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**CALCULATION OF THE TEMPERATURE DISTRIBUTION
IN THE ANTIPROTON TARGET**

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Abstract

The temperature distribution in the antiproton source target as a result of the heating by the proton beam is calculated for typical values of the transverse beam size.

In this note I present the calculation of the temperature distribution in the copper target as a result of the heating by the incident proton beam. In particular, I consider the temperature distribution in a copper disc of given thickness (= the length of the target) and infinite radius which is initially at the room temperature and at $t = 0$ hit by 1.72×10^{12} 120 GeV protons.

The heat capacity at constant pressure of copper is $C_P = a + (b \times 10^{-3})T$ where the coefficients are usually given in units cal/g mole and have values $a = 5.41$ and $b = 1.5$. The total energy deposited is equal to the integral of the heat capacity over the range of temperatures – here from the room temperature (298°K) to the temperature T to which the given volume element is heated:

$$\epsilon = \int_{298}^T C_P dT = \frac{1}{2} b \times 10^{-3} T^2 + aT - A, \quad (1)$$

where $A = 1678.78$.

On the other hand, ϵ can be numerically evaluated by using simulation codes, from the knowledge of the beam spot size σ , the number and the energy of the incident protons and the copper atomic weight. In Fig. 1 results of a calculation of C. Bhat [1] for a target of 7 cm length are shown for two different beam spot sizes.

For the values of ϵ of Fig.1 the corresponding values of temperature are obtained by solving Eq. (1) for T :

$$T = 10^3 \frac{-a + \sqrt{a^2 + 2 \times 10^{-3} b (A + \epsilon)}}{b}. \quad (2)$$

Here, ϵ has to be expressed in cal/g mole in order to obtain T in °K. For copper (atomic weight 63.55), $1\text{GeV/g} = 2.43 \times 10^{-9}$ cal/g mole.

The energy deposition numbers of Fig. 1 translate now into the temperature distribution of Fig. 2. Because of axial symmetry the temperature is a function of z and r only. This is the temperature distribution at time $t = 0$. The temperature distribution $T(r, z, t)$ at later times must then be computed from the heat equation

$$\frac{1}{r} \left(\frac{\partial}{\partial r} r \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} r \frac{\partial T}{\partial z} \right) = \frac{1}{\kappa} \frac{\partial T}{\partial t} \quad (3)$$

for the given initial distribution. This obviously can only be done numerically and can be quite tedious. I want to show now that the results of interest can actually be obtained analytically by replacing the original problem with a soluble one.

First, note that the z -component of the temperature gradient (Fig. 2) is much smaller than the radial one. This is in particular true for small r , which is the hottest region in the target and therefore the most interesting one. By neglecting the z -component of the temperature gradient, we simplify the problem considerably. $T(r, z, t)$ becomes just $T(r, t)$ and Eq. (3) simplifies to

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} = \frac{1}{\kappa} \frac{\partial T}{\partial t} \quad (4)$$

For the front ($z = 0$) surface of the target the radial temperature distributions are given in the histograms of Figs. 3 and 4. The next step is to realize that these distributions can be closely approximated with the Gaussian one which is the natural distribution for the heat equation. The Gaussians shown in Figs. 3 and 4 are $1118e^{-7r^2}$ and $2090e^{-23r^2}$.

The solution of Eq. (4) with the initial δ -function heating, *i.e.* $T(r, 0) = N\delta(r)$ is

$$T(r, t) = \frac{N}{4\pi\kappa t} e^{-r^2/4\kappa t}, \quad (3)$$

where $\kappa = k/C\rho$, k the thermal conductivity, C the heat capacity and ρ the density. For copper $\kappa = 1.16 \text{ cm}^2/\text{sec}$.

All that remains to be done is to determine N and $t = t_0$ such that $T(r, t_0)$ equals the Gaussians of Figs. 3 and 4. The physical meaning of this trick is that the temperature distributions of Figs. 3 and 4, although in reality being created instantaneously at $t = 0$, can be thought of as originating from a δ -function heating $N\delta(r)$ at time $t = -t_0$. The temperature distributions at any time t is then obtained from Eq. (3) by simply replacing t by $t_0 + t$. For $\sigma = 0.15\text{mm}$ (Fig.3), $N = 489^\circ\text{K mm}^2$ and $t_0 = 300\mu\text{sec}$ and for $\sigma = 0.10\text{mm}$ (Fig.4), $N = 152.7^\circ\text{K mm}^2$ and $t_0 = 90\mu\text{sec}$.

