PNote #465

September 5, 1985

Bumps and aperture in the Accumulator

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A brief description is given from a theoretical point of view for the formula used to create a local bump. This is general and can be applied to any machine.

Bumps: The purpose is to create a local deformation somewhere in the machine without pertubing the rest of the closed orbit.

We can achieve this through several means.

- Trim dipoles
- Shunts on dipoles
- Moving the quadrupoles

One creates a pertubation (angle θ_1) at point A and another one (angel θ_2) at point B which cancel the previous effects. If there are several betatron oscillations between A and B which is often the case, then we can use three or four (or more!) magnets.

Transfer matrix through any section

According to the solution of the equation of motion, we know that the transfer matrix between point A (s_1) and point B, (s_2) is

$$M_{21} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}\,(\cos\psi_{21} + \alpha_1 \sin\psi_{21}) & \sqrt{\beta_1\beta_2}\,\sin\psi_{21} \\ \frac{-1}{\sqrt{\beta_1\beta_2}}\left[(1+\alpha_1\alpha_2)\sin\psi_{21} + (\alpha_2-\alpha_1)\cos\psi_{21}\right] & \sqrt{\frac{\beta_1}{\beta_2}}\,(\cos\psi_{21} - \alpha_2\sin\psi_{21}) \end{pmatrix} \quad (1)$$

β and α being the Twiss parameters and ψ_{21} the phase advance between 1 and 2.

Let us assume:

X_i = position of beam at point i

X'_i = angle of beam at point i

Approximations used

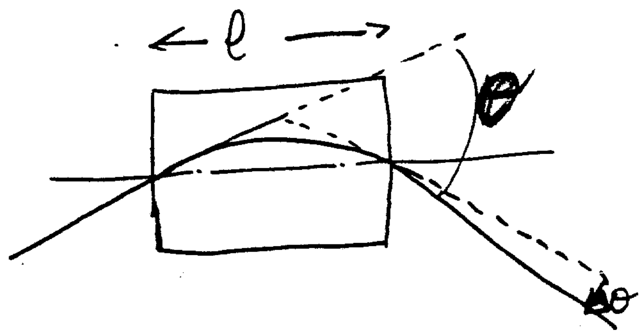

Fig 1

The deflection angle $\Delta\theta = \frac{\Delta B \ell}{B\rho}$ is small compared to the bending angle θ. Then we can consider the dipole as a thin magnet. At the entrance plane, the angle is 0 and at the exit plane it becomes θ (first order approximation).

Bumps with 3 dipole magnets

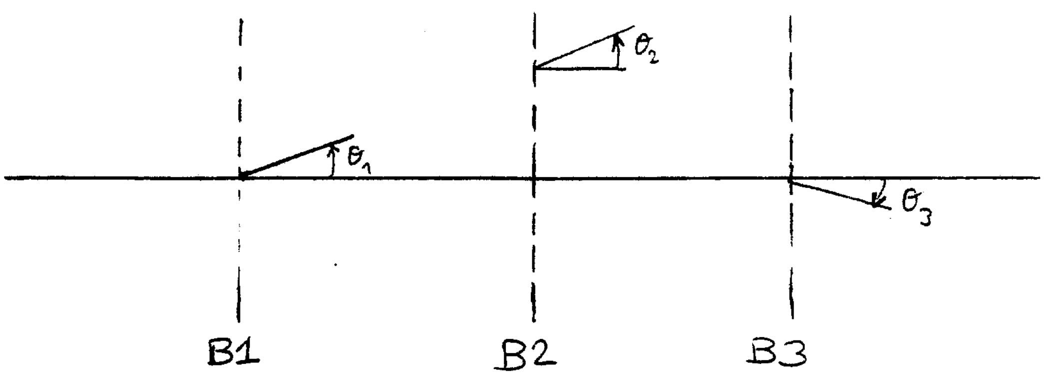

Fig 2

Entering plane 1, we have $\begin{pmatrix} X_0 \\ X'_0 \end{pmatrix}$

Exiting plane 1, we have $\begin{pmatrix} X_1 \\ X'_1 \end{pmatrix}$

$$\begin{pmatrix} X_0 \\ X'_0 \end{pmatrix}_{\substack{\text{in} \\ 1}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} X_1 \\ X'_1 \end{pmatrix}_{\substack{\text{out} \\ 2}} = \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} \tag{2}$$

