Recalculation of some impedances of the accumulator

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(Sept. 1986)

Summary

The impedances of steps along beam pipe and those of bellows (for the accumulator) have been ~~redo~~ rederived. The results are:

1. For one section of the accumulator, due to steps (at low frequencies),

$$Z_L/n = j\,0.059\,\Omega$$
$$Z_\perp = j\,5.9\times10^3\,\Omega/m$$

2. Bellows (also for 1 section only)

(a) low frequencies

$$Z_L/n = j\,0.090\,\Omega$$
$$Z_\perp = j\,6.5\times10^3\,\Omega/m$$

(b) high frequencies

There is a broad resonance near 6.5 GHz

$$Z_L/n = 0.30\,\Omega \quad \text{(shunt impedance)}$$
$$Z_\perp = 2.5\times10^4\,\Omega/m \quad \text{(shunt impedance)}$$

I. Steps along Vacuum chamber

I follow the drawing, the steps or changes in crosssection of vacuum chamber differ from what I assumed in the earlier report. The new steps are:

C : circular 4.56 cm internal radius

S : Star

rect. (large) 3.5 x 9.8 cm²

rect. (small) 3.5 x 22.5 cm²

Oval

C gate valve C short bellow C Q1 S C Q2 S C Q3 S C long bellow B1 rect (small) short bellow C

pump C long bellow C Q4 S C Q5 S C long bellow C long bellow C Q6 S C long bellow C Q7 S C

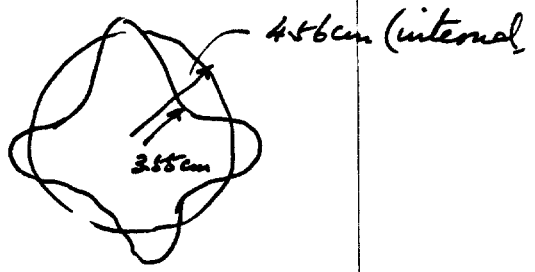
gate valve pump C long bellow B7 rect (small) short bellow C Q8 S B8 rect (small) short bellow C Q9 S

C long bellow ~~(oval)~~ B9 rect (large) short bellow (oval) Q10 S B10 rect (large) long bellow (oval) Q11 Q12 oval short bellow (oval)

Q13 Q14 oval long bellow (oval) gate valve oval circle

Steps formed by beam-pipe transitions

1. 17 star to circle



$b_{max} = 4.56\,cm.$ $b_{min} = 3.55\,cm.$

$$Z_\perp/n = j\,\frac{Z_0 g}{2\pi R}\ln\frac{b_{max}}{b_{min}}$$

$$Z_\perp = j\,\frac{g Z_0}{2\pi}\left(\frac{1}{b_{min}^2} - \frac{1}{b_{max}^2}\right)$$

g = effective length across the step where magnetic field of low frequencies can be trapped

Take $g = \frac{1}{2}(b_{max} - b_{min}) \simeq 0.5\,cm.$

$\bar{R} = 75.45\,m.$

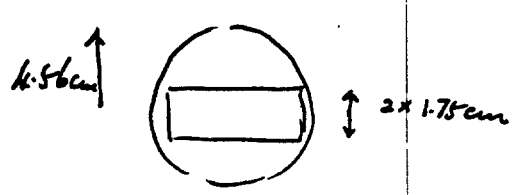
$$\therefore\ Z_\perp/n = j\,\frac{377\times 0.5}{2\pi\times 75.45}\ln\frac{4.56}{3.55} = j\,0.00100\,\Omega \longrightarrow j\,0.017\,\Omega$$

$$Z_\perp = j\,\frac{377\times 0.005}{2\pi}\left(\frac{1}{.0355^2} - \frac{1}{.0456^2}\right) = j\,93.8\,\Omega/m \longrightarrow \underbrace{j\,1594\,\Omega/m}_{17\text{ steps}}$$

Only $\frac{1}{2}$ of the circumference has step so take another factor of $\frac{1}{2}$

$$\boxed{\begin{aligned} Z_\perp/n &= j\,0.0085\,\Omega \\ Z_\perp &= j\,797\,\Omega/m \end{aligned}}$$

2. 6 circular - rect. (small).



$b_{max} = 4.56\,cm.$ $b_{min} = 1.75\,cm.$

$g \sim 1.41\,cm.$

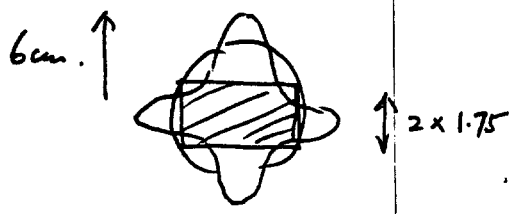
$$Z_{L/n} = j\frac{377\times1.41}{2\pi\times7545}\ln\frac{4.56}{1.75} = j0.0107\,\Omega \longrightarrow j0.064\,\Omega$$

$$Z_{\perp} = j\frac{377\times0.0141}{2\pi}\left(\frac{1}{0.0175^2}-\frac{1}{0.0456^2}\right) = j\,2356\,\Omega/m \longrightarrow j\,14134\,\Omega/m$$

Again take a factor of $\frac{1}{2}$

$$\boxed{\begin{aligned} Z_L/n &= j\,0.032\,\Omega \\ Z_{\perp} &= j\,7067\,\Omega/m \end{aligned}}$$

3. 1 Star-rect (small)



$b_{max} = 6.00$ cm. $b_{min} = 1.75$ cm.

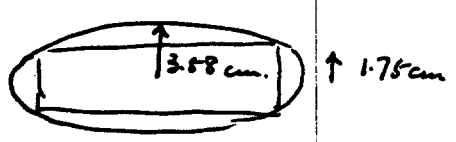
$\therefore\ q \sim 2.12$ cm.

$Z_L/n = j\,0.0208\,\Omega \qquad Z_{\perp} = j\,3800\,\Omega/m$

Again take a factor of $\frac{1}{2}$

$$\boxed{\begin{aligned} Z_L/n &= j\,0.0104\,\Omega \\ Z_{\perp} &= j\,1900\,\Omega/m \end{aligned}}$$

4. 3 oval-rect (large)



$b_{max} = 3.58$ cm. $b_{min} = 1.75$ cm.

$q \sim 0.92$ cm.

$Z_L/n = j\,0.00524\,\Omega \longrightarrow j\,0.0147\,\Omega$

$Z_{\perp} = j\,1372\,\Omega/m \longrightarrow j\,4115\,\Omega/m$

factor of $\frac{1}{2}$ $\Longrightarrow$ $\boxed{\begin{aligned} Z_L/n &= j\,0.0079\,\Omega \\ Z_{\perp} &= j\,2058\,\Omega/m \end{aligned}}$

Total

$$Z_\perp/n = j(0.0085 + 0.032 + 0.0104 + 0.0079)\,\Omega = j0.0688\,\Omega$$

$$Z_\perp = j(797 + 7067 + 1900 + 2058)\,\Omega/m = j\,11822\,\Omega/m.$$

The formula used for $Z_\perp$ when compared with the numerical formula $Z_\perp = j\,\frac{Z_0}{2\pi b_{max}}\,\frac{(s-1)^2}{0.90+0.78s}$ is 2 times too big for s from 1 up to at least 3

The formula used for Z_0/n when compared with the numerical formula $Z_\parallel/n = j\,\frac{.241}{2}\,\frac{b_{min}}{R}\,Z_0\,\frac{(s-1)^2}{s+0.412}$ is very well from $s=1$ up to at least 2.

Thus we need to divide our result for $Z_\perp$ by $\frac{1}{2}$ again.