The temperature distributions for $t = 0, 0.05, 0.1, 0.2, 0.3$ and 0.5 msec are shown in Figs.5 ($\sigma = 0.15\text{mm}$) and 6 ($\sigma = 0.1\text{mm}$). At the core ($r = 0$), the temperature as a function of time is

$$T(0, t) = \frac{335.5}{0.3 + t[\text{msec}]}^\circ\text{K} \quad \text{for } \sigma = 0.15\text{mm},$$

and

$$T(0, t) = \frac{188}{0.09 + t[\text{msec}]}^\circ\text{K} \quad \text{for } \sigma = 0.1\text{mm}.$$

REFERENCE

1. C. Bhat, PBAR Note # 480, 1989.

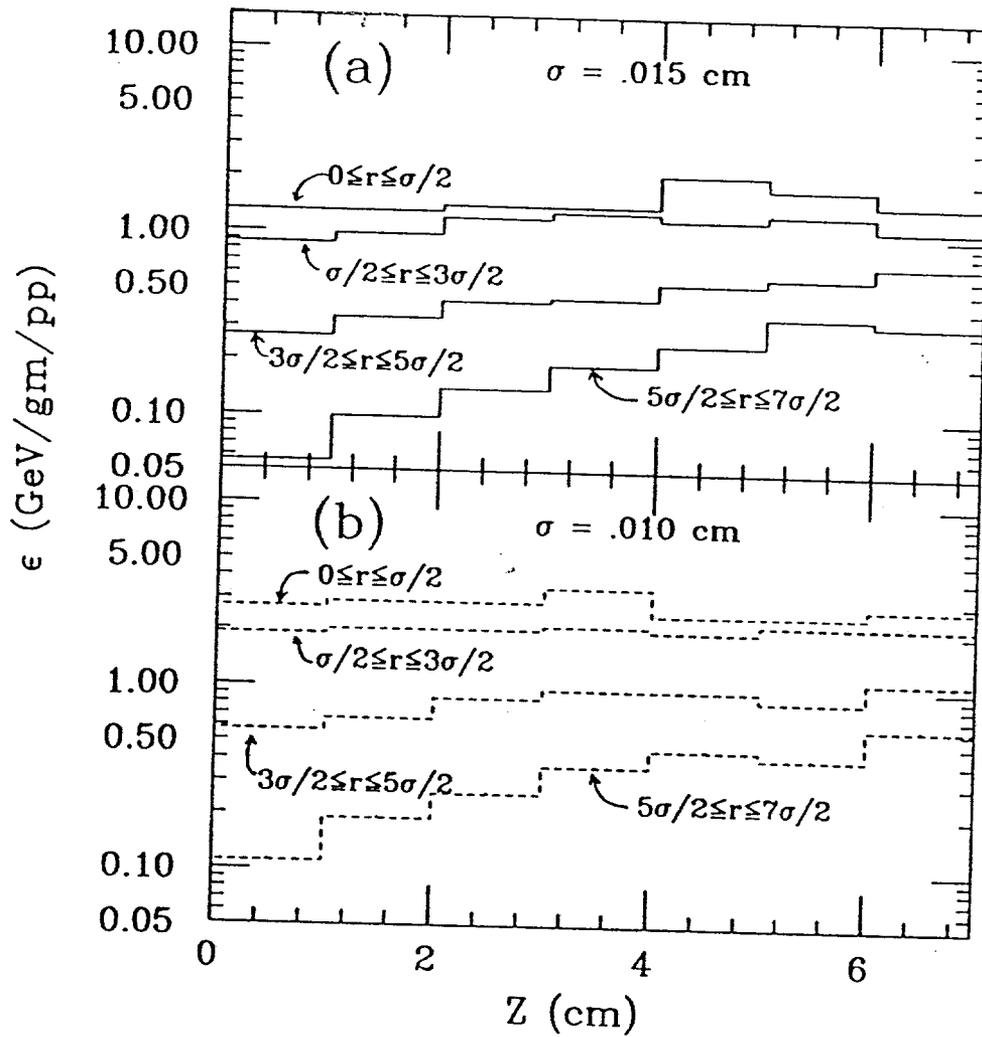


Fig. 1. Energy density distribution in the 7cm long target for beam spot sizes (a) $\sigma=0.015\text{cm}$ and (b) $\sigma=0.010\text{cm}$. Data only up to $r=7\sigma/2$ is shown.