These are the vector values according to the kick given in plane 1 by the bend B1. We continue, writing the vector value,s in the other 2 planes. If M_{21} is the transfer matrix between 1 and 2, we have:

$$\begin{pmatrix} X_2 \\ X'_2 \end{pmatrix}_{\substack{\text{in} \\ 2}} = M_{21} \begin{pmatrix} X_1 \\ X'_1 \end{pmatrix} \qquad \begin{pmatrix} X_3 \\ X'_3 \end{pmatrix}_{\substack{\text{out} \\ 2}} = \begin{pmatrix} X_2 \\ X'_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} X_4 \\ X'_4 \end{pmatrix}_{\substack{\text{in} \\ 3}} = M_{32} \begin{pmatrix} X_3 \\ X'_3 \end{pmatrix} \qquad \begin{pmatrix} X_5 \\ X'_5 \end{pmatrix}_{\substack{\text{out} \\ 3}} = \begin{pmatrix} X_4 \\ X'_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} \tag{4}$$

The vector $\begin{pmatrix} X'_5 \\ X'_5 \end{pmatrix}$ should be equal to zero for a local bump. Using (2) and (3) to develope (4), we obtain

$$\begin{pmatrix} X_5 \\ X'_5 \end{pmatrix} = M_{32} \left[M_{21} \begin{pmatrix} X_1 \\ X'_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} \tag{5}$$

Equation (5) becomes

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{32}\, M_{21} \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + M_{32} \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{31} \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + M_{32} \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} \tag{7}$$

Take the coefficients of the matrix equation (1), we find, for position

$$0 = \theta_1 \sqrt{\beta_1 \beta_3}\ \sin \psi_{31} + \theta_2 \sqrt{\beta_2 \beta_3}\ \sin \psi_{32} \tag{8}$$

and for angle

$$0 = \theta_1 \sqrt{\frac{\beta_1}{\beta_3}}\ (\cos\ _{31} - \alpha_3 \sin \gamma_{31}) + \theta_2 \sqrt{\frac{\beta_2}{\beta_3}}\ (\cos \psi_{32} - \alpha_3 \sin \psi_{32}) + \theta_3 \tag{9}$$

Then we can derive

$$\theta_2 = - \theta_1 \sqrt{\frac{\beta_1}{\beta_2}}\ \frac{\sin \psi_{31}}{\sin \psi_{32}} \tag{10}$$

and

$$\theta_3 = \theta_1 \sqrt{\frac{\beta_1}{\beta_3}}\ \frac{\sin \psi_{21}}{\sin \psi_{32}} \tag{11}$$

At the point 2, the position and angle of the beam are

$$\chi_2 = \theta_1 \sqrt{\beta_1 \beta_2}\ \sin \psi_{21} \tag{12}$$

$$\chi'_2 = \theta_1 \sqrt{\frac{\beta_1}{\beta_2}}\ (\cos \psi_{21} - \alpha_2 \sin \psi_{21}) \tag{13}$$

According to which type of magnet we create the pertubations with, we must fine the relationship between $\Delta\theta_i$ and the current variation ΔI (in amps) for trims and shunt dipoles or between $\Delta\theta_i$ and the displacement Δd (in mm) for quadrupoles.

$$\Delta\theta_i = \frac{\mu N \ell}{g(B\rho)}\ \Delta I \qquad \text{for dipoles} \tag{14}$$

$$\Delta\theta_i = K\ell\ \ \Delta d \qquad \text{for quadrupoles} \tag{15}$$

ℓ effective length of magnet (m)

μ permeability (H/m)

N mumber of turns

g gap of dipole (m)

$B\rho$ rigidity (m)

K strength of quadrupole (m^{-2})

Examples of bumps in the Accumulator

Using the vertical trims A2 V9, A2 V6, A2 V6 we have

Magnet	(m^{-1}) strength	β_y(m)	ψ_y (rad)
A2V9	-0.2744	18.227	11.673
A2V6	0.14902	23.991	12.861
A2V6	0.26409	22.403	15.183

The Angle θ_1 for + 1 mm of displacement in A2V6 is,

$$\theta_1 = \chi_2 \sqrt{\frac{\beta_6}{\beta^9}} \frac{\chi_2}{\beta_6\beta_9 \sin \psi_{69}} \qquad (16)$$

$$\theta_1 = 0.051 \text{ mrad}$$

$$\theta_2 = + 0.022 \text{ mrad}$$

$$\theta_3 = 0.059 \text{ mrad}$$

The characteritics of these trim vertical magnets are:

$N = 720$
$\ell = 0{,}254$ m (10")
$g = 0{,}114$ m (4.5")
$B\rho = 29$ T.m (290 kg.m)

With these characteristics, we found

$$\Delta I = 14{,}42 \, \Delta\theta_i \qquad (17)$$

by applying relationship (14).

