



Fermilab

\bar{p} Note #380

End Field of Small-Aperture Quadrupoles

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April 18, 1984

The importance of magnet end fields for both dipoles and quadrupoles has been fully explained in \bar{p} Note #313, "Tunes with Fringe Field of Magnets - Numerical Integration of Exact Equations". A request for the field measurement near magnet ends was made by me in November '83 in a memo sent to Bruce Brown, Dave Harding and Bob Peters. The memo is attached here as Appendix. Recently, Bob Peters has done a very good measurement of the end field (gradient) of a small-aperture quadrupole, SQC#147, using a gradient-type Hall probe. In the near future, he will make similar measurements on large-aperture quadrupoles and small- and large-aperture dipoles. Meanwhile, this note is simply a short summary of his results. Details of the measurement itself should be available from him and the use of these measured end fields in the calculation of tunes will be reported (I hope) by Sandro Ruggiero (debuncher) and by Steve Holms (accumulator) once data for other magnet ends become available.

From Eq. (A-5), we see that the field gradient on the median plane ($Y=0$) takes the form

$$G(X,Z) \equiv \partial B_Y / \partial X \propto f_2(Z) - (1/4)f_2''(Z) \cdot X^2 + \dots$$

Since the higher-order terms are ignored and the effects of current-carrying conductors are not included in this expression, the value of $|X|$ should not be too large. The over-all characteristics of the end field can be seen in Fig. 1 which was produced by Bob Peters. Near $Z=0$, $f_2(Z)$ has the inflection point, $f_2''(Z)=0$, and the field gradient becomes independent of X . For $Z<0$ (inside), $f_2''(Z)<0$ so that $G(X,Z)>G(0,Z)$ while for $Z>0$ (outside), $G(X,Z)<G(0,Z)$. This property of the gradient is shown in Fig. 2 although one can see it in Fig. 1 as well. The measured values of $f_2(Z)$ are listed in Table 1 together with $f_2'(Z)$ and $f_2''(Z)$ which are obtained in two different manners. The agreement of column (3) with column (4), and column (2) with column (5), should be considered as satisfactory for our purpose. A possible representation of $f_2(Z)$ for the integration of equations of motion is

$$f_2(Z) = \{ 1. + 0.011 (Z+3'')^4 \}^{-1} ; \quad -3'' \leq Z \leq +5''$$

Table 1.

column (1): data at $X=0$, normalized to $f_2(-4'')=1$.
 column (2): $f_2''(Z)$ obtained from data at $-2'' \leq X \leq 2''$, least-squares.
 column (3): f_2' of column (2) integrated with $f_2'(-4'')=0$
 column (4): numerical derivative of $f_2(Z)$, column (1)
 column (5): numerical derivative of $f_2'(Z)$, column (4)
 column (6): $f_2(Z)$ evaluated with the expression given on p. 1

	(1)	(2)	(3)	(4)	(5)	(6)
Z	f_2	f_2''	f_2'	f_2'	f_2''	f_2
-4''	1.000	-0.017	0.	0.		
-3''	.999	-0.015	-0.016	-0.007		1.
-2.5''	.995	-0.026	-0.026	-0.013	-0.029	.999
-2''	.985	-0.056	-0.047	-0.036	-0.092	.989
-1.5''	.952	-0.142	-0.096	-0.106	-0.193	.947
-1''	.872	-0.261	-0.198	-0.230	-0.266	.850
-0.5''	.725	-0.196	-0.312	-0.347	-0.142	.699
0.	.541	0.062	-0.345	-0.358	0.074	.529
0.5''	.379	0.150	-0.293	-0.286	0.182	.377
1''	.258	0.123	-0.224	-0.195	0.170	.262
1.5''	.182	0.086	-0.171	-0.125	0.105	.181
2''	.128	0.056	-0.136	-0.087	0.063	.127
2.5''	.093	0.036	-0.113	-0.059	0.045	.090
3''	.068	0.026	-0.097	-0.042	0.027	.066
3.5''	.050					.048
4''	.037					.036
5''	.019					.022