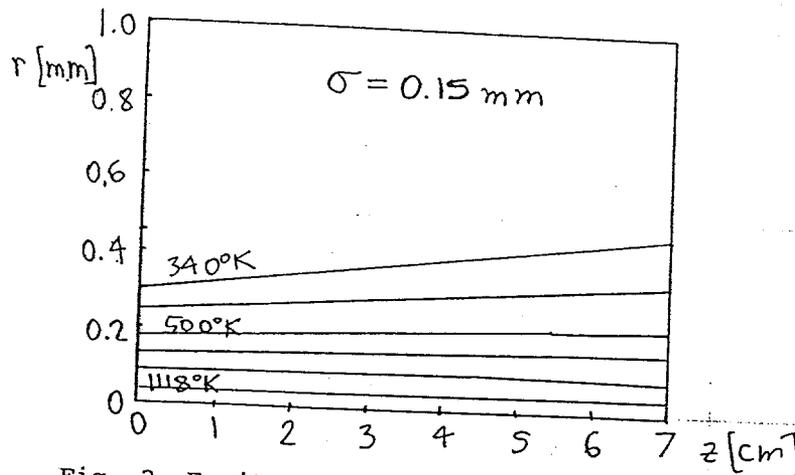


Fig. 2 Equitemperature lines for the energy deposition of Fig.1.

