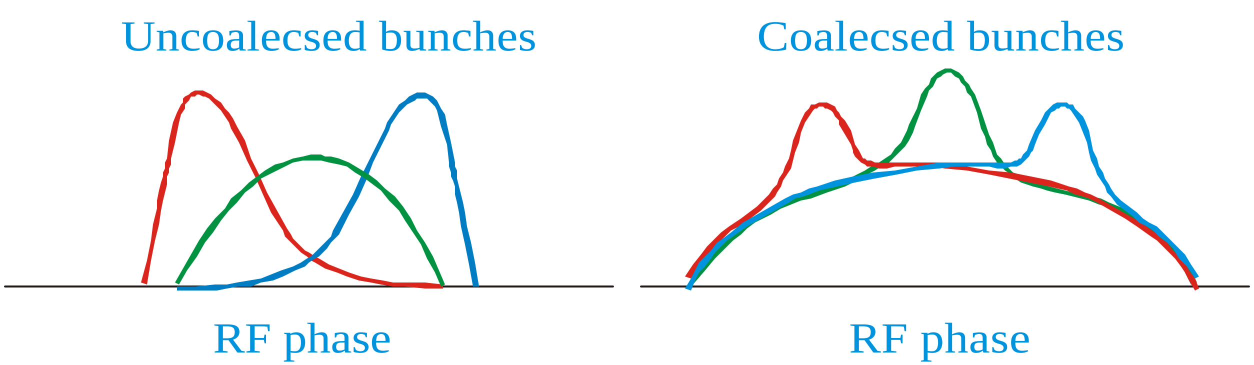

$N_b = 6\cdot10^{12}$, $B = 0.3$,
$\varepsilon_{||} = 50$ eV s, $\Delta p/p = 7\cdot10^{-4}$,
$\nu' = -2$

## Conclusions and plans

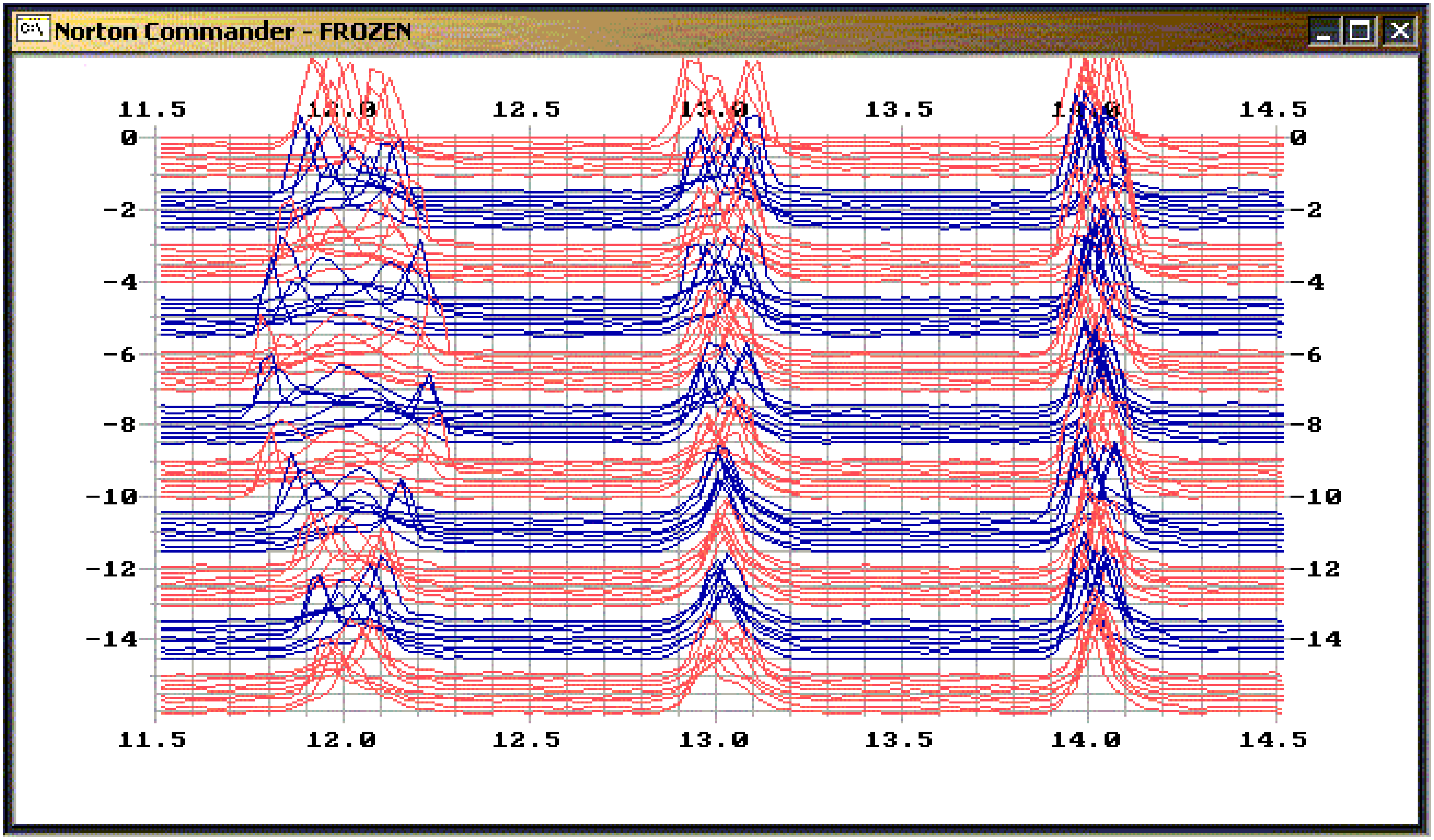
- Presently, instabilities do not produce severe limitations on the collider luminosity
- Transverse instabilities in Tevatron and Recycler are well understood
  - We plan further reduction Tevatron chromaticity
    - Introduction of cubic (octupole) non-linearities is main direction
    - Further improvements of head-tail damper may be required if we will encounter operational difficulties with octupoles
  - To suppress Recycler instability we plan to built two band transverse damper: 10 kHz – 10 MHz and 10 MHz – ≥200 MHz
- Longitudinal instability in Tevatron is presently stabilized by bunch-by-bunch damper
  - To make shorter bunches we need better understanding how it works and how it can be suppressed

