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INTERMODULATION DISTORTION IN THE STOCHASTIC COOLING SYSTEM

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Abstract

Nonlinearities in the amplifiers used in the stochastic cooling system cause intermodulation distortion in the signal applied to the beam. I study here the core heating due to the intermodulation distortion in the stack tail system. The width of the core is obtained as a function of the intermodulation strength in the stack tail system and of the gain in the core cooling system by solving Fokker–Planck equation both analytically and numerically.

1. THEORY

In the stochastic cooling system the signal from the pickup is amplified and sent to the kicker which applies it to the beam. If the amplifier circuit were linear, the signal at the kicker would be identical with the one at the pickup. The real amplifier, however, always has nonlinear gain which causes the so called intermodulation distortion. It distorts the signal at the frequency ω by adding to it an integral over all the sums and differences of pairs of the frequencies in the passband (second order intermodulation distortion), the sums and differences of groups of three frequencies in the passband (third order intermodulation distortion) *etc.* As a consequence, the intermodulation distortion in the stack tail system heats the core by "filling up" the notches at the Schottky frequencies corresponding to the core energy. This contributes to the D term (the heating term) in the Fokker–Planck equation[2] describing the beam profile. The effect of this heating on the core can be estimated by solving the Fokker–Planck equation:

$$\frac{\partial \Psi}{\partial t} = -\frac{\partial}{\partial E}[F(E)\Psi(E) - D(E)\frac{\partial \Psi}{\partial E}]. \tag{4}$$

Stationary (or asymptotic) stack profile $\Psi(E) = \lim_{t\to\infty}\Psi(E,t)$ will be achieved when the cooling term balances out the heating term. It can be obtained by taking the time derivative in this equation equal to 0, or

$$F(E)\Psi(E) - D(E)\frac{d\Psi}{dE} = K = \text{const.} \tag{5}$$

Since the intermodulation distortion heats the core more or less evenly, we will take D independent of energy. Eq. (5) can be written as

$$d\Psi - \frac{1}{D}[F(E)\Psi(E) + K]dE = 0.$$

This equation can be integrated by means of the integrating factor $\lambda = e^{-\frac{1}{D}\int F(E)dE}$. In the vicinity of the central energy E_0, $F(E) \approx f(E_0 - E)$, f positive constant, and the integrating factor is $\lambda = e^{\frac{f}{2D}(E-E_0)^2}$. The solution is now obtained in the form $u(E,\Psi)$ =const, with

$$\frac{\partial u}{\partial \Psi} = \lambda,$$

and

$$\frac{\partial u}{\partial E} = -\frac{\lambda}{D}[F(E)\Psi(E) + K].$$

From the first of these equations we have

$$u(E,\Psi) = e^{\frac{f}{2D}(E-E_0)^2}\Psi + g(E),$$

and the unknown function g is to be fixed from the second equation, which gives

$$g'(E) = -\frac{K}{D}e^{\frac{f}{2D}(E-E_0)^2}.$$

The solution for Ψ is then

$$\Psi(E) = [\text{const} - g(E)]e^{-\frac{f}{2D}(E-E_0)^2}.$$

If $K > 0$, g' grows beyond bounds as $E \to \pm\infty$ and for some energy Ψ becomes negative, which is not allowed since Ψ is the probability density. If $K < 0$, g' goes in the other direction and Ψ becomes non normalizable. We conclude that the only allowed value for K is 0, *i. e.* $g(E)$ =const, and we finally obtain for Ψ the Gaussian solution:

$$\Psi(E) = Ae^{-\frac{f}{2D}(E-E_0)^2}. \tag{6}$$

The constant A is obtained from the normalization condition $\int \Psi(E)dE$ = total number of particles in the stack.

We need to determine how the width of this distribution changes with D in dB. The width of the distribution σ is $\sqrt{D/f}$. Define $\tilde{D} = 10 \log \frac{D}{D_0}$ the increase in dB in heating over some reference level D_0. The core width increases as

$$\sigma = \sigma_0 \times 10^{\frac{\tilde{D}}{20}}, \tag{7}$$

where σ_0 is the width for $D = D_0$. Thus we expect the core width to grow exponentially with the strength of intermodulation distortion expressed in dB. This prediction is in excellent agreement with the more realistic numerical solutions of the Fokker-Planck equation, Figs. 5 to 8.

2. NUMERICAL INTEGRATION OF FOKKER-PLANCK EQUATION

In order to obtain more quantitative results we compute the beam profile using the Fermilab Stochastic Cooling Code[3] for a range of values of the D_1 term at the core energy and for various values of injection rate. The system with no intermodulation distortion is defined as 0 dB. For the present values of the injection rate ($1.5 \times 10^7 \bar{p}/2.2$sec) I compute the beam profile over 10 hours of stacking. The final core width in MeV as a function of the intermodulation strength is shown in Fig. 5. In the range between 5 and 10 dB we expect widening of the core by a factor between 1.5 and 2.