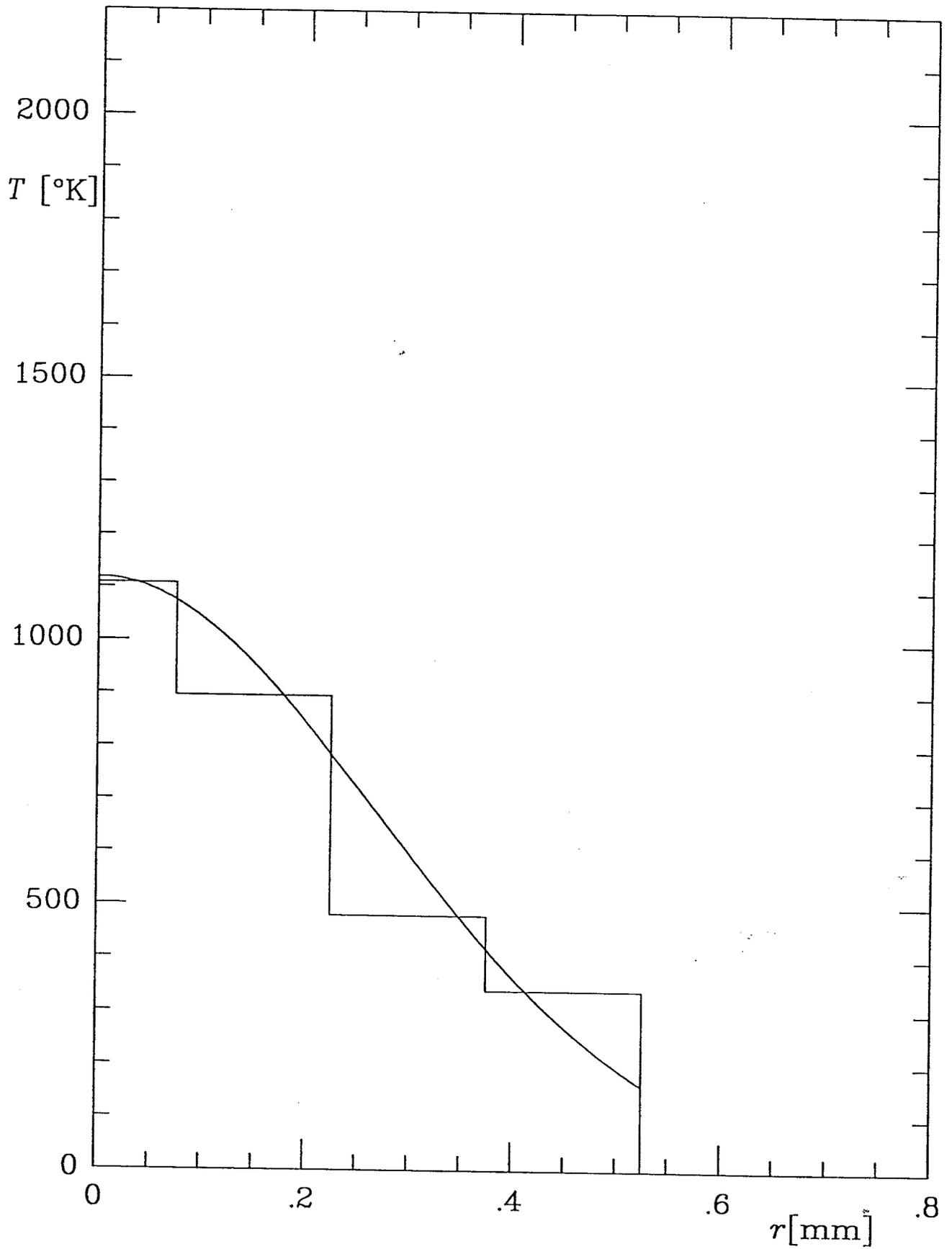


Fig. 3