Fig. 1 End field of SQC147 measured by R. Peters

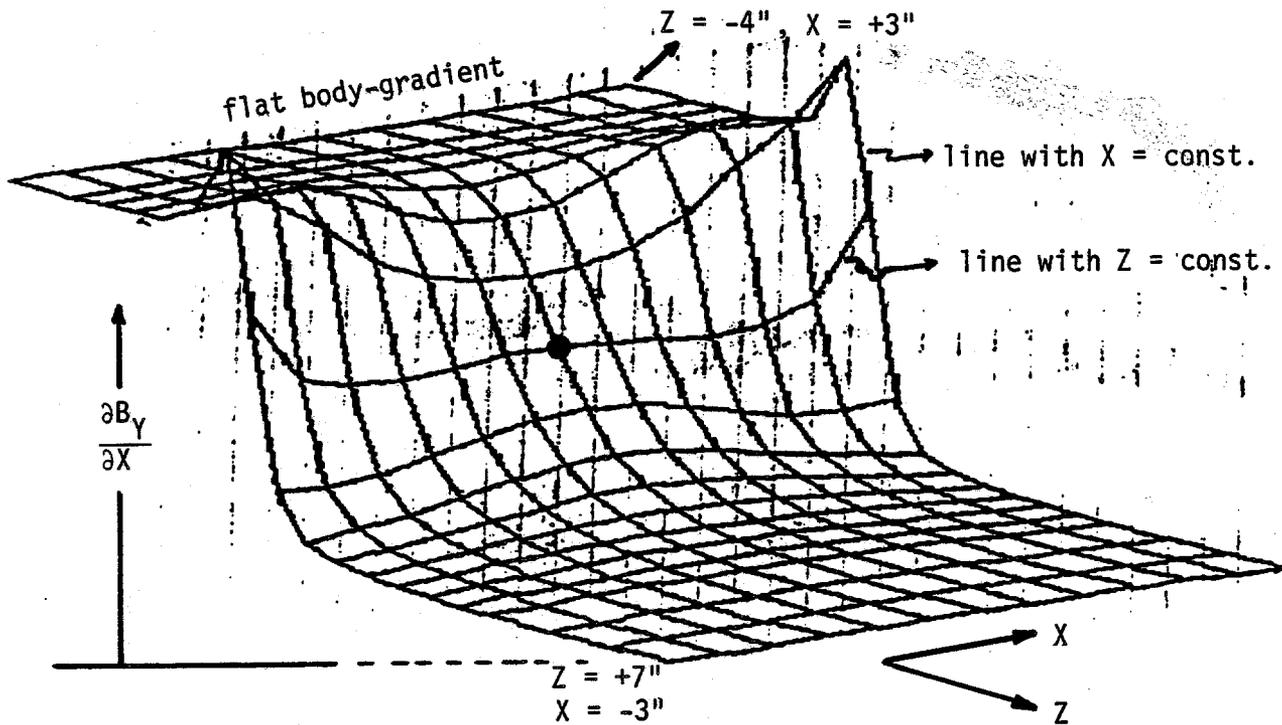
$X = -3.0''$ (0.5'') $3.0''$; $Y = 0$ (median plane)