Therefore for all steps along beam pipes

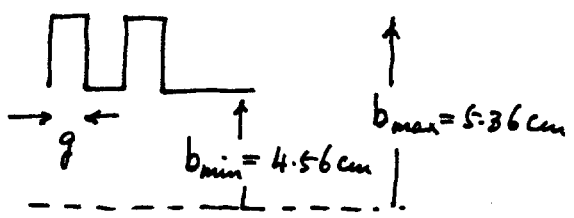
$$\boxed{\begin{aligned} \frac{Z_\perp}{n} &= j0.059\,\Omega \\ Z_\perp &= j5.9\times10^3\,\Omega/m \end{aligned}}$$

II. Bellows

7 long circular, 20 corrugations each = 140 corr.
5 short circular, 4 corrugations each = 20 corr.
160

2 long oval 20 corrugations each = 40 corr.
2 short oval 4 corrugations each = 8 corr.
48

Circular bellow



Oval bellow

$b_{max} = 4.48$ cm
$b_{min} = 3.68$ cm

In each case period of corrugation = 0.64 cm
or $g = 0.32$ cm

IIa Low frequencies

$$\frac{Z_{\parallel}}{n} = j\,\frac{Z_0 g}{2\pi R}\ln\frac{b_{max}}{b_{min}}$$

$$\therefore\quad Z_{\parallel} = j f\,\frac{Z_0 g}{2\pi R}\,\frac{b_{max}}{b_{min}} \qquad f = \text{freq.}$$

} valid if $\frac{g}{b_{min}} \ll \pi$

$$Z_{\perp} = j\,\frac{Z_0 g}{\pi b_{min}^2}\,\frac{s^2-1}{s^2+1} \qquad s = \frac{b_{max}}{b_{min}}$$

valid if $\frac{g}{b_{min}} \ll \frac{\pi^2}{32}$

(not so well satisfied)

For one ripple

		circular	Oval	
$Z_{\parallel}/f$	theory	$j\,0.650\,\Omega$	$j\,0.791\,\Omega$	f in GHz
	TBCI	$j\,0.65\,\Omega$	$j\,0.79\,\Omega$	
$Z_{\perp}$	theory	$j\,29.6\,\Omega/m$	$j\,55.1\,\Omega/m$	
	TBCI	$j\,26\,\Omega/m$	$j\,48\,\Omega/m$	

Agreement is good. We use the TBCI values

$$\frac{Z_{\parallel}}{n} = \frac{Z_{\parallel}}{f} f_0 = \frac{Z_{\parallel}}{f}\frac{c}{2\pi R} \qquad R = 75.45\ m$$

$$\therefore \quad \frac{Z_{\parallel}}{n} = j4.11\times10^{-4}\,\Omega \ \text{(circular)} \qquad j5.00\times10^{-4}\,\Omega \ \text{(Oval)}$$

$$Z_{\perp} = j\,26\,\Omega/m \ \text{(circular)} \qquad j48\,\Omega/m \ \text{(Oval)}$$

For all bellows 160 corrugations of circular
48 " of oval

$$\frac{Z_{\parallel}}{n} = j\,10^{-4}(4.11\times160 + 5.00\times48) = j\,0.090\,\Omega$$

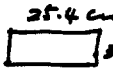
$$Z_{\perp} = j\,(26\times160 + 48\times48) = j\,0.0065\ M\Omega/m$$

II (b) <u>Resonances of bellows</u> (see Fig. following)

We see broad resonances for the 2 types of bellows in $m=0$ & $m=1$ modes around 6.5 GHz

<u>Circular</u> (cut off freq = 2.5 GHz)

9 ripples $Z_\perp = 124\,\Omega \longrightarrow 2204\,\Omega$
$Z_\perp = 900\,\Omega/m \longrightarrow 16000\,\Omega/m$ } for 160 ripples

<u>Oval</u> (Cut off freq = 4.3 GHz assuming rect. 

$f_{cut\,off} = \frac{c}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^{\frac{1}{2}}$)

9 ripples $Z_\perp = 60\,\Omega \longrightarrow 853\,\Omega$
$Z_\perp = 1700\,\Omega/m \longrightarrow 9100\,\Omega/m$ } 48 ripples

$\therefore$ Total $Z_\perp = 3060\,\Omega$ $Z_\perp = 25{,}000\,\Omega/m$

at ~6.5 GHz harmonic $n = \frac{2\pi f_R R}{c} = \frac{2\pi \times 6.5}{30} \times 7545$

$\cong 10270$

$$\therefore \quad \boxed{\begin{aligned} Z_\perp/n &= 0.30\,\Omega \\ Z_\perp &= 2.5\times10^4\,\Omega/m \end{aligned}}$$

Figure Captions

Fig 1. Longitudinal wake, real & imaginary longitudinal impedance for 9 corrugations of circular bellow

Fig 2. same as fig. 1 but in transverse mode

Fig 3. Same as fig 1 but for 9 corrugations of oval bellow

Fig 4. Same as fig 2 but for 9 corrugations of oval bellow.