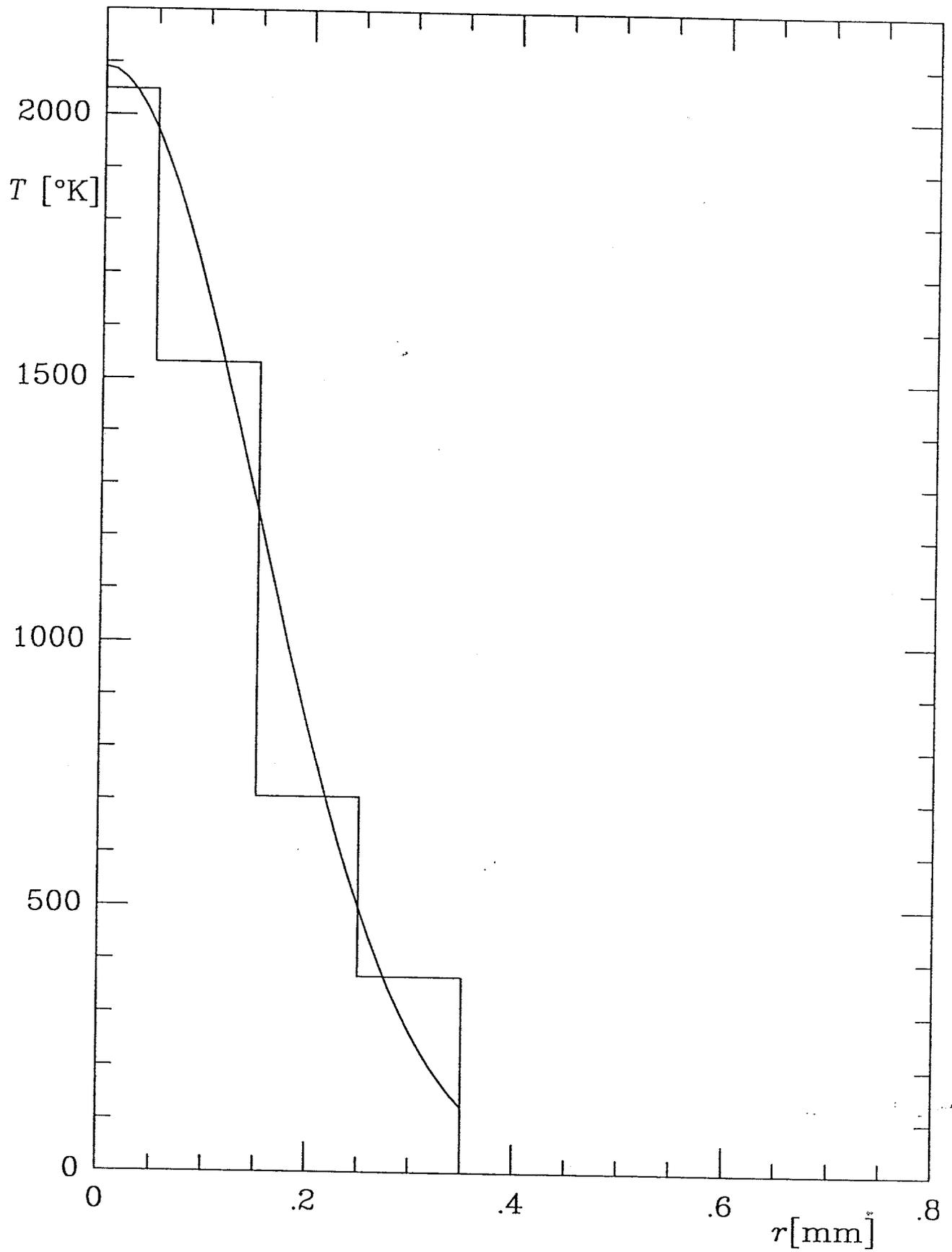


Fig. 4

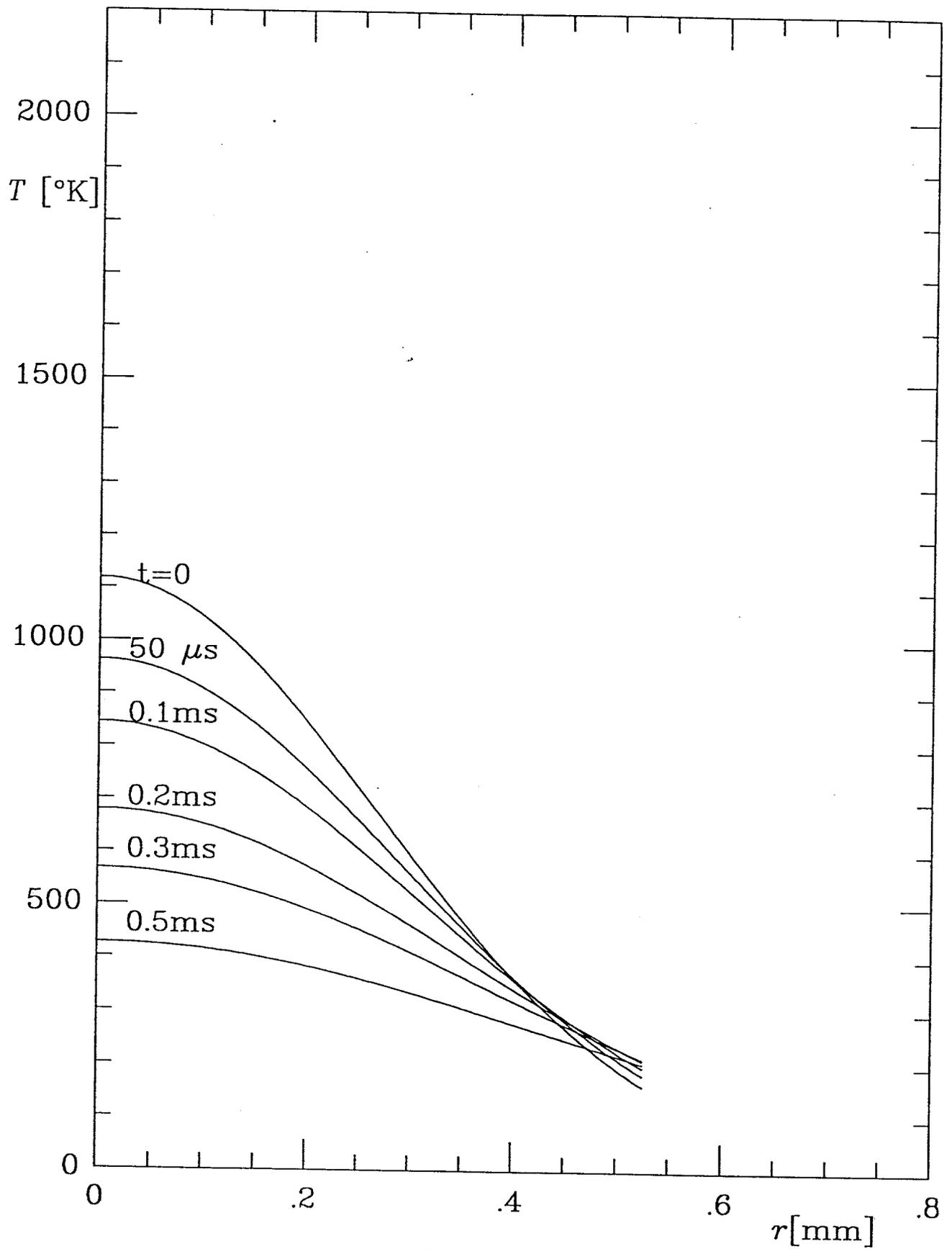