That means for 1 mrad, ΔI should vary by approximetely 15 A.

Bumps with 4 dipoles magnets

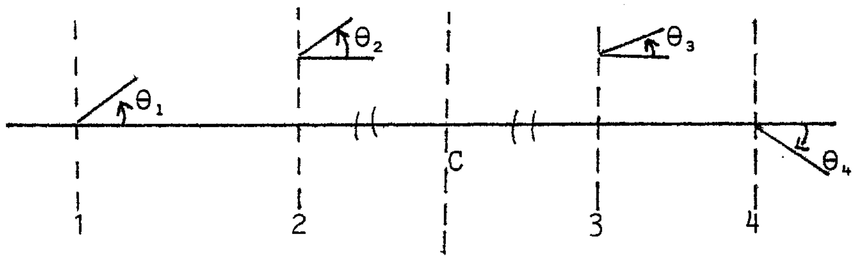

Fig 3

By applying the same method as for 3 bumps, we obtain:

$$\begin{pmatrix} X_0 \\ X'_0 \end{pmatrix}_{\text{in}_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} X_1 \\ X'_1 \end{pmatrix}_{\text{out}_1} = \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} \tag{18}$$

$$\begin{pmatrix} X_2 \\ X'_2 \end{pmatrix}_{\text{in}_2} = M_{21} \begin{pmatrix} X_1 \\ X'_1 \end{pmatrix} \qquad \begin{pmatrix} X_3 \\ X'_3 \end{pmatrix}_{\text{out}_2} = \begin{pmatrix} X_2 \\ X'_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} \tag{19}$$

$$\begin{pmatrix} X_4 \\ X'_4 \end{pmatrix}_{\text{in}_3} = M_{32} \begin{pmatrix} X_3 \\ X'_3 \end{pmatrix} \qquad \begin{pmatrix} X_5 \\ X'_5 \end{pmatrix}_{\text{out}_3} = \begin{pmatrix} X_4 \\ X'_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} X_6 \\ X'_6 \end{pmatrix}_{\text{in}_4} = M_{43} \begin{pmatrix} X_5 \\ X'_5 \end{pmatrix} \qquad \begin{pmatrix} X_7 \\ X'_7 \end{pmatrix}_{\text{out}_4} = \begin{pmatrix} X_6 \\ X'_6 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_4 \end{pmatrix} \tag{21}$$

For a closed orbit and local bump, we must have

$$\begin{pmatrix} x_7 \\ x'_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{22}$$

A transfer matrix for them all gives

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{43}\left[M_{32}\left[M_{21}\begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ \theta_4 \end{pmatrix} \tag{23}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{43}\ M_{32}\ M_{21}\begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + M_{43}\ M_{32}\begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} + M_{43}\begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_4 \end{pmatrix} \tag{24}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{41}\begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + M_{42}\begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} + M_{43}\begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_4 \end{pmatrix} \tag{25}$$

The Equation for position gives

$$\theta_1\sqrt{\beta_1\beta_4}\ \sin\psi_{41} + \theta_2\sqrt{\beta_2\beta_4}\ \sin\psi_{42} + \theta_3\sqrt{\beta_3\beta_4}\ \sin\psi_{43} = 0 \tag{26}$$

The Equation for angle gives:

$$\theta_1\sqrt{\frac{\beta_1}{\beta_4}}\left(\cos\psi_{41} - \alpha_4\sin\psi_{41}\right) + \theta_2\sqrt{\frac{\beta_2}{\beta_4}}\left(\cos\psi_{42} - \alpha_4\sin\psi_{42}\right) \tag{27}$$

$$+ \theta_3\sqrt{\frac{\beta_3}{\beta_4}}\left(\cos\psi_{43} - \alpha_4\sin\psi_{43}\right) + \theta_4 = 0$$

eliminating $\sqrt{\beta_4}$ in (26) and by looking at the coefficient of α_4 in (27) which is the same as (26), we can deduce:

$$\boxed{\begin{aligned} &\theta_1 \sqrt{\beta_1} \sin \psi_{41} + \theta_2 \sqrt{\beta_2} \sin \psi_{42} + \theta_3 \sqrt{\beta_3} \sin \psi_{43} = 0 && (28) \\ \text{and } &\theta_1 \sqrt{\beta_1} \cos \psi_{41} + \theta_2 \sqrt{\beta_2} \cos \psi_{42} + \theta_3 \sqrt{\beta_3} \cos \psi_{43} + \theta_4 \sqrt{\beta_4} = 0 && (29) \end{aligned}}$$

These equations are the conditions to get a local bump for a given closed orbit.