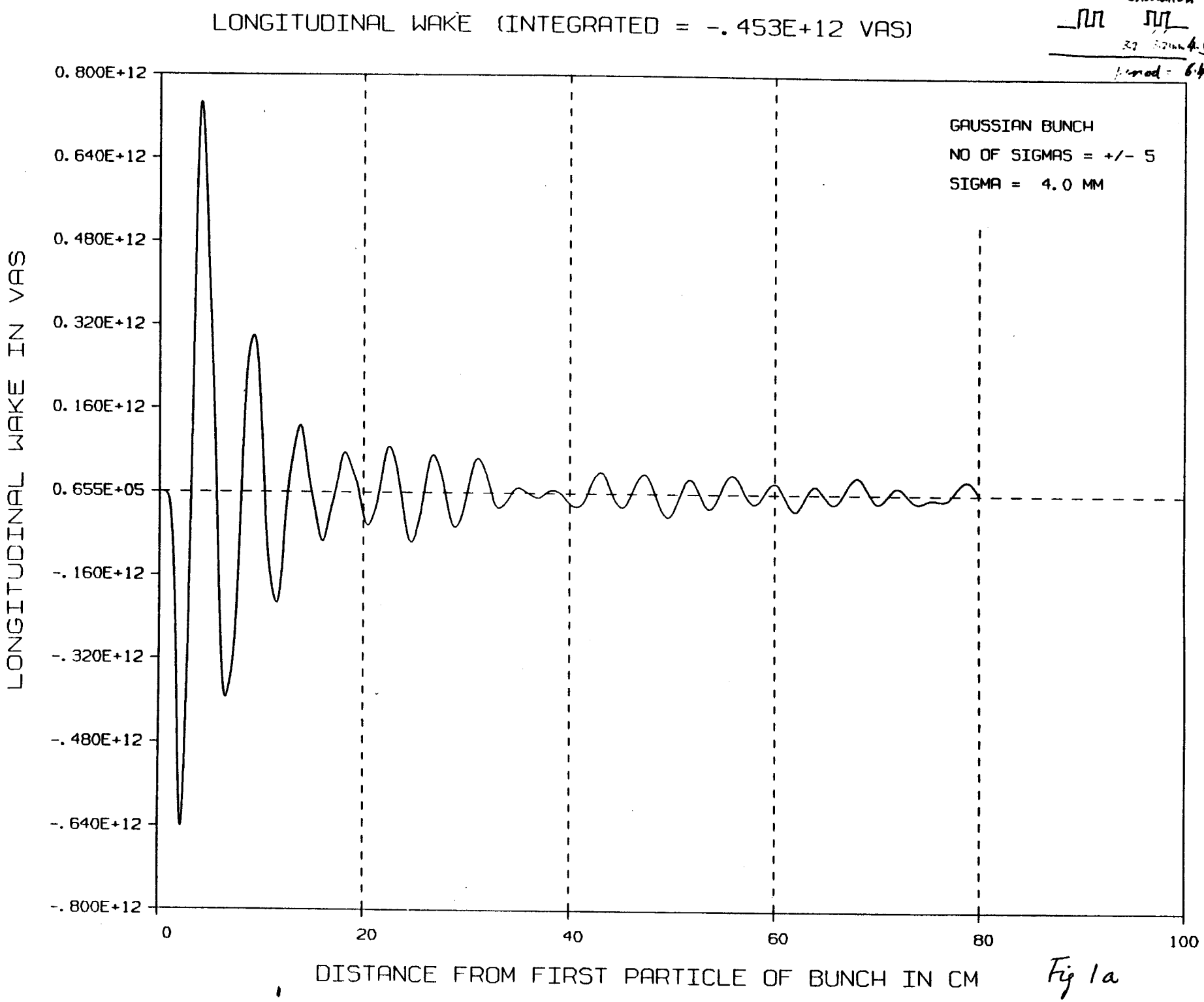
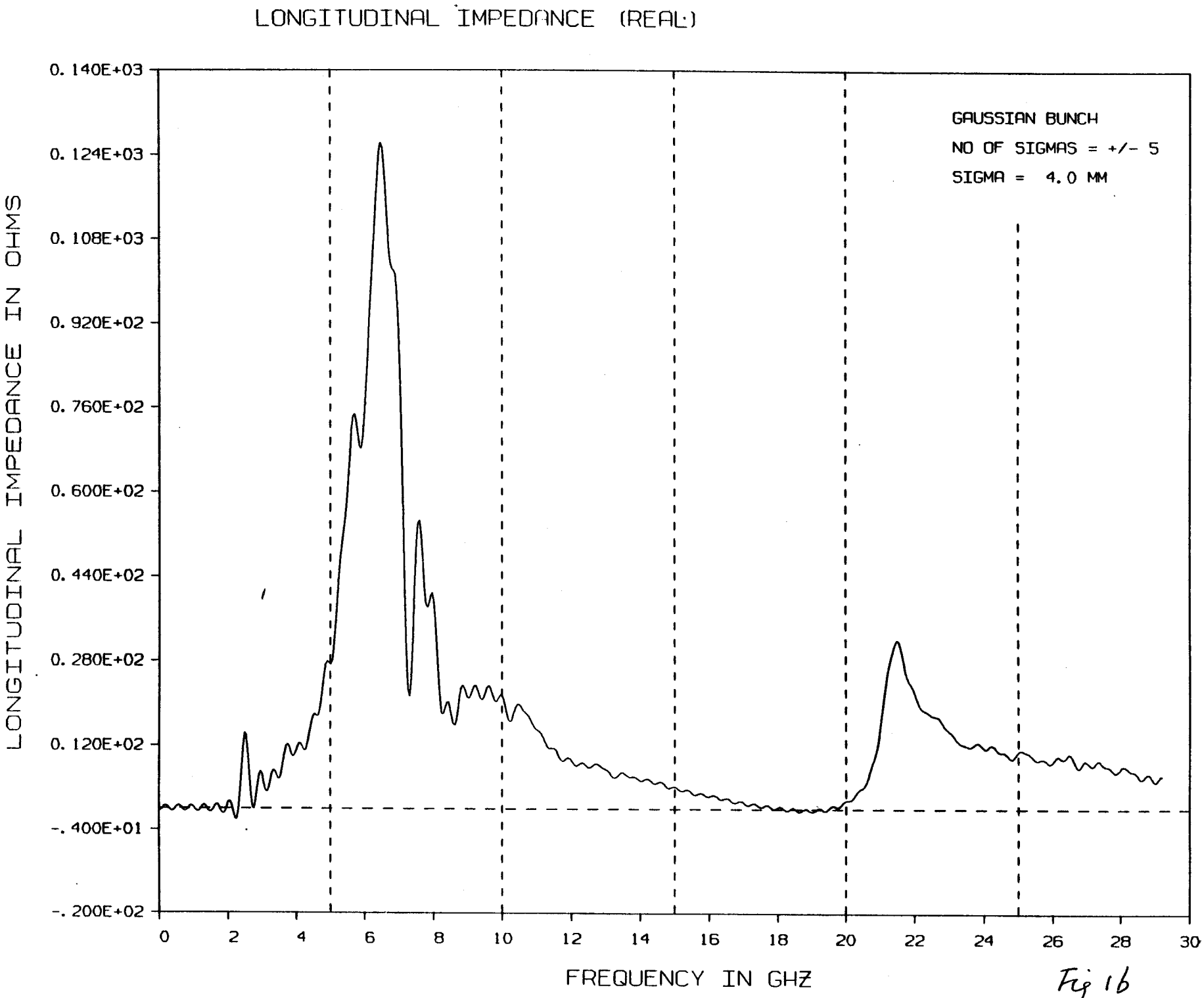

LONGITUDINAL WAKE (INTEGRATED = -.453E+12 VAS)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
LONGITUDINAL WAKE IN VAS
0.800E+12
0.640E+12
0.480E+12
0.320E+12
0.160E+12
0.655E+05
-.160E+12
-.320E+12
-.480E+12
-.640E+12
-.800E+12
0
20
40
60
80
100
DISTANCE FROM FIRST PARTICLE OF BUNCH IN CM

Fig 1a


LONGITUDINAL IMPEDANCE (REAL)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
LONGITUDINAL IMPEDANCE IN OHMS
0.140E+03
0.124E+03
0.108E+03
0.920E+02
0.760E+02
0.600E+02
0.440E+02
0.280E+02
0.120E+02
-.400E+01
-.200E+02
0
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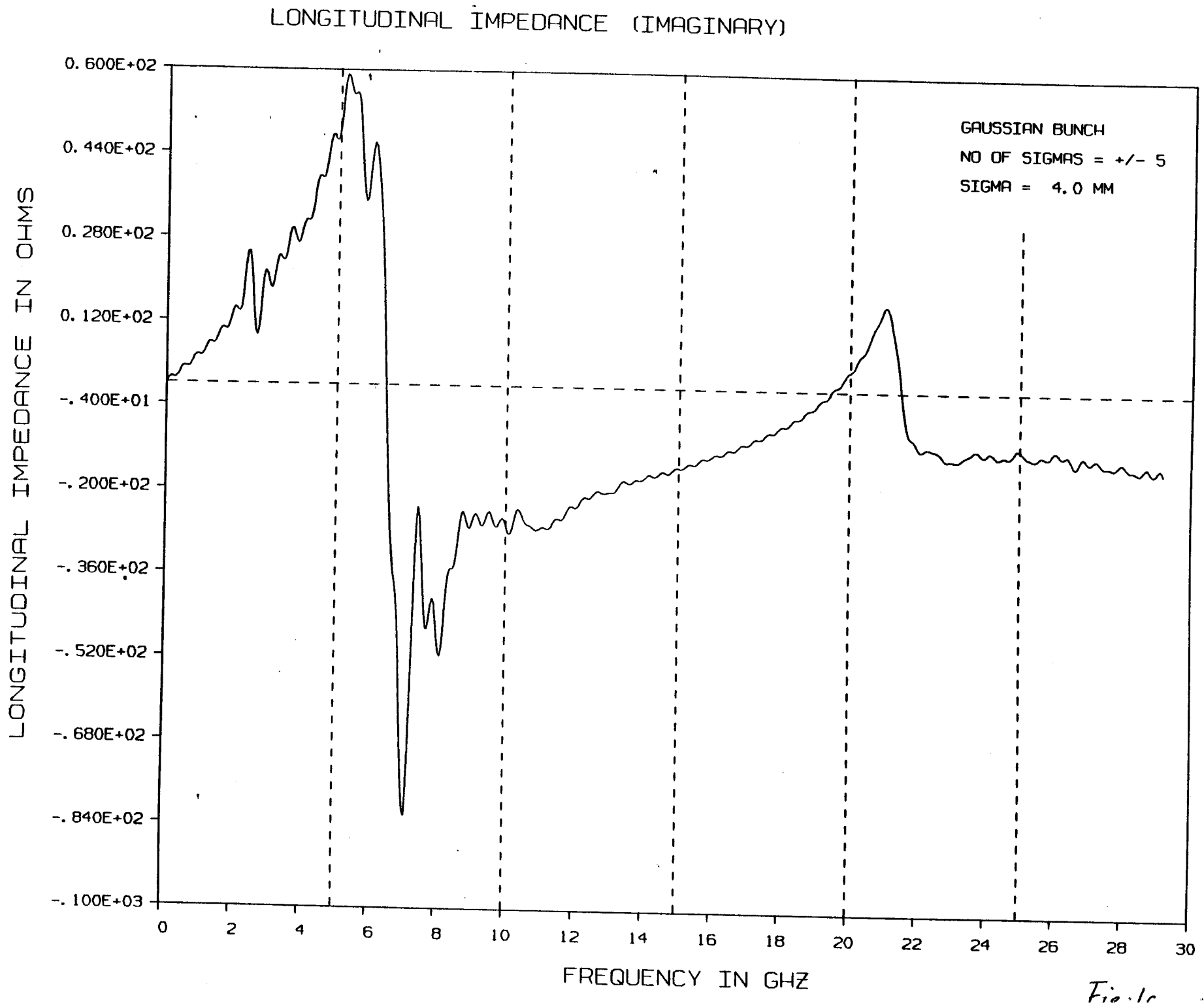
FREQUENCY IN GHZ