# Bunch dancing in Tevatron[2]

- Long-term coherent synchrotron oscillations of proton bunches are observed in Tevatron.
- Bunch shape at the oscillations differs for uncoalesced and coalesced bunches.
  - Uncoales. bunches – osc. persist for hours
  - Coales. – oscillations decay in ~5 min.
- Longitudinal bunch-by-bunch damper accelerates the damping




[2] a) Ronald Moore, *et.al.*, Longitudinal Bunch Dynamics in the Tevatron
b) C.Y. Tan, *et. al.*, The Tevatron Bunch by Bunch Longitudinal Dampers



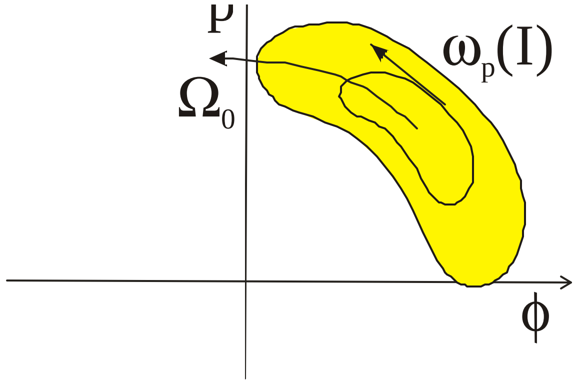

- Without coherent interactions the synchrotron tune spread would damp oscillations within seconds

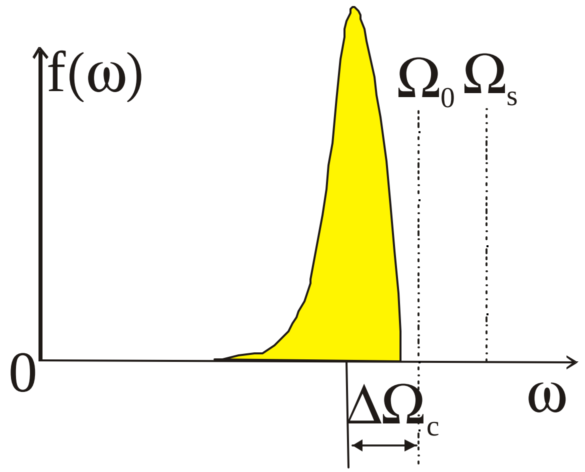
- Effect of **inductive** longit. impedance separates coherent and incoherent tunes and prevents decoherence at[3]

$$|\Delta\Omega| > \delta\Omega_c$$

  - where $\delta\Omega$ is synchrotron tune spread,
  - $\Delta\Omega_c$ - coherent tune shift produced by the impedance.
- For Tevatron at 150 GeV it yields:

$$|Z/n|[\Omega] > 2 \cdot 10^{11} f^{5}/N \approx 1\ \Omega$$




[3] V.Balbekov, S.Ivanov, 1991