Another approch has been made by R. E. Peters starting from equation (24). We multiply it by $[M_{43}\ M_{32}]^{-1}$. Then we get

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{21} \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} + [M_{32}]^{-1} \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix} + [M_{42}]^{-1} \begin{pmatrix} 0 \\ \theta_4 \end{pmatrix} \qquad (30)$$

We obtain for position and angle respectively the following equations:

$$\boxed{\begin{aligned} &\theta_1 \sqrt{\beta_1} \sin \psi_{21} - \theta_3 \sqrt{\beta_3} \sin \psi_{32} - \theta_4 \sqrt{\beta_4} \sin \psi_{42} = 0 && (31) \\ \text{and } &\theta_1 \sqrt{\beta_1} \cos \psi_{21} + \theta_2 \sqrt{\beta_2} + \theta_3 \sqrt{\beta_3} \cos \psi_{32} + \theta_4 \sqrt{\beta_4} \cos \psi_{42} = 0 && (32) \end{aligned}}$$

It is easy to go from (31) (32) to (28) (29) and vice versa. If we multiply (31) by ($\cos \psi_{42}$) and (32) by ($\sin \psi_{42}$) and add both, we get exactly, equation (28)

If we multiply (31) by $(\frac{1}{\sqrt{\beta_4}} \cos \psi_{42})$ and (32) by $(\frac{1}{\sqrt{\beta_4}} \sin \psi_{42})$ and

subtract both, we get exactly, equation (29). We also can calculate the beam position and angle at the entrance of each plane 2, 3, 4 according to the transfer matrix M_{21}, M_{32} and M_{43}.

Calaculation of O's in general case

Let y_2 be the beam displacement in dipole 2 and y_3 in dipole 3. From equations (18) and (19) we find

$$\chi_2 = y_2 = \theta_1\sqrt{\beta_1\beta_2}\,\sin\psi_{21}$$

$$\theta_1 = \frac{y_2}{\sqrt{\beta_1\beta_2}\,\sin\psi_{21}} \tag{33}$$

From equation (20), the beam position in dipole 3 is:

$$\chi_4 = y_3 = \theta_1\sqrt{\beta_1\beta_3}\,\sin\psi_{31} + \theta_2\sqrt{\beta_2\beta_3}\,\sin\psi_{32} \tag{34}$$

θ_1 and y_3 being known, we can calculate θ_2 from (34). However we can express it as function of y_2 and y_3. Then

$$\theta_2 = \frac{\frac{y_3}{\sqrt{\beta_3}}\sin\psi_{21} - \frac{y_2}{\sqrt{\beta_2}}\sin\psi_{31}}{\sqrt{\beta_2}\,\sin\psi_{21}\,\sin\psi_{32}} \tag{35}$$

From equation (28), we can calculate θ_3 as a function of θ_1 and θ_2. We can also give an expression with y_2 and y_3.

$$\theta_3 = \frac{\frac{y_2}{\sqrt{\beta_2}}\sin\psi_{43} - \frac{y_3}{\sqrt{\beta_3}}\sin\psi_{42}}{\sqrt{\beta_3}\,\sin\psi_{32}\,\sin\psi_{43}} \tag{36}$$

From equation (29), we can calculate θ_4 as a function of θ_1, θ_2 and θ_3. We give also an expression with y_2 and y_3.

We find

$$\theta_4 = \frac{y_3}{\sqrt{\beta_3\beta_4}\ \sin\psi_{43}} \tag{37}$$

This is a confirmation of recipocity with respect to the beam direction. Equation (37) is the reciprocal of equation (33). If $y_3 = 0$ in (35) and (36), we find again (10) and (11).

Bump at arbitrary point "C" (between 2 and 3)

$$\begin{pmatrix} \chi_C \\ \chi'_C \end{pmatrix} + Mc_2 \begin{pmatrix} \chi_3 \\ \chi'_3 \end{pmatrix} = Mc_2 \left[M_{21} \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} \right] \tag{38}$$

$$\begin{pmatrix} \chi_C \\ \chi'_C \end{pmatrix} = Mc_1 \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} + Mc_2 \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} \tag{39}$$

The equation for position is:

$$\chi_C = \theta_1 \sqrt{\beta_1 \beta_C} \sin \psi_{C1} + \theta_2 \sqrt{\beta_2 \beta_C} \sin \psi_{C2} \tag{40}$$

The equation for angle is:

$$\chi'_C = \theta_1 \sqrt{\frac{\beta_1}{\beta_C}} \left(\cos \psi_{C1} - \alpha_C \sin \psi_{C1}\right) + \theta_2 \sqrt{\frac{\beta_2}{\beta_C}} \left(\cos \psi_{C2} - \alpha_C \sin \psi_{C2}\right) \tag{41}$$

At point C it is then possible to have either a pure displacement bump ($\chi'_C = 0$) or a pure angular bump ($\chi_C = 0$).