Fig. 5

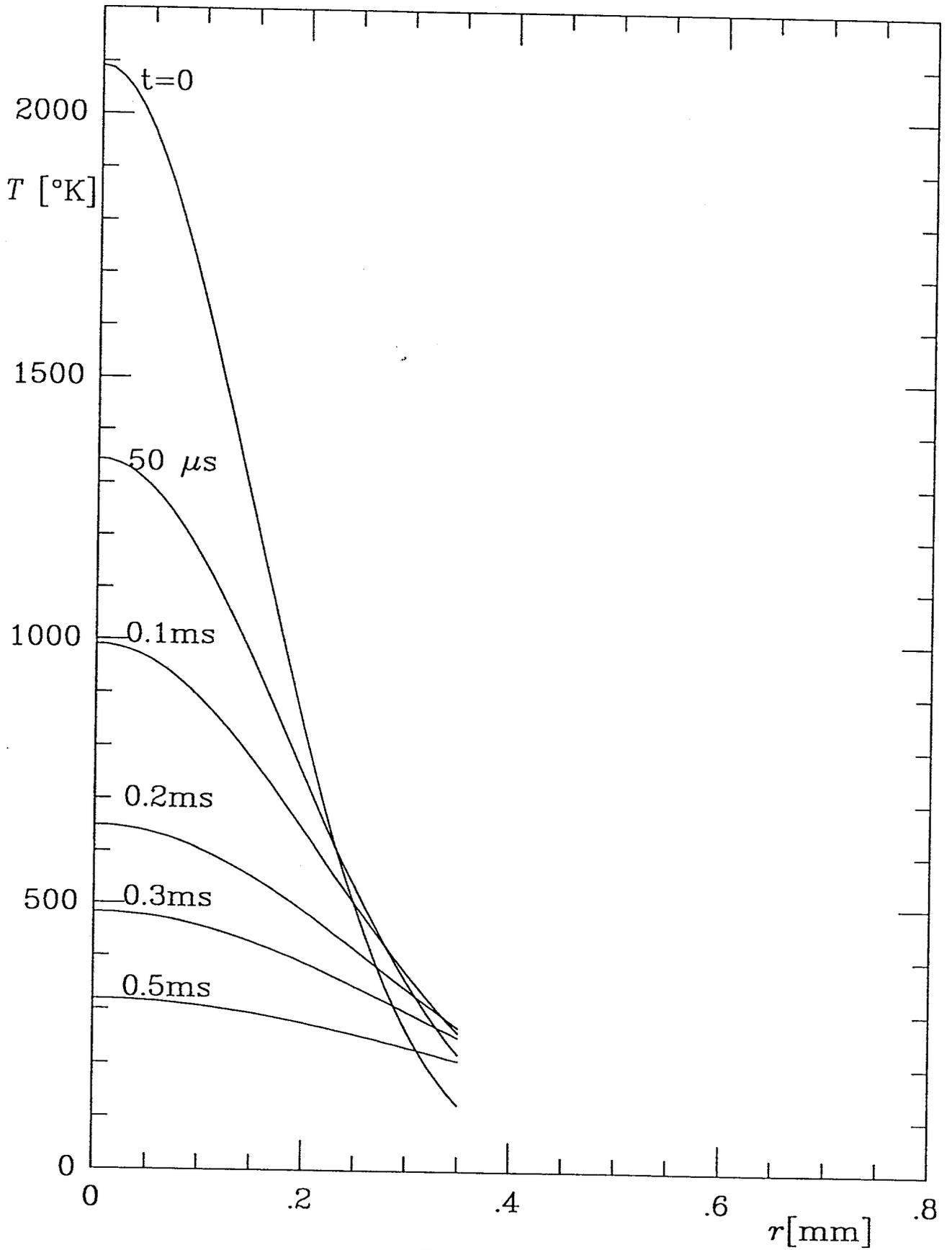


Fig. 6