Fig 1b


LONGITUDINAL IMPEDANCE (IMAGINARY)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
LONGITUDINAL IMPEDANCE IN OHMS
0.600E+02
0.440E+02
0.280E+02
0.120E+02
-.400E+01
-.200E+02
-.360E+02
-.520E+02
-.680E+02
-.840E+02
-.100E+03
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FREQUENCY IN GHZ

Fig. 1c

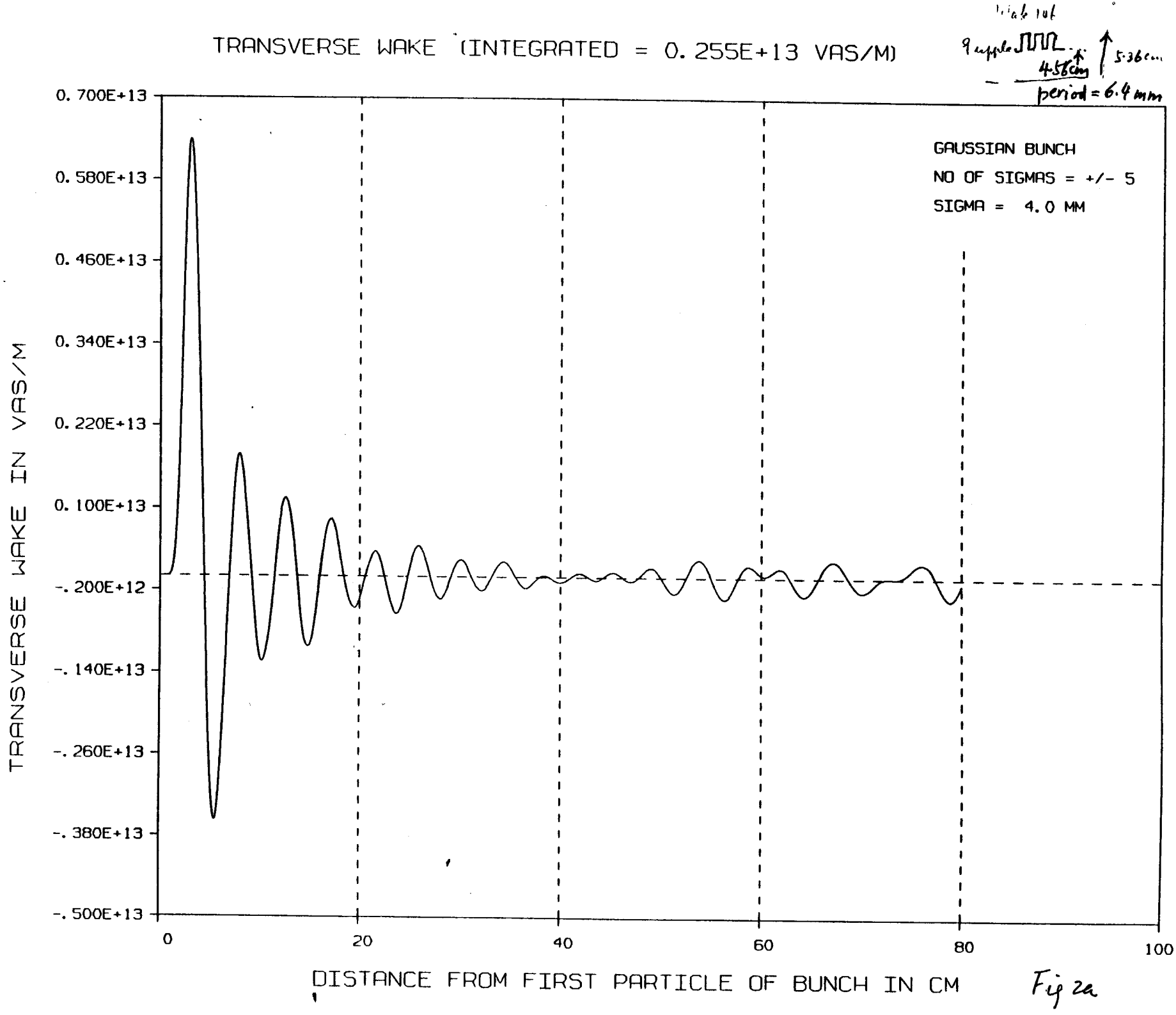
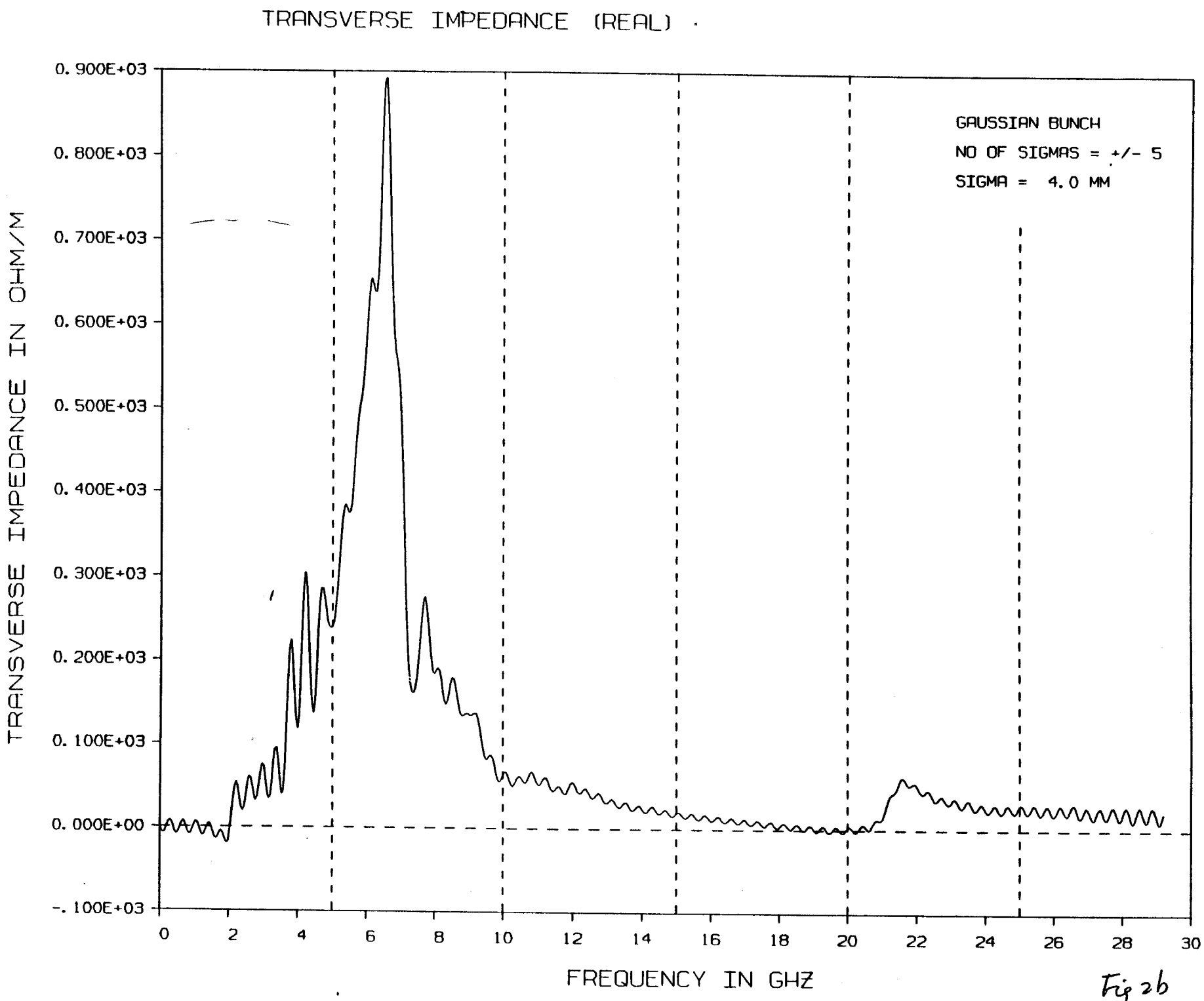