Pure orbit displacement $(\chi'_C = 0)$

From (41), we can find θ_2 as a function of θ_1, and from (40) we can find the position χ_C as a function of θ_1:

$$\boxed{\theta_2 = -\,\theta_1 \sqrt{\frac{\beta_1}{\beta_2}}\;\frac{\cos\psi_{C1} - \alpha_C \sin\psi_{C1}}{\cos\psi_{C2} - \alpha_C \sin\psi_{C2}}} \tag{42}$$

$$\boxed{\chi_C = \theta_1 \sqrt{\beta_1\beta_C}\;\frac{\sin\psi_{21}}{\cos\psi_{C2} - \alpha_C \sin\psi_{C2}}} \tag{43}$$

because $\sin(\psi_{C1} - \psi_{C2}) = \sin\psi_{21}$

From (28), we find θ_3 and from (29) we find θ_4. After sumplification:

$$\boxed{\theta_3 = -\,\theta_1 \sqrt{\frac{\beta_1}{\beta_3}}\;\frac{\sin\psi_{21}}{\sin\psi_{43}}\;\frac{(\cos\psi_{4C} + \alpha_C \sin\psi_{4C})}{(\cos\psi_{C2} - \alpha_C \sin\psi_{C2})}} \tag{44}$$

$$\boxed{\theta_4 = \theta_1 \sqrt{\frac{\beta_1}{\beta_4}}\;\frac{\sin\psi_{21}}{\sin\psi_{43}}\;\frac{(\cos\psi_{3C} + \alpha_C \sin\psi_{3C})}{(\cos\psi_{C2} - \alpha_C \sin\psi_{C2})}} \tag{45}$$

Pure orbit angle $(\chi_C = 0)$

From (40), we can find θ_2 as a function of θ_1 and from (41), we can find the angle χ'_C as a function of θ_1:

$$\boxed{\theta_2 = -\,\theta_1 \sqrt{\frac{\beta_1}{\beta_2}}\ \frac{\sin \psi_{C1}}{\sin \psi_{C2}}} \tag{46}$$

and

$$\boxed{\chi'_C = -\,\theta_1 \sqrt{\frac{\beta_1}{\beta_C}}\ \frac{\sin \psi_{21}}{\sin \psi_{C2}}} \tag{47}$$

As before, from (28) and (29), we get

$$\boxed{\theta_3 = \theta_1 \sqrt{\frac{\beta_1}{\beta_3}}\ \frac{\sin \psi_{21}\, \sin \psi_{4C}}{\sin \psi_{43}\, \sin \psi_{C2}}} \tag{48}$$

$$\boxed{\theta_4 = -\,\theta_1 \sqrt{\frac{\beta_1}{\beta_4}}\ \frac{\sin \psi_{21}\, \sin \psi_{3C}}{\sin \psi_{43}\, \sin \psi_{C2}}} \tag{49}$$

where $\sin(\psi_{43} - \psi_{4C}) = -\sin \psi_{3C}$ has been used for simplication.

Bump with 3 quadrupoles

We consider the general case shown on figure 4. If a quadrupole is focusing in the horizontal plane then it is defocusing in the vertical plane. We must specify in the figure which plane is considered. We can apply equations (2) to (7)
where θ_1 is replaced by $K_1\ d_1$
θ_2 " " " " $K_2\ d_2$
θ_3 " " " " $K_3\ d_3$

K_1 and K_3 being negative for a focusing qradrupole in the horizontal plane.

Equation (7) becomes:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{31} \begin{pmatrix} 0 \\ K_1 d_1 \end{pmatrix} + M_{32} \begin{pmatrix} 0 \\ K_2 d_2 \end{pmatrix} + \begin{pmatrix} 0 \\ K_3 d_3 \end{pmatrix} \qquad (50)$$

We can develop this matrix equation or we can replace θ_i by $K_i d_i$ in equation (10) to (13).