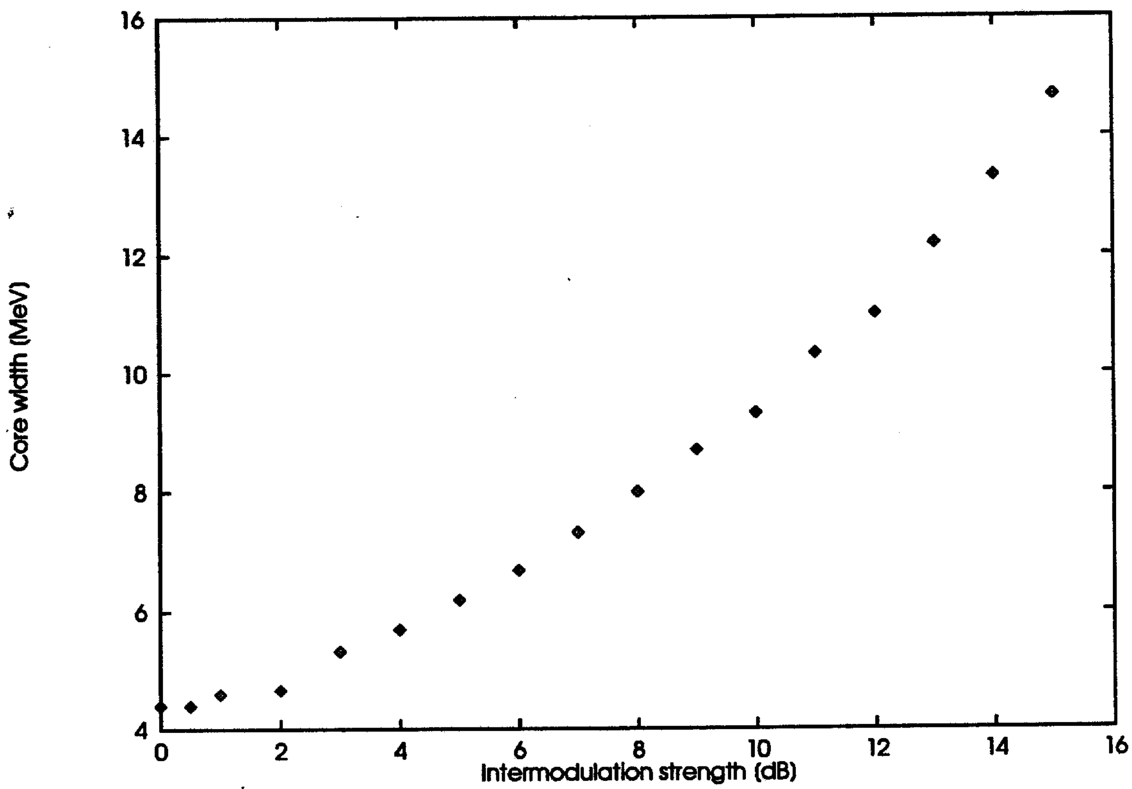

Fig. 5 The core width after 10 hours of stacking as a function of the intermodulation strength.

In the future we expect the injection rate to increase from the present $1.5 \times 10^7/2.2$sec to about $2 \times 10^7/2$sec, and since the extraction is done during stacking, the stacking can go on for more than a week at the time. The question is what is the effect of the additional core heating on the beam profile on the time scale of 100 hours. Fig. 6 shows the core width after 10 and after 100 hours as a function of intermodulation strength.

