Fermilab

PBAR #437

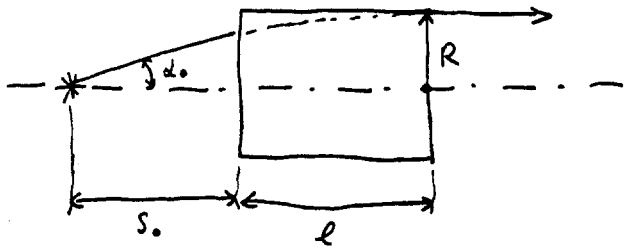
DESIGN PARAMETERS FOR LONG (PLASMA?) - LENSES

P. SIEVERS

NOVEMBER 15, 1984

Design Parameters for Long (PLASMA?) - Lenses

assume point → parallel optics



$$\alpha_0 = R\sqrt{k}\,\sin\sqrt{k}\,l$$

$$s_0 = \frac{1}{\sqrt{k}\,\mathrm{tg}\sqrt{k}\,l}$$

a) For $\sqrt{k}\,l \leq 45°$: $\sin\sqrt{k}\,l \approx \sqrt{k}\,l$ (±10%)

$$\alpha_0 \approx R K l$$

$$K \sim \frac{I_{eff}}{R^2}$$, I_{eff} : current through lens

$$\rightarrow \underline{I \approx \alpha_0 R/l}$$

b) Minimum current for a given α_0: Threshold current I_k :

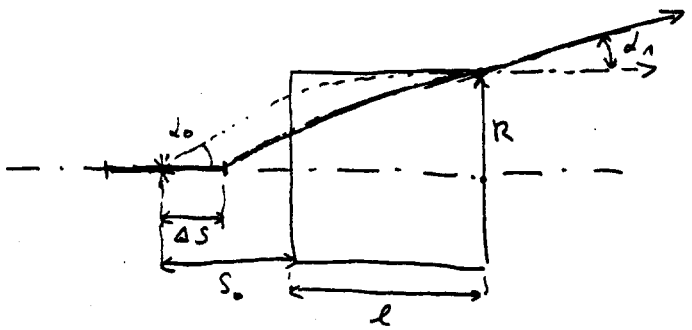
$$\sin\sqrt{K}\,l = \pi/2$$

$$\alpha_0 = R\sqrt{k}$$

$$\rightarrow \underline{I_k \approx \alpha_0^2}$$ (independent of R)

however $s_0 \approx 0$

c) "Geometric aberration" due to axially extended source (target length):



replacing $\sin \Gamma\ell$, $\cos \Gamma\ell$ and Γ in the transfer matrix by S_0, α_0 as defined above:

$$\alpha_1 = \frac{\alpha_0^2 \Delta S}{R\left[1 - \frac{\alpha_0^2 S_0}{R^2} \Delta S\right]}$$

for small S_0:

$$\alpha_1 \approx \frac{\alpha_0^2 \Delta S}{R}$$

"Emittance" at exit of lens:

$$\bar{\varepsilon} \sim \alpha_1 \cdot R \approx \alpha_0^2 \Delta S \text{ (as expected from } \bar{\varepsilon} \text{ at target end)}$$

d) Design parameters for FERMI-Li-Lens:

$P_0 = 8.9$ GeV/c

$R = 0.01$ m

$I = 500$ kA

$g = 1000$ T/m

$\sqrt{k} = 5.8\ m^{-1}$

$\varphi = 49.9°$ $(\varphi = \sqrt{k} \cdot l)$

$S_0 = 0.145$ m

$\alpha_0 = 44.4$ mrad

$\Delta S = 0.025$ m

$\alpha_1 = 4.93 \cdot 1.077 = 5.31$ mrad

e) Parameters for a FERMI-PLASMA-Lens:

assume that acceptance is not changed, ~~ie~~.

I assume: $\underline{\underline{\alpha_0 = 45 \text{ mrad}}}$

Fig 1 shows as a function of the length of the lens l the required current I and the resulting focal distance S_0 for various radii R:

e.g. for $l = 0.15$ m ~~to~~ and $R = 1.$ cm →

$S_0 = 0.14$ m, $I = 500$ kA (FERMI-Li-Lens)

or for $l = 0.60$ m and $R = 2.$ cm →

$S_0 = 0.085$ m, $I = 320$ kA

The threshold current is 300 kA

f) Fig 2 shows the same story, but I and l as a function of S_0 for $\alpha_0 = 45$ mrad and $R = 1$ cm.
It also shows that α_1 does not change rapidly with S_0

g) Fig 3 shows the same plot as in Fig 1, however for a smaller acceptance, i.e. $\alpha_0 = 35$ mrad:
I and S_0 for $R = 1$ cm resp. $R = 1.5$ cm as a function of l.
The threshold current for this acceptance is 180 kA.