TRANSVERSE WAKE (INTEGRATED = 0.255E+13 VAS/M)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
TRANSVERSE WAKE IN VAS/M
0.700E+13
0.580E+13
0.460E+13
0.340E+13
0.220E+13
0.100E+13
-.200E+12
-.140E+13
-.260E+13
-.380E+13
-.500E+13
0
20
40
60
80
100
DISTANCE FROM FIRST PARTICLE OF BUNCH IN CM
period = 6.4 mm

Fig 2a


TRANSVERSE IMPEDANCE (REAL)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
TRANSVERSE IMPEDANCE IN OHM/M
0.900E+03
0.800E+03
0.700E+03
0.600E+03
0.500E+03
0.400E+03
0.300E+03
0.200E+03
0.100E+03
0.000E+00
-.100E+03
0
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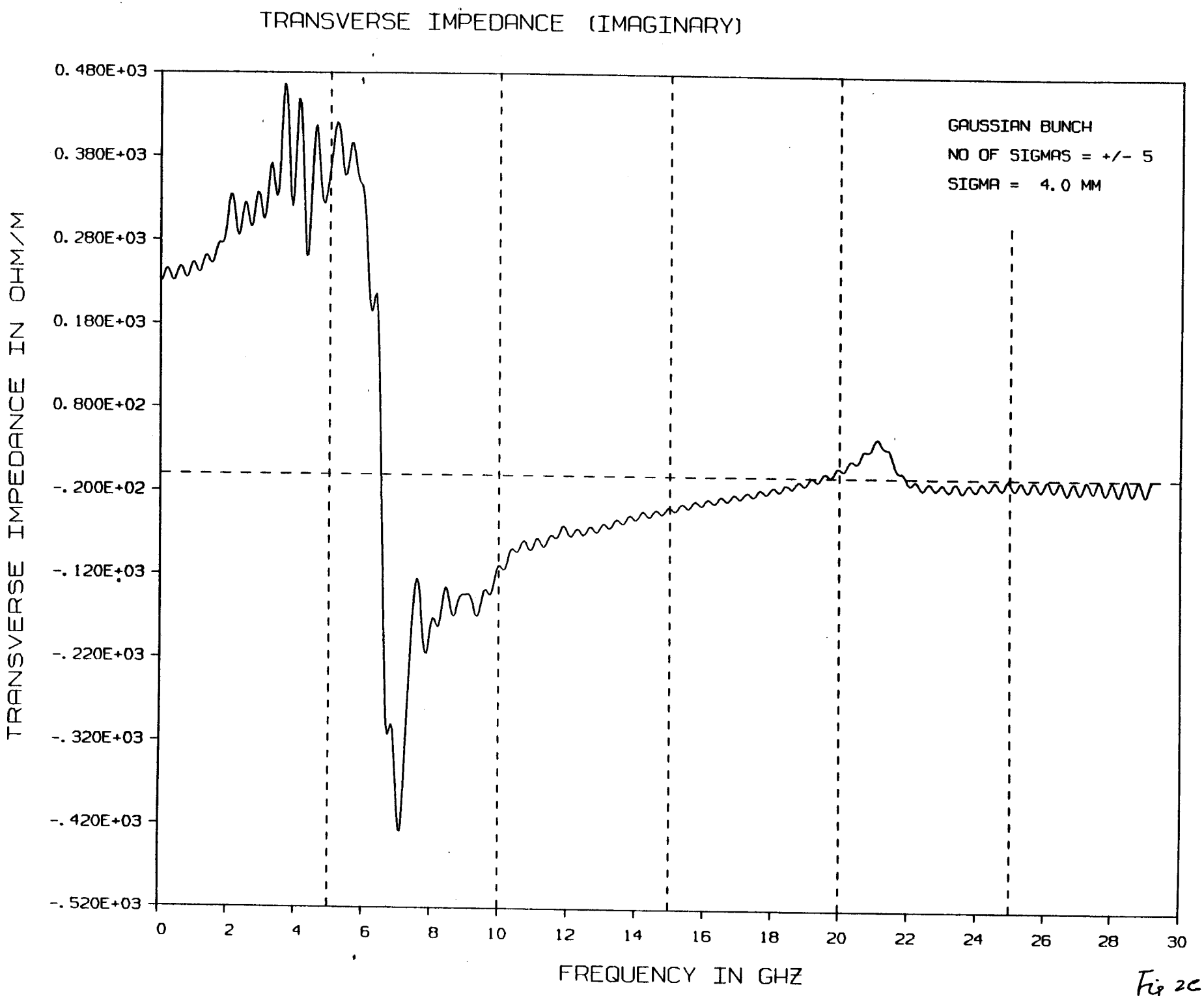
FREQUENCY IN GHZ

Fig 2b


TRANSVERSE IMPEDANCE (IMAGINARY)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
TRANSVERSE IMPEDANCE IN OHM/M
0.480E+03
0.380E+03
0.280E+03
0.180E+03
0.800E+02
-.200E+02
-.120E+03
-.220E+03
-.320E+03
-.420E+03
-.520E+03
0
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FREQUENCY IN GHZ