$Z =$ from $-4''$ to $+7''$

pole tip diameter = $3.5''$

Field with quadrupole symmetry

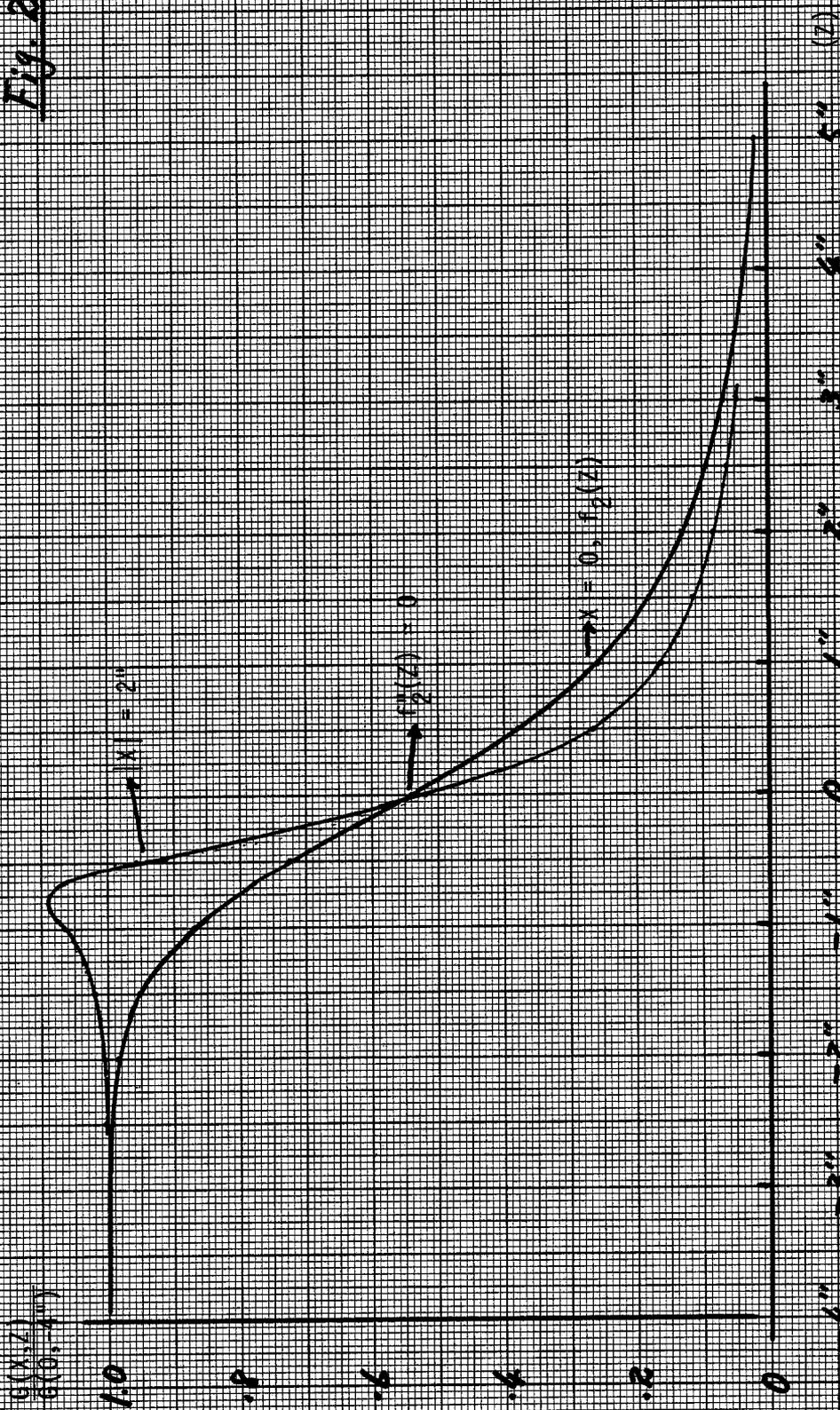
$$B_Y \propto X f_2(Z) - \frac{1}{12} (X^3 + 3XY^2) f_2''(Z) + \dots$$



Gradient $\partial V(X,Z)/\partial X$ on the median plane ($Y=0$), small-aperture quadrupole S00147

(no iron beyond $Z=0$)

Fig. 2





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APPENDIX

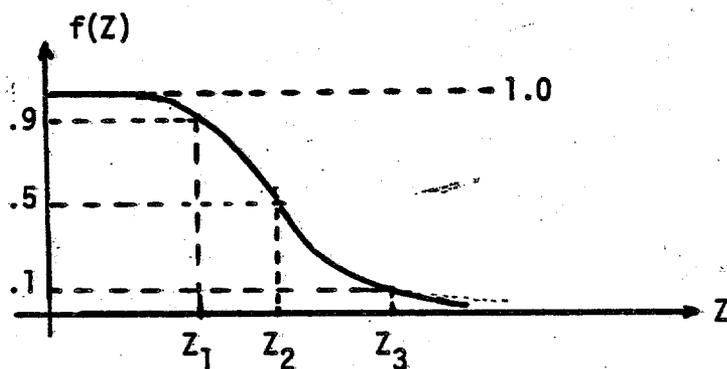
November 21, 1983

To: B. Brown, D. Harding, and R. Peters

From: S. Ohnuma

Field Measurements near Magnet Ends

The importance of knowing the field fall-off near magnet ends in calculating the machine tunes has been stated in my report, \bar{p} Note #313. I should like you to start thinking about measurements of the endfield and, in planning the measurements, I hope you would find this note to be of some use. Measurements should be made for the final version of prototype endpacks. I understand there will be altogether six for ring magnets, two for quadrupoles and four for dipoles. Since my interest is only in the shape of the fall-off, the absolute value is not really important. At the same time, I should like to have the relative accuracy such that a representation of the fall-off with at least three parameters is meaningful. These parameters could be, for example, three values of coordinate Z where the fall-off function $f(Z)$ takes the values 0.9, 0.5 and 0.1.