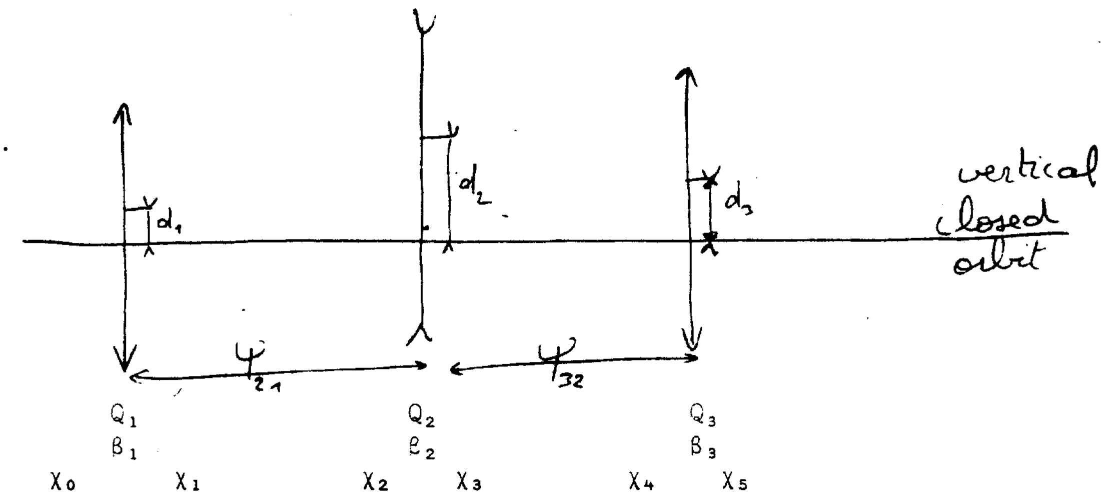

Fig 4. Quad displaced in vertical plane

We find

$$\boxed{d_2 = - d_1 \frac{K_1}{K_2} \sqrt{\frac{\beta_1}{\beta_2}} \frac{\sin \psi_{31}}{\sin \psi_{32}}} \qquad (51)$$

$$\boxed{d_3 = d_1 \frac{K_1}{K_3} \sqrt{\frac{\beta_1}{\beta_3}} \frac{\sin \psi_{21}}{\sin \psi_{32}}} \qquad (52)$$

At the quadrupole 2, the position and the angle of the beam are:

$$\boxed{\chi_2 = K_1 d_1 \sqrt{\beta_1 \beta_2} \sin \psi_{21}} \qquad (53)$$

$$\boxed{\chi'_2 = K_1 d_1 \sqrt{\frac{\beta_1}{\beta_2}} (\cos \psi_{21} - \alpha_2 \sin \psi_{21})} \qquad (54)$$