Fig 2c

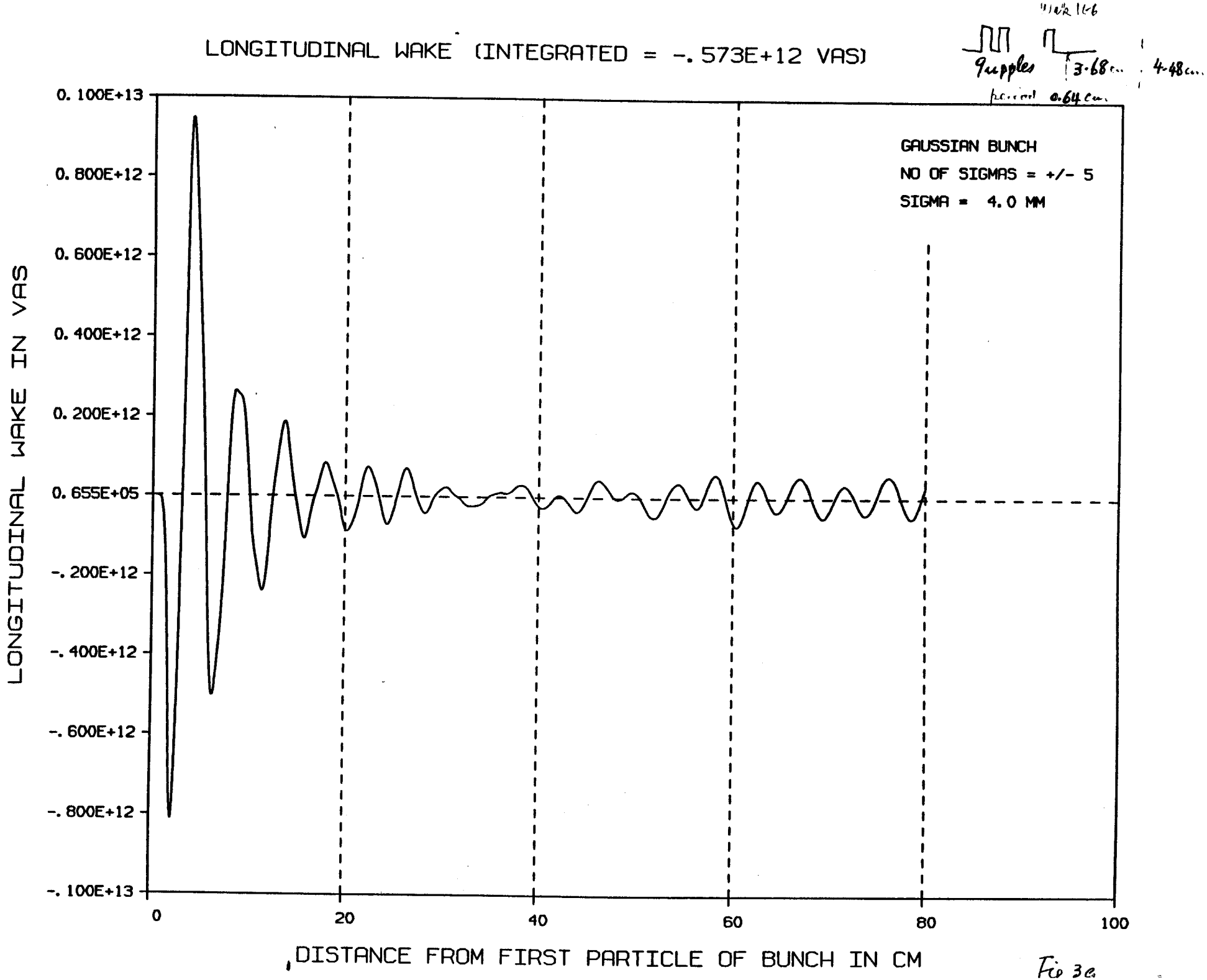
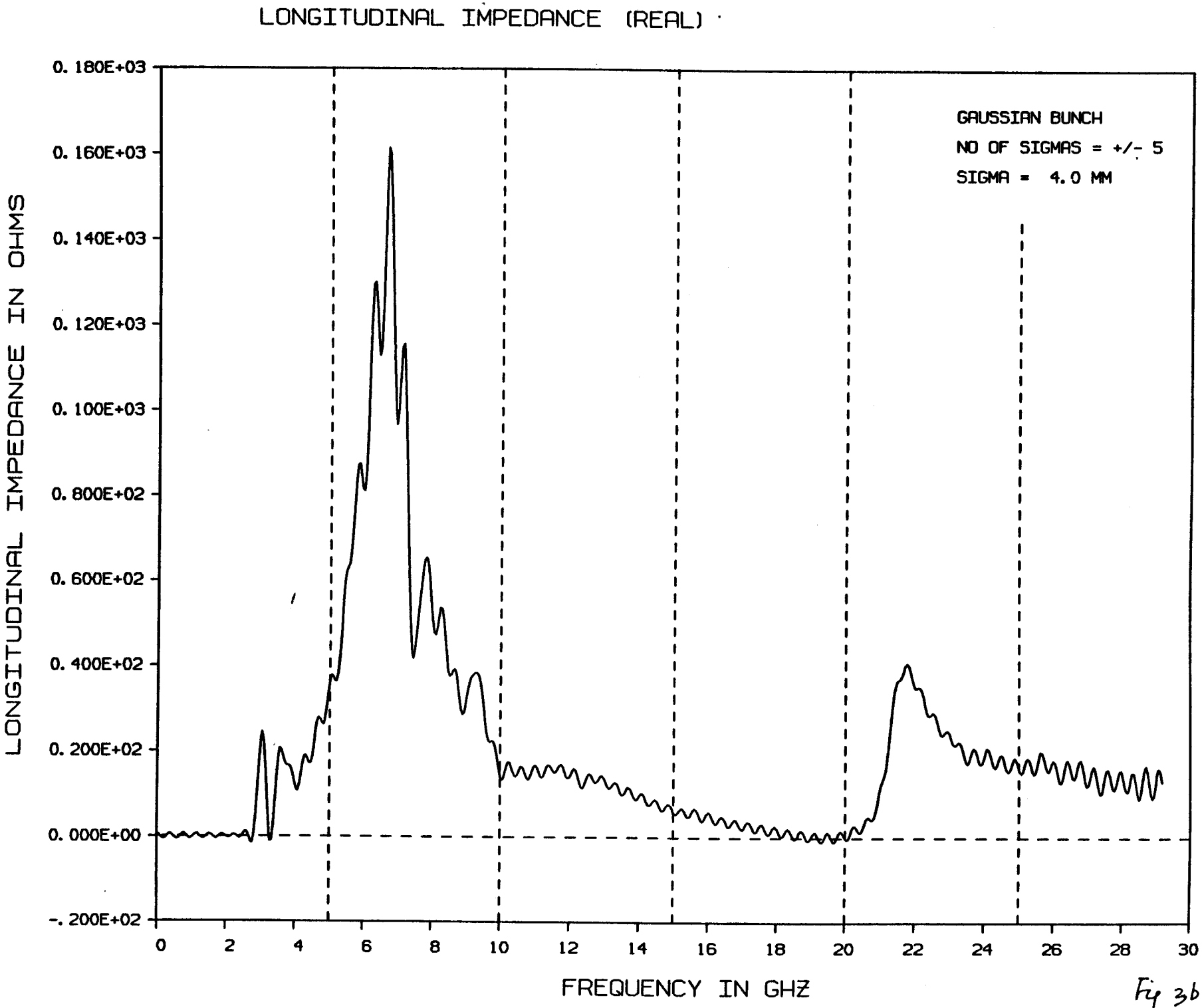

LONGITUDINAL WAKE (INTEGRATED = -.573E+12 VAS)
9 ripples
3.68 cm
4.48 cm
period 0.64 cm
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
LONGITUDINAL WAKE IN VAS
0.100E+13
0.800E+12
0.600E+12
0.400E+12
0.200E+12
0.655E+05
-.200E+12
-.400E+12
-.600E+12
-.800E+12
-.100E+13
0
20
40
60
80
100
DISTANCE FROM FIRST PARTICLE OF BUNCH IN CM