If $f(Z)$ reverses its sign along the Z direction, we may need more than three parameters.

In the cylindrical coordinate system (r, θ, Z) , the general form of the static potential $\phi(r, \theta, Z)$ which satisfies the Laplace equation

$$\nabla^2 \phi(r, \theta, Z) = 0 \quad (\text{A-1})$$

is

$$\phi(r, \theta, Z) = -\sum_{n=1}^{\infty} \frac{n! r^n \sin(n\theta + \alpha_n)}{4^m m!(n+m)!} \sum_{m=0}^{\infty} \frac{(-)^m r^{2m}}{4^m m!(n+m)!} f_n^{(2m)}(Z) \quad (\text{A-2})$$

where $f_n^{(2m)}(Z) \equiv \frac{d^{(2m)}}{dZ^{2m}} f_n(Z)$, the $(2m)$ th derivative of $f_n(Z)$.

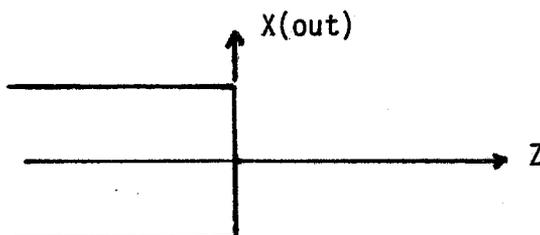
Since we are interested in the systematic field common to all magnets (and not in the fluctuation from magnet to magnet), we take only the normal component: $\alpha_n = 0$ for all n 's. In the expression above, skew components are proportional to $\sin(\alpha_n)$. If, in the coordinate system one is using, the field has a certain symmetry, the summation over n does not take all values of n . For example,

dipole symmetry: $n = 1, 3, 5$, etc.

quadrupole symmetry: $n = 2, 6, 10$, etc.

A. Quadrupole ends

The reference orbit is always normal to the quadrupole ends so that the quadrupole symmetry is maintained in (X, Y, Z) coordinate system:



$$X = r \cos \theta$$

$$Y = r \sin \theta$$

$$Y > 0 \text{ for up.}$$

Because of the quadrupole symmetry, we should retain all terms with $n = 2, 6, 10$, etc. However, we ignore here all terms of the order r^6 or higher. That is, we keep terms with $n = 2, m = 0$ and 1 only: (factor 2 ignored)

$$\phi(X, Y, Z) = -XY \{ f_2(Z) - \frac{1}{12} (X^2 + Y^2) f_2''(Z) \} \quad (A-3)$$

so that

$$B_X = - \frac{\partial \phi}{\partial X} = Y f_2(Z) - \frac{1}{12} (3X^2 Y + Y^3) f_2''(Z), \quad (A-4)$$

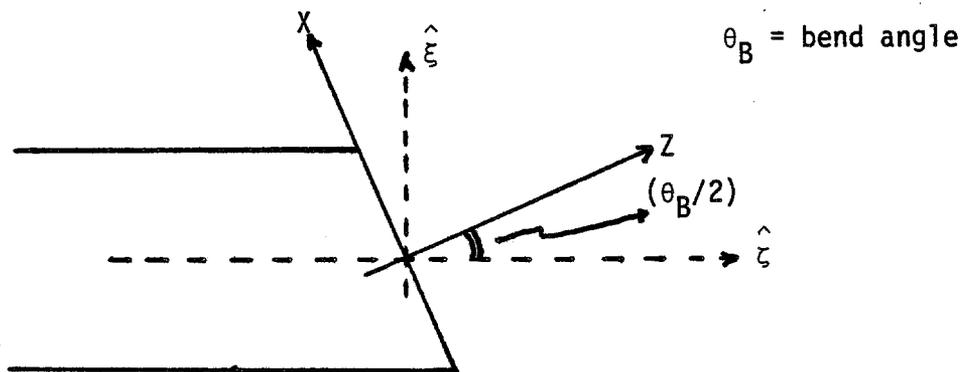
$$B_Y = - \frac{\partial \phi}{\partial Y} = X f_2(Z) - \frac{1}{12} (X^3 + 3XY^2) f_2''(Z), \quad (A-5)$$

$$B_Z = - \frac{\partial \phi}{\partial Z} = XY f_2'(Z) - \frac{1}{12} (X^3 Y + XY^3) f_2'''(Z). \quad (A-6)$$

1. Measure B_Y on $Y = 0$ plane to find $f_2(Z)$ and $f_2''(Z)$.
2. Measure B_X on $X = 0$ plane to find $f_2(Z)$ and $f_2''(Z)$.
3. Measure B_Z on $X = Y$ plane to find $f_2'(Z)$ and $f_2'''(Z)$.