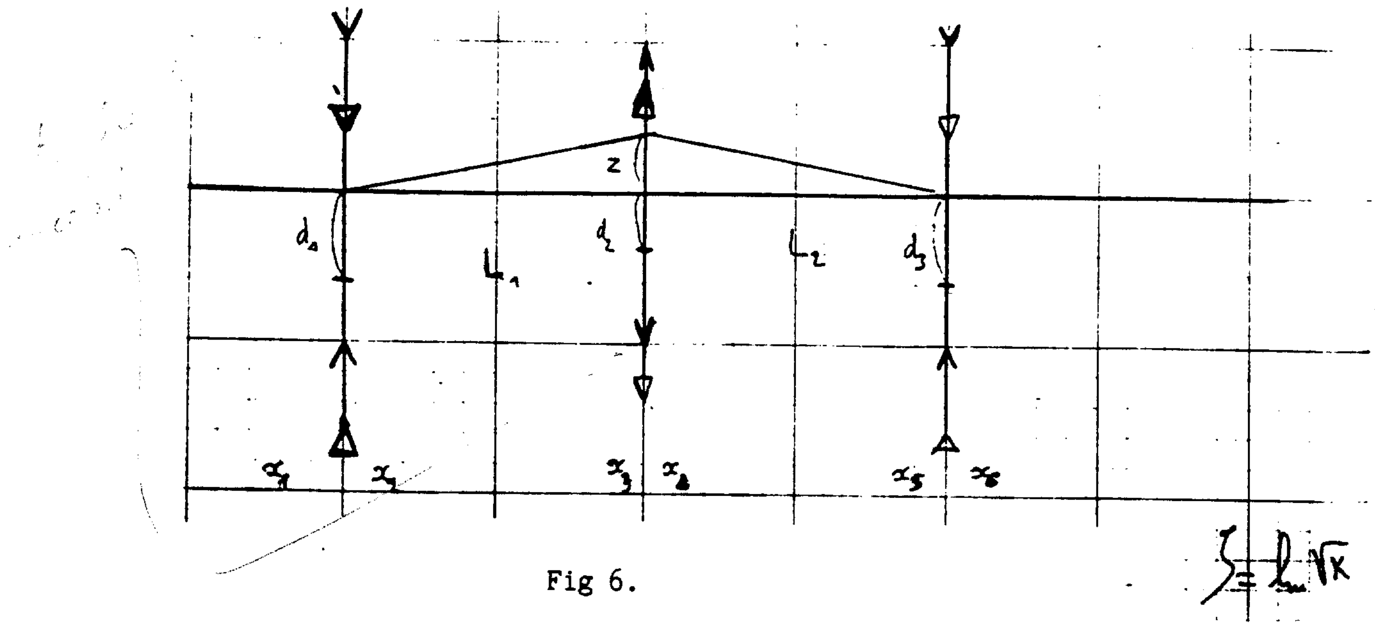

Fig 6.

then computed from the TPOT generated Twiss parameters of the lattice. The fractional part of the tune of a given particle for a given turn is then calculated from the phase advance from the previous turn.

Figures 1-15 are y tune distributions for runs with different values of the Neutralization factor N. Each particle has been weighted by its "emittance" with the whole distribution then renormalized to the total number of particles. This distribution should then be like the power distribution that one gets from a spectrum analyzer of a transverse Schottky signal. The beam current is 100 mAmp and the emittance is 2π mm-mrad. Figures 16-30 are semilog plots of the same distributions. All figures have a bin size in tune of 0.0002 unless noted on the plot. Table 1 summarizes the mean and variance of these distributions.

Table 2 summarizes the means, peaks, and maximum tunes of these distributions. The peaks are my estimation of what they are. The maximums are determined by integrating the histograms until the given percentage (either 99% or 99.9%) of total is reached. Figures 31-34 are plots of these tunes versus the Neutralization factor N.

The averages appear to be linear in N. The peaks are not as nice. The distributions are non-Gaussian and asymmetric with an increasing tail at higher tune and an increasing width as N increases. Estimating the peaks without fitting to a known distribution is therefore, given the statistics, somewhat subjective. Also given the distributions the peaks need not be linear in N.

The maximums appear to be linear above $N = 0.20$. Below that one gets into the smear caused by the momentum distribution and the discreetness of the calculation. The two maximums also have different slopes. The question therefore arises what is the true "maximum". Equation 4 of the introduction gives a value of 0.0399.

But how do these slopes vary with the Current I and the Emittance E? Table 3 and Figure 35 show how they vary with the Current I. Table 4 and Figure 36 show how they vary with the Emittance E. As can be seen, the slopes are linear with the current I and with 1/E as expected from Equation 4.

Application to the Accumulator

Consider the 3 quadrupoles A1Q1 (Q_1)

A1Q2 (Q_2)

A1Q3 (Q_3)

We want a bump 1 mm up in A1Q2. Thus, χ_2 = 1mm

	Q_1	Q_2	Q_3
$K(m^{-2})$	0,361	-0,361	0,360
β (m)	17,96	32,64	13,41
ψ	0,9374	0,9973	1,0872

ψ_{32} = 0,0899 ψ_{31} = 0,1498 ψ_{21} = 0,0599

d_1 = 1,91 mm

d_2 = 2,36 mm

d_3 = 1,48 mm

Bumps with 4 quadrupoles

As before, we can apply equation (25) for 4 quadrupoles where θ is replaced by Kd

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{41} \begin{pmatrix} 0 \\ K_1 d_1 \end{pmatrix} + M_{42} \begin{pmatrix} 0 \\ K_2 d_2 \end{pmatrix} + M_{43} \begin{pmatrix} 0 \\ K_3 d_3 \end{pmatrix} + \begin{pmatrix} 0 \\ K_4 d_4 \end{pmatrix} \tag{55}$$

The Equation for position gives

$$K_1 d_1 \sqrt{\beta_1 \beta_4} \sin \psi_{41} + K_2 d_2 \sqrt{\beta_2 \beta_4} \sin \psi_{42} + K_3 d_3 \sqrt{\beta_3 \beta_4} \sin \psi_{43} = 0$$

$$\boxed{K_1 d_1 \sqrt{\beta_1} \sin \psi_{41} + K_2 d_2 \sqrt{\beta_2} \sin \psi_{42} + K_3 d_3 \sqrt{\beta_3} \sin \psi_{43} = 0} \tag{56}$$

The equation for angle gives

$$\boxed{K_1 d_1 \sqrt{\beta_1} \cos \psi_{41} + K_2 d_2 \sqrt{\beta_2} \cos \psi_{42} + K_3 d_3 \sqrt{\beta_3} \cos \psi_{43} + K_4 d_4 \sqrt{\beta_4} = 0} \tag{57}$$