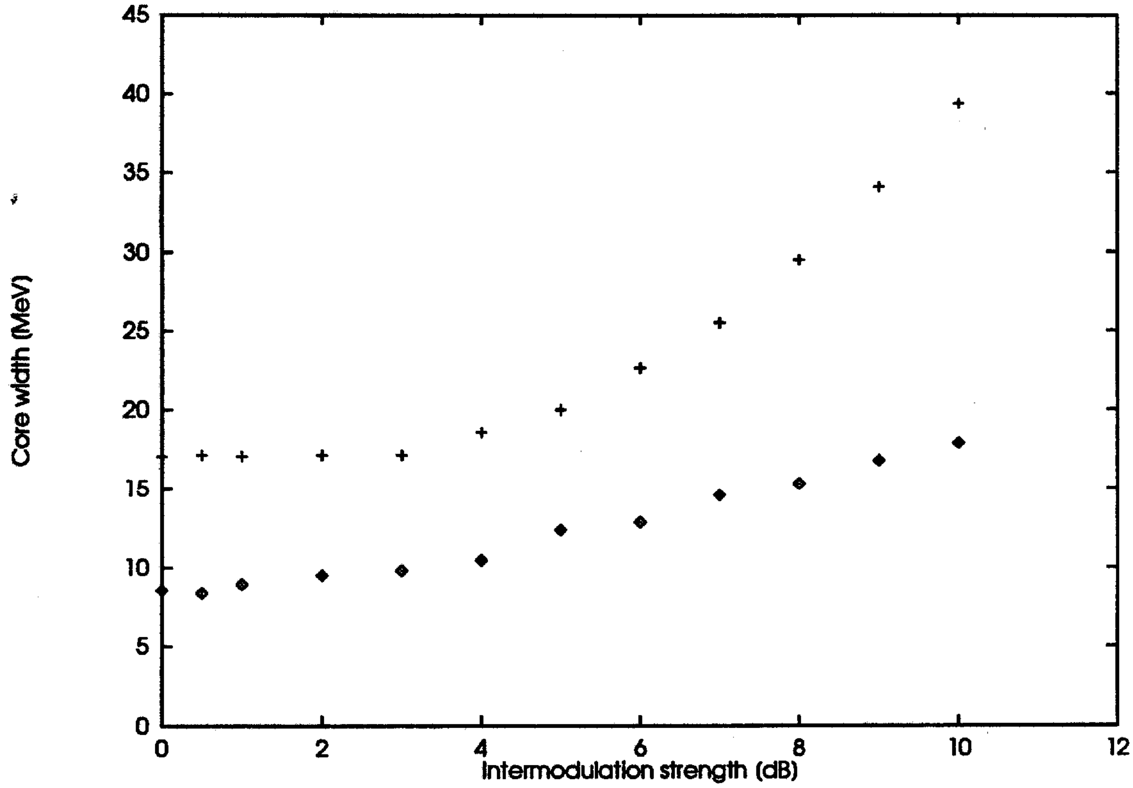

Fig. 6 The core width after 10 and 100 hours of stacking as a function of intermodulation strength.

Can we compensate the core heating from the stack tail system by increasing the gain in the core cooling system? The computation indicates that this might be the case. We have done a series of computations for the values of intermodulation distortion of 4, 6, 8 and 10 dB and the core cooling gain increased 4, 6 and 8 times. The results are shown in Fig. 7 for 10 hrs of stacking and in Fig 8 for 100 hrs. Clearly, there is a significant improvement in the stack profile.