Fig 3a


LONGITUDINAL IMPEDANCE (REAL)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
LONGITUDINAL IMPEDANCE IN OHMS
0.180E+03
0.160E+03
0.140E+03
0.120E+03
0.100E+03
0.800E+02
0.600E+02
0.400E+02
0.200E+02
0.000E+00
-.200E+02
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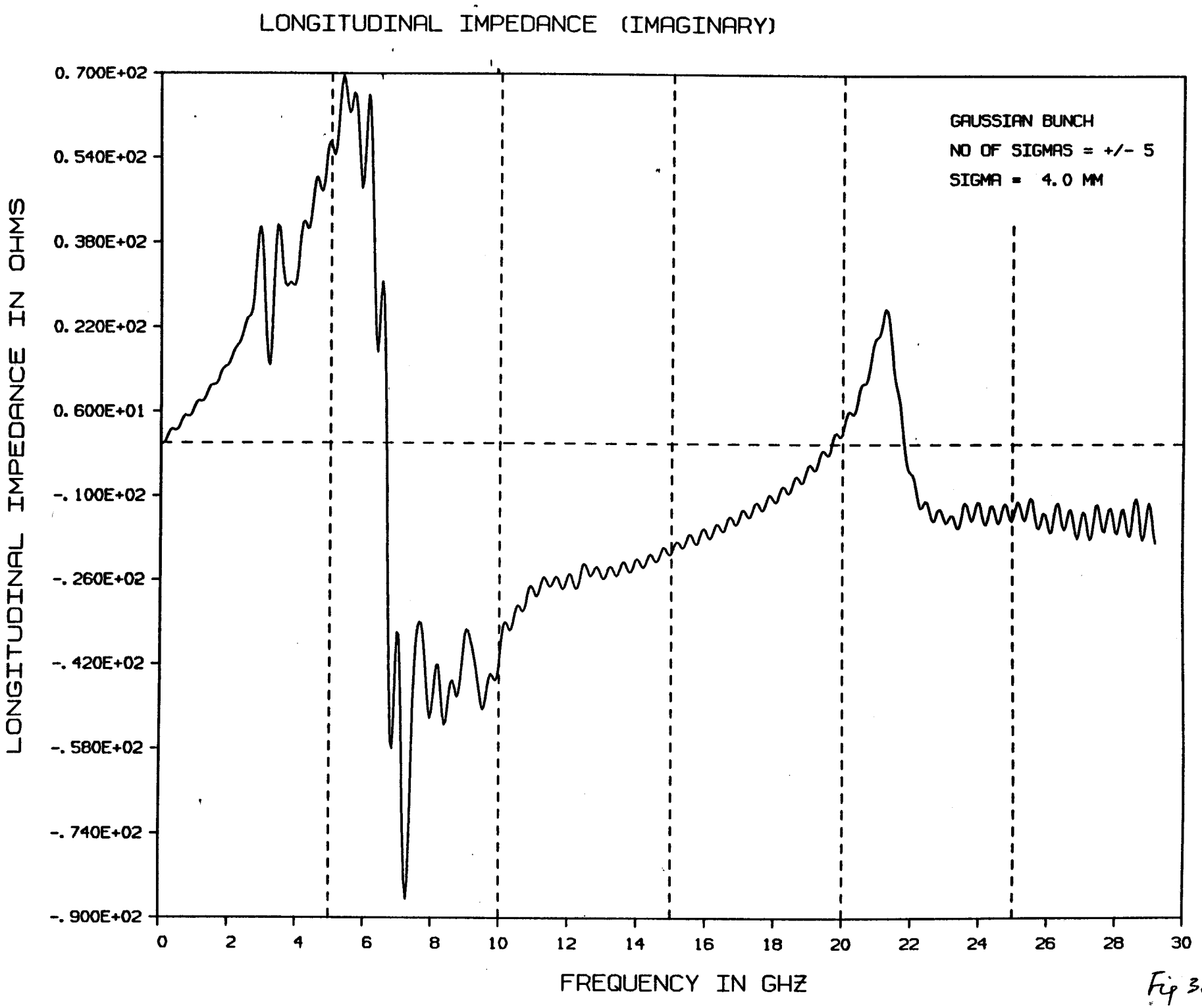
FREQUENCY IN GHZ

Fig 3b


LONGITUDINAL IMPEDANCE (IMAGINARY)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
LONGITUDINAL IMPEDANCE IN OHMS
0.700E+02
0.540E+02
0.380E+02
0.220E+02
0.600E+01
-.100E+02
-.260E+02
-.420E+02
-.580E+02
-.740E+02
-.900E+02
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FREQUENCY IN GHZ

Fig 3c

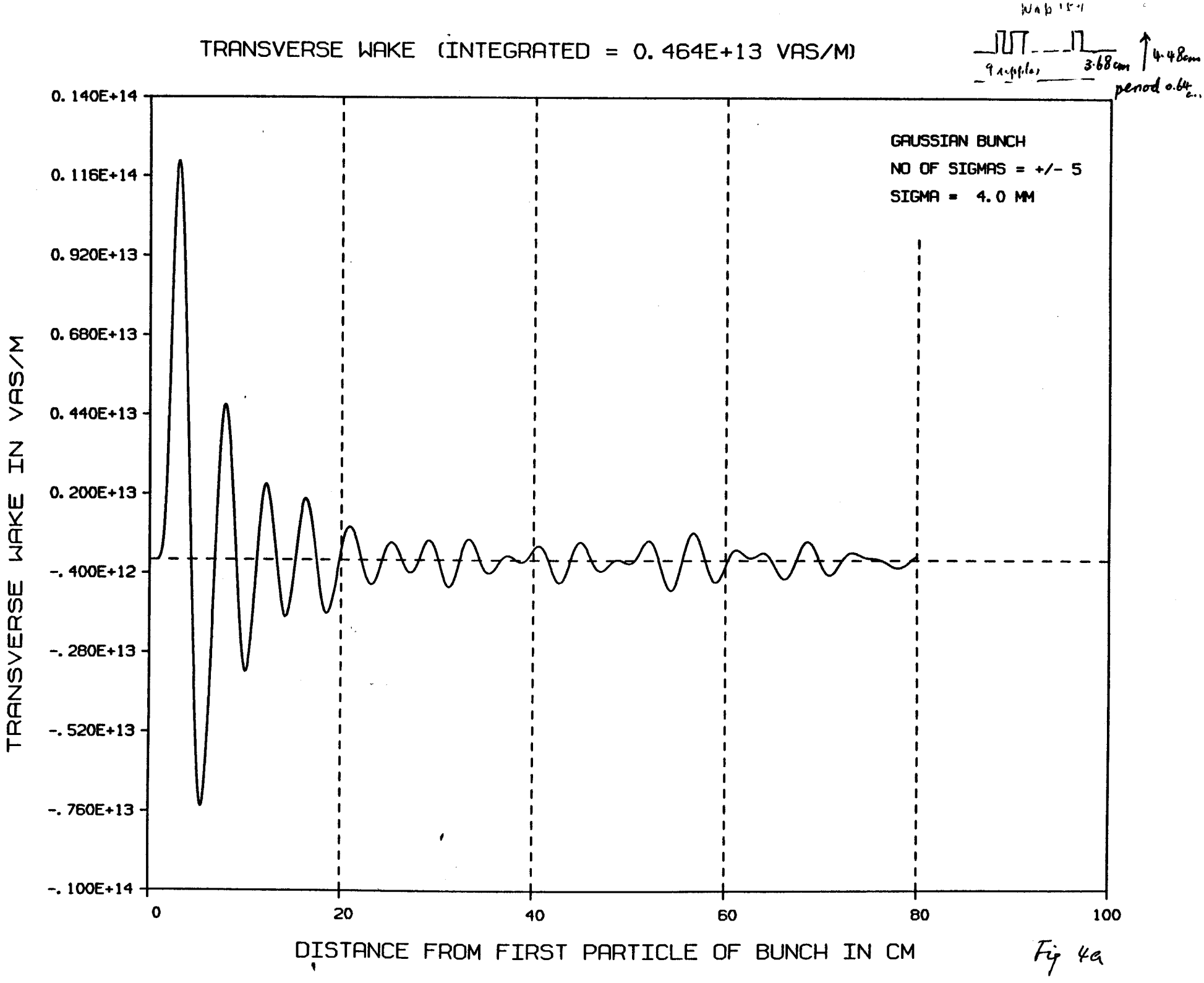
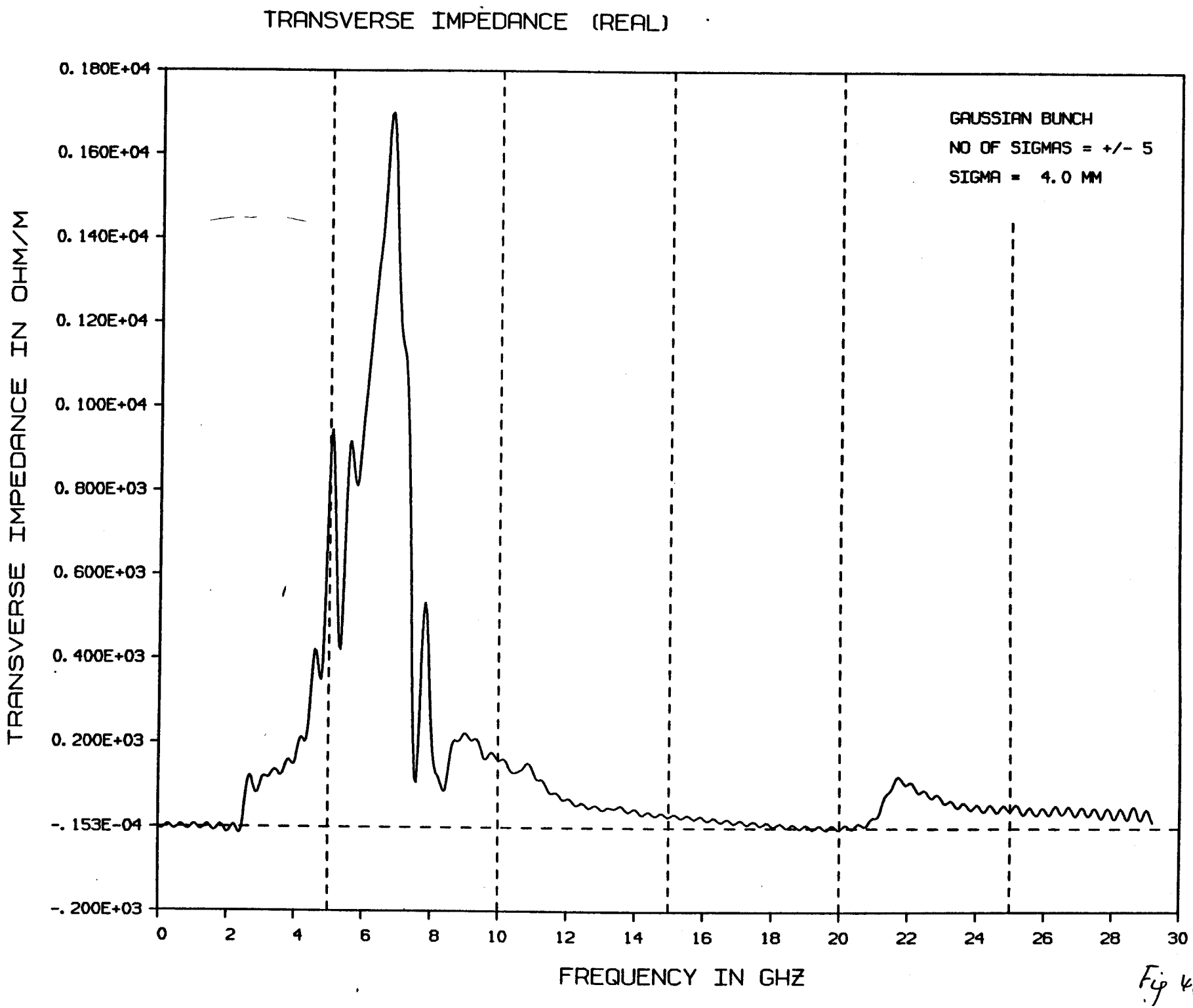