Calculation of d's in the general case

Let y_2 be the beam displacement in quadrupole 2 and y_3 in quadrupole 3. From equations (18) and (19), where θ is replaced by Kd, we find

$$\chi_2 = y_2 = K_1 d_1 \sqrt{\beta_1 \beta_2} \sin \psi_{21}$$

$$\boxed{d_1 = \frac{y_2}{K_1 \sqrt{\beta_1 \beta_2} \sin \psi_{21}}} \qquad (58)$$

Following the same method as for the dipole, we find:

$$\boxed{d_2 = \frac{\frac{y_3}{\sqrt{\beta_3}} \sin \psi_{21} - \frac{y_2}{\sqrt{\beta_2}} \sin \psi_{31}}{K_2 \sqrt{\beta_2} \sin \psi_{21} \sin \psi_{32}}} \qquad (59)$$

$$\boxed{d_3 = \frac{\frac{y_2}{\sqrt{\beta_2}} \sin \psi_{43} - \frac{y_3}{\sqrt{\beta_3}} \sin \psi_{42}}{K_3 \sqrt{\beta_3} \sin \psi_{32} \sin \psi_{43}}} \qquad (60)$$

$$\boxed{d_4 = \frac{y_3}{K_4 \sqrt{\beta_3 \beta_4} \sin \psi_{43}}} \qquad (61)$$

Bump at an Arbitray Point "C" (between Q2 and Q3)

Following the same method as for the dipole, equations (40) and (41) become:

$$X_C = K_1 d_1 \sqrt{\beta_1 \beta_C} \sin \psi_{C1} + K_2 d_2 \sqrt{\beta_2 \beta_C} \sin \psi_{C2} \tag{62}$$

$$X'_C = K_1 d_1 \sqrt{\frac{\beta_1}{\beta_C}} (\cos \psi_{C1} - \alpha_C \sin \psi_{C1}) + K_2 d_2 \sqrt{\frac{\beta_2}{\beta_C}} (\cos \psi_{C2} - \alpha_C \sin \psi_{C2}) \tag{63}$$

All equations (42) to (49) are the same except that we should replace θ_i by $K_i d_i$.

Aperture

The accumulator has been designed for an aperture of 10π mm mrad in both planes.

However the measured acceptances are

$$\varepsilon_H = 4\pi \quad \text{mm.mrad}$$

$$\varepsilon_V = 5\pi \quad \text{mm.mrad}$$

Table 1 gives the calculated aperture where possible restrictions can exist. Taking into account the β variations at each place, there are no restrictions according to know physical apertures. Nevertheless, we can see from table 1, the stochastic cooling devices could limit the vertical acceptances and the Lambertson magnet could limit the horizontal acceptance. Schottky P.U.'s could give limitations in both planes.

An error in the alignement of these devices, or in the closed orbit or in β values would explain these restrictions.

Table 1

	Names	Physical Dimensions				Acceptance	
		X	Y	βx	βy	Calculated	
		mm	mm	m	m	$\frac{\varepsilon x}{\pi}$	$\frac{\varepsilon y}{\pi}$
A10	H Schottkey PU V Schottky PU	15 15	15 15	8.8 → 16 8.8 → 16	7.2 → 17 7.2 → 17	25 → 14 25 → 14	31 → 13 31 → 13
A10 A50	μwave absorbers	19	19	8.8 → 16	7.2 → 17	41 → 22	50 → 21
A10 A50	Dampers			8.8 → 16	7.2 → 17		
A2Q16	Lambertson	18	17.46	13 → 23	7 → 8.5	25 → 14	35
A1Q4	Septum	20 (out) 29.7(in)	19.8	13.9	6.6	28	60
A1Q14	Inject. Kicker			8.6	8.2		
A2Q16	Extract Kicker			15.4	15.3		
A50	RF ARF1 ARF2 ARF3	 49.15 46 46	 49.15 46 46	 12 20 20	 27 9 9	 200 100 100	 88 200 200
Stochastic Cooling:				β variations			
A10		300/2	30/2	15.9	15.9	no restr.	14
A20		300/2	30/2	8.11	8.03	no restr.	28
A30		300/2	30/2	15.9	15.9	no restr.	14
A60		300/2	30/2	8.11	8.03	no restr.	28

Acknolegement:

W. Kells read this note and made several useful comments. I had a very friendly support from J. Peoples and his group and it was a great pleasure to participate to the commissioning of TeV 1.