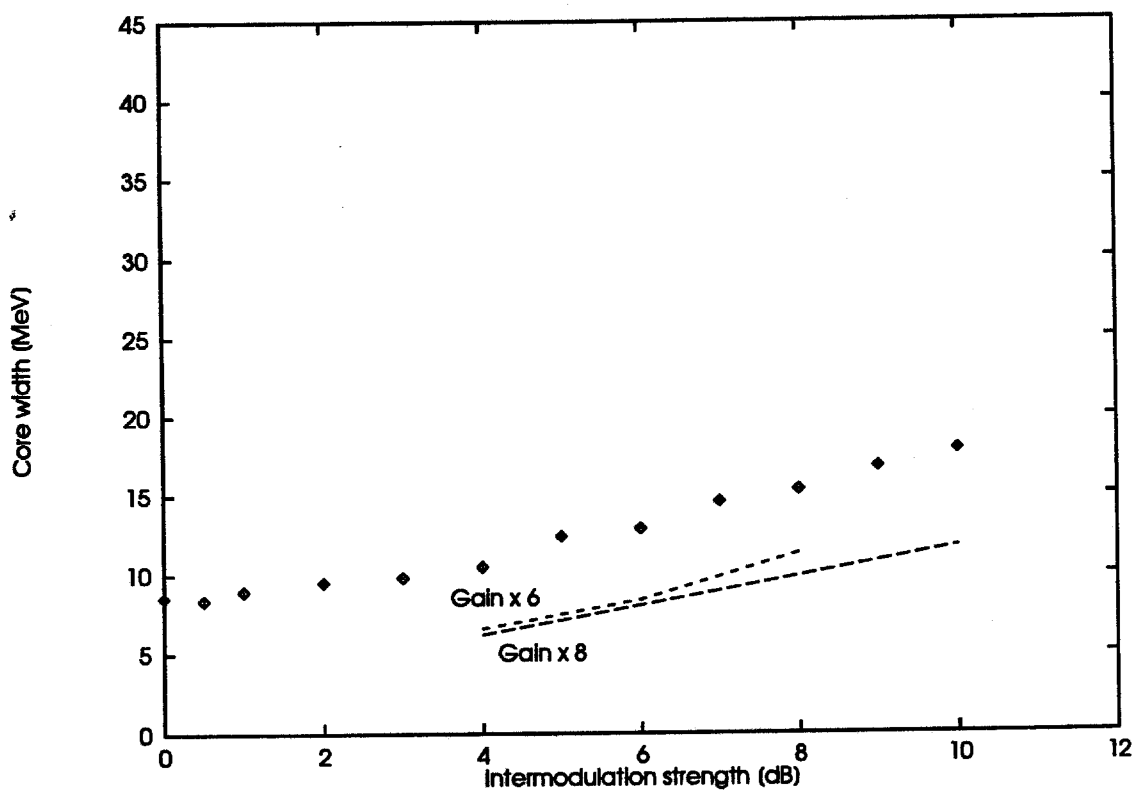

Fig. 7 The core with after 10 hr of stacking with no gain increase (points) and with the gain increased 6 and 8 times.

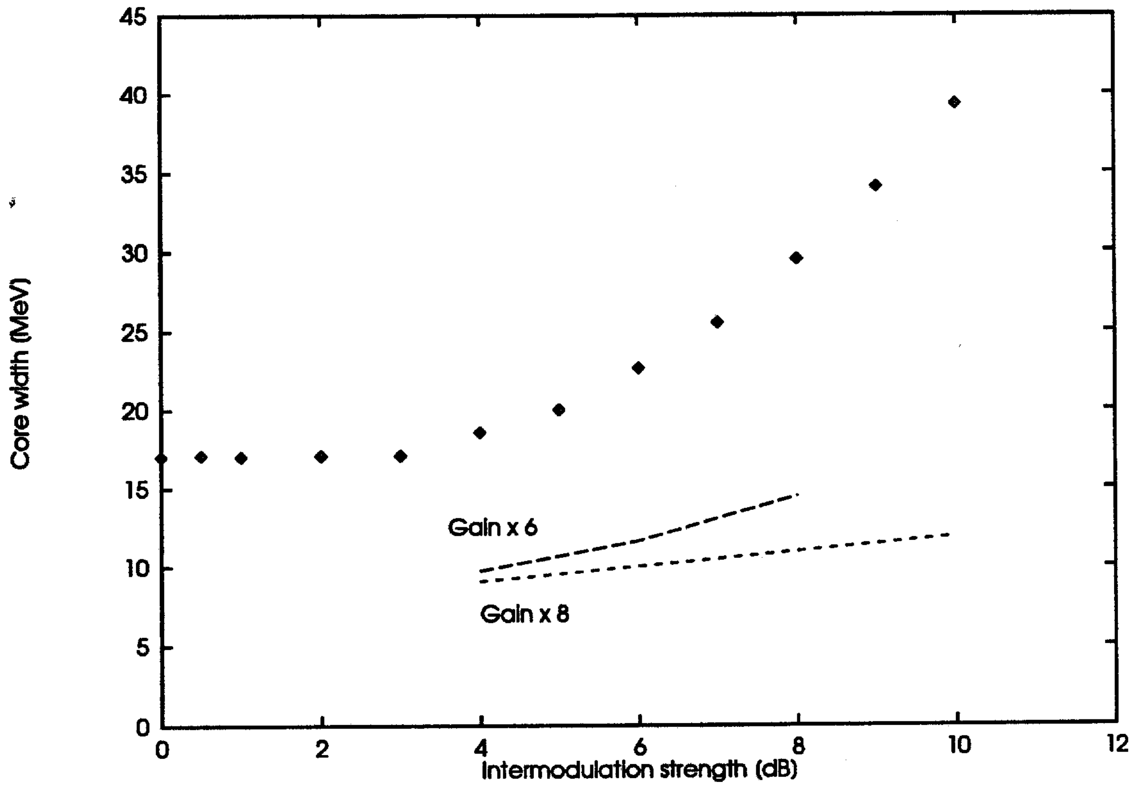

Fig. 8 The core with after 100 hr of stacking with no gain increase (points) and with the gain increased 6 and 8 times.

The conclusion is that for the intermodulation distortion of ~ 5 dB we can keep the core width under control by turning up the core cooling gain. It is clear that this process cannot be continued too far, but we expect this to work well for the range of values of interest.

REFERENCES

[1] Tevatron Design Report

[2] D. Mohl, G. Petrucci, L. Thorndahl and S. van der Meer, *Physics Reports* **58** (1980) 73.

[3] J. Marriner and V. Visnjic, PBAR Note 498, 1991.