TRANSVERSE WAKE (INTEGRATED = 0.464E+13 VAS/M)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
TRANSVERSE WAKE IN VAS/M
0.140E+14
0.116E+14
0.920E+13
0.680E+13
0.440E+13
0.200E+13
-.400E+12
-.280E+13
-.520E+13
-.760E+13
-.100E+14
0
20
40
60
80
100
DISTANCE FROM FIRST PARTICLE OF BUNCH IN CM
9 ripples
3.68 cm
4.48 cm
period 0.64

Fig 4a


TRANSVERSE IMPEDANCE (REAL)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
TRANSVERSE IMPEDANCE IN OHM/M
0.180E+04
0.160E+04
0.140E+04
0.120E+04
0.100E+04
0.800E+03
0.600E+03
0.400E+03
0.200E+03
-.153E-04
-.200E+03
0
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FREQUENCY IN GHZ

Fig 4b

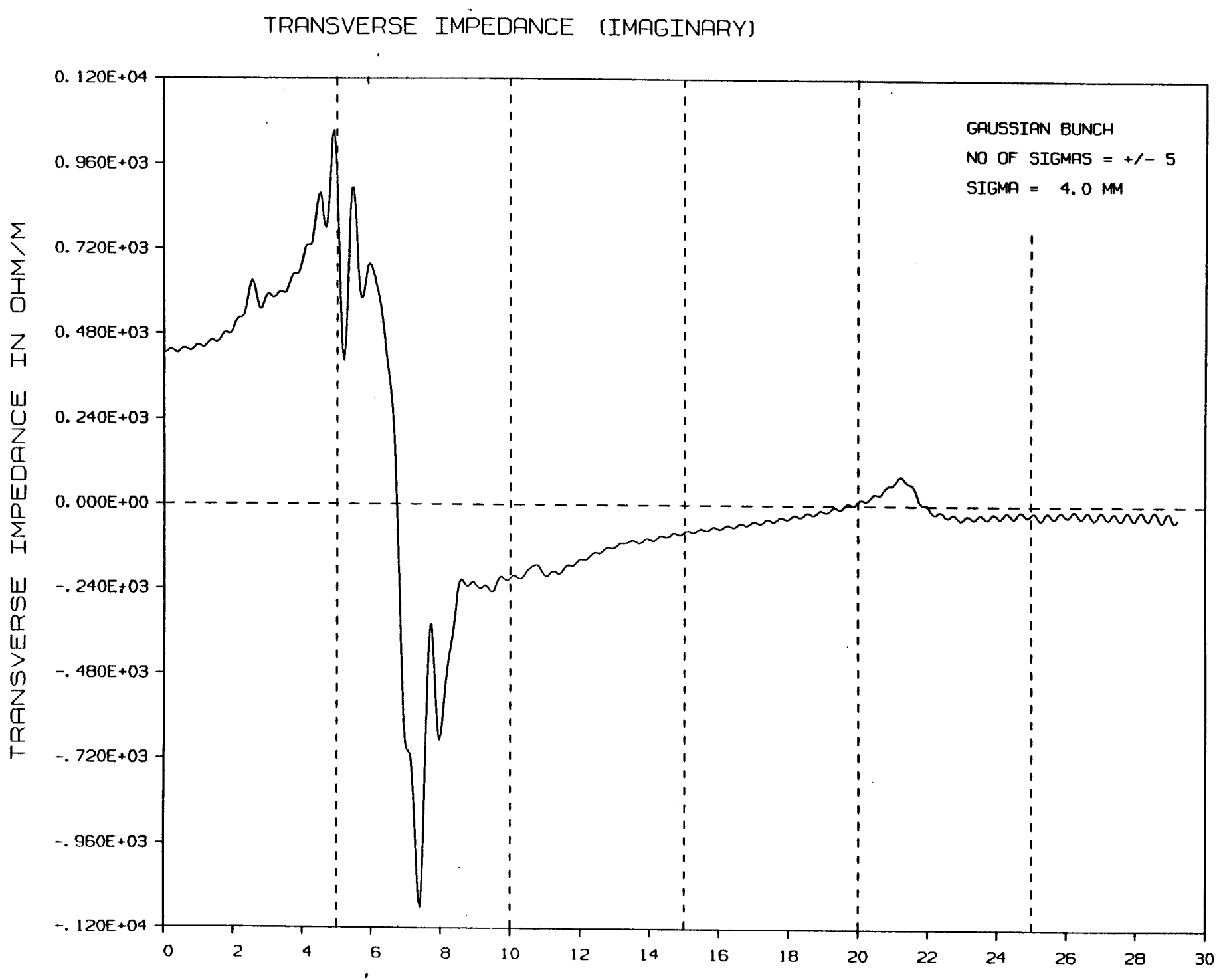
TRANSVERSE IMPEDANCE (IMAGINARY)
GAUSSIAN BUNCH
NO OF SIGMAS = +/- 5
SIGMA = 4.0 MM
TRANSVERSE IMPEDANCE IN OHM/M
0.120E+04
0.960E+03
0.720E+03
0.480E+03
0.240E+03
0.000E+00
-.240E+03
-.480E+03
-.720E+03
-.960E+03
-.120E+04
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
FREQUENCY IN GHZ

Fig 4c