Conclusion: The minimum "Threshold"-current is only defined by the acceptance (i.e. α_0) and not by the radius (of the lens). Instead of pushing for current, one should push for length and to some extent for radius. The latter allows for practical focal distances to locate the target. The target length is of importance for the emittance but barely for the parameters of the lens.
A long, low current plasma lens is only of interest if big diameter beams ($R \sim 2$ cm) can be matched into the beam line. If not, ~~high~~ short, high current PLASMA or Li-lenses are equivalent, (except for absorption, scattering ??).

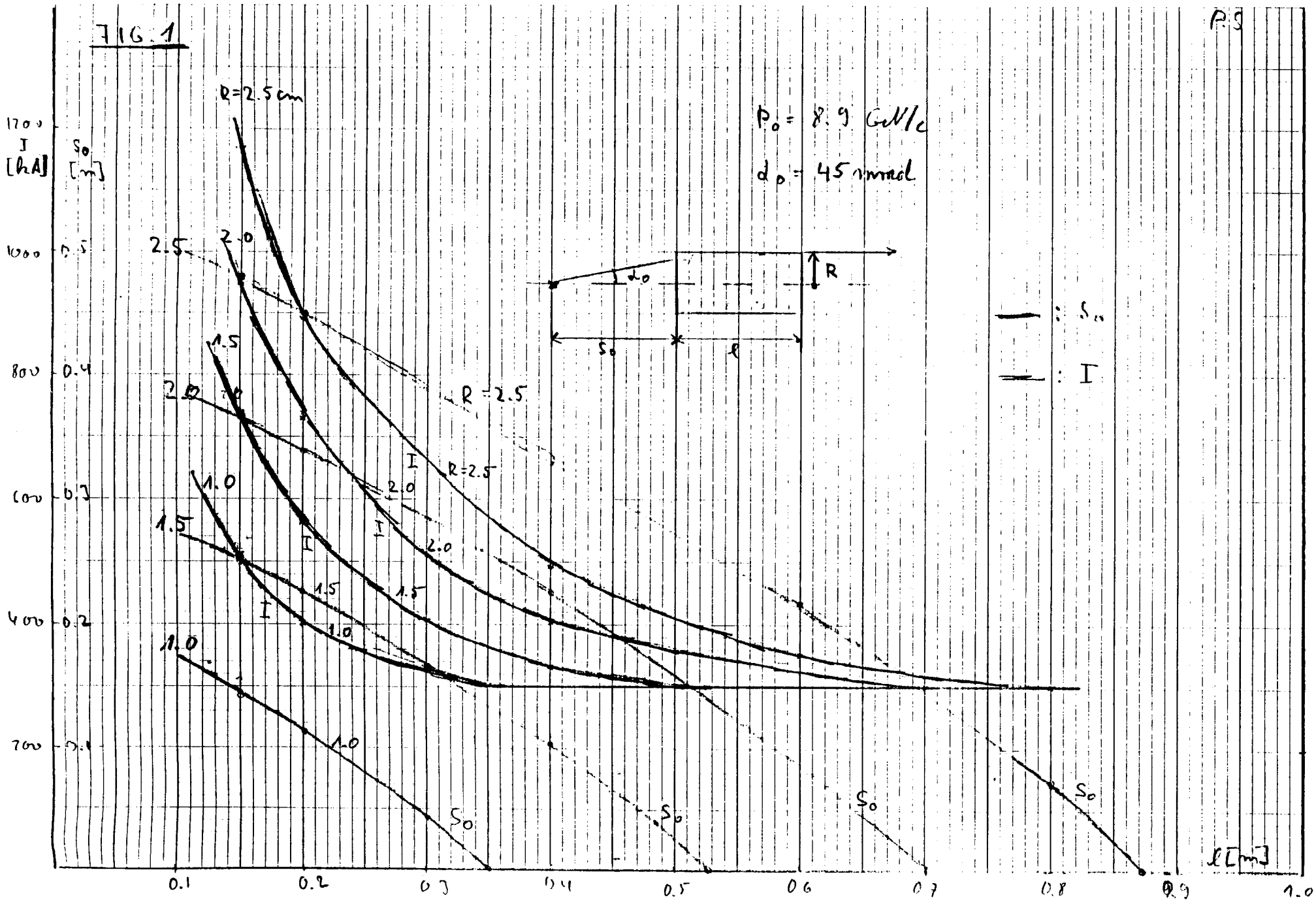
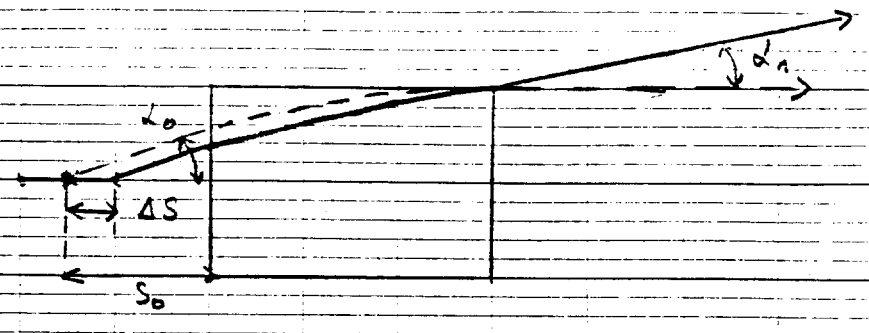


FIG. 1
P.S
P_0 = 8.9 GeV/c
d_0 = 45 mrad
R=2.5 cm
R=2.5
I
S_0
α_0
R
S_0
ℓ
: S_0
: I
I [kA]
S_0 [m]
1700
1000
800
600
400
200
0.5
0.4
0.3
0.2
0.1
2.5
2.0
1.5
1.0
ℓ [m]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

FIG. 2



$$\alpha_1 = \frac{\alpha_0^2 \, \Delta S}{R\left[1 - \left(\frac{\alpha_0}{R \cdot S_0}\right)^2 \frac{\Delta S}{S_0}\right]}$$

$\alpha_0 = 45$ mrad

$R = 0.01$ m

$\Delta S = 0.025$ m

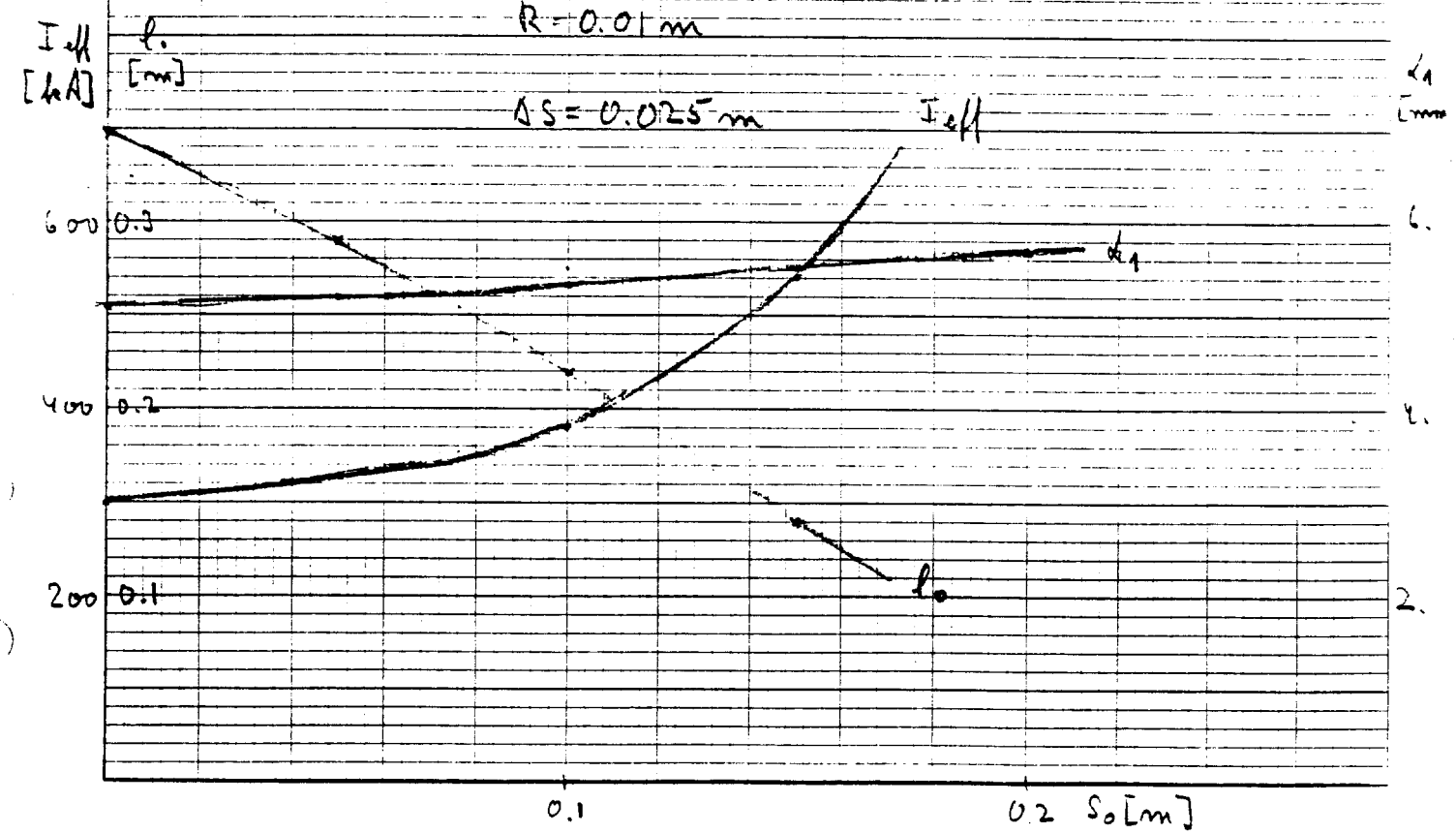
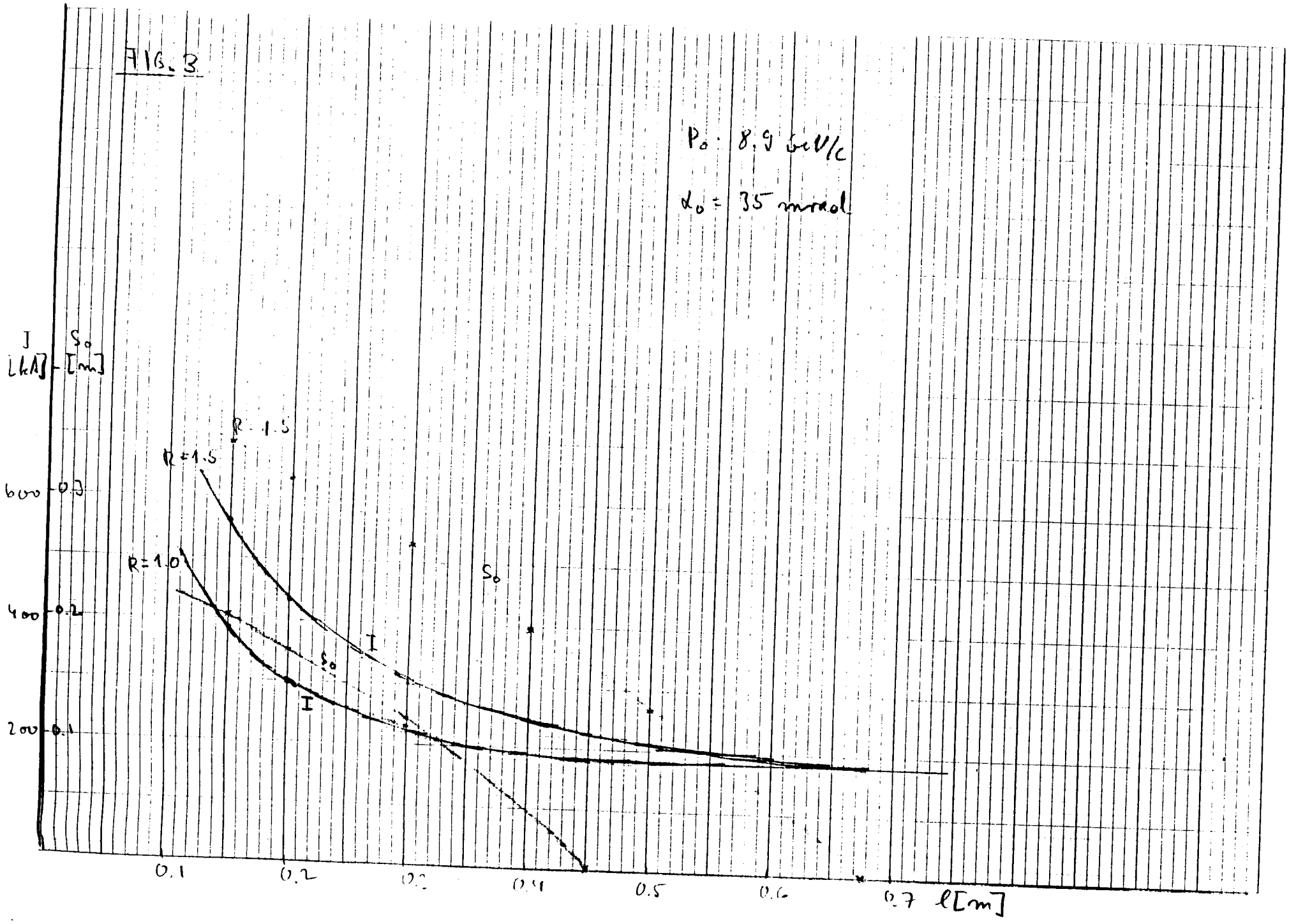


FIG. 3


P_0 = 8.9 GeV/c
α_0 = 35 mrad
J [kA]
S_0 [mm]
600
0.3
400
0.2
200
0.1
R = 1.5
R = 1.5
R = 1.0
S_0
S_0
I
I
0.1
0.2
0.3
0.4
0.5
0.6
0.7
ℓ [m]

46 0860