B. Dipole ends

For dipoles, the reference orbit is not normal to the magnet ends and we have to consider two different coordinate systems, (X, Y, Z) and $(\hat{\xi}, Y, \hat{\zeta})$.



a) (X,Y,Z) system

The dipole symmetry still exists and we should take terms with $n = 1, 3, 5$, etc. We keep terms with $n=1$ ($m=0,1$) and $n=3$ ($m=0$) only, ignoring terms of the order r^5 or higher:

$$\phi(X,Y,Z) = -Y \left\{ f_1(Z) - \frac{1}{8} (X^2+Y^2) f_1''(Z) \right\} - (3X^2Y-Y^3) f_3(Z) \quad (A-7)$$

$$B_X = - \frac{\partial \phi}{\partial X} = (6f_3 - \frac{1}{4} f_1'') XY, \quad (A-8)$$

$$B_Y = - \frac{\partial \phi}{\partial Y} = f_1 + (3f_3 - \frac{1}{8} f_1'') X^2 - (3f_3 + \frac{3}{8} f_1'') Y^2, \quad (A-9)$$

$$B_Z = - \frac{\partial \phi}{\partial Z} = f_1' Y + (3f_3' - \frac{1}{8} f_1''') X^2 Y - (f_3' + \frac{1}{8} f_1''') Y^3 \quad (A-10)$$

1. Measure B_Y on $Y = 0$ plane to find $f_1(Z)$ and $(3f_3 - f_1''/8)$.
2. Measure B_Y on $X = 0$ plane to find $f_1(Z)$ and $(f_3 + f_1''/8)$.
3. Measure B_Z on $X = 0$ plane to find $f_1'(Z)$ and $(f_3' + f_1'''/8)$.

b) ($\hat{\xi}, Y, \hat{\zeta}$) system

Measurements in this system are redundant if measurements in (X,Y,Z) system are possible. Nevertheless, it will be nice to have them for the consistency check. Since the magnet end is not normal to $\hat{\zeta}$ -axis, the dipole symmetry is broken. We have to take $n = 1, 2, 3$, etc. for the potential in this coordinate system. Keep terms with $n=1$ ($m=0,1$), $n=2$ ($m=0,1$) and $n=3$ ($m=0$):

$$\begin{aligned} \phi(\hat{\xi}, Y, \hat{\zeta}) = & -Y \left\{ g_1(\zeta) - \frac{1}{8} (\xi^2 + Y^2) g_1''(\zeta) \right\} - 2\xi Y \left\{ g_2(\zeta) - \frac{1}{12} (\xi^2 + Y^2) g_2''(\zeta) \right\} \\ & - (3\xi^2 Y - Y^3) g_3(\zeta) \end{aligned} \quad (A-11)$$

$$B_{\hat{\xi}} = 2g_2(\zeta)Y + (6g_3 - g_1''/4)\xi Y - \frac{1}{6} (3\xi^2 Y + Y^3) g_2''(\zeta), \quad (A-12)$$

$$\begin{aligned}
 B_Y = & g_1(\zeta) + (3g_3 - \frac{1}{8} g_1'')\xi^2 - (3g_3 + \frac{3}{8} g_1'')Y^2 \\
 & + 2\xi g_2(\zeta) - \frac{1}{6} (\xi^3 + 3\xi Y^2) g_2''(\zeta),
 \end{aligned}
 \tag{A-13}$$

$$\begin{aligned}
 B_{\hat{\zeta}} = & g_1'(\zeta)Y + (3g_3' - \frac{1}{8} g_1''')\xi^2 Y - (g_3' + \frac{1}{8} g_1''')Y^3 \\
 & + 2\xi Y g_2'(\zeta) - \frac{1}{6} (\xi^3 Y + \xi Y^3) g_2'''(\zeta).
 \end{aligned}
 \tag{A-14}$$

1. Measure B_Y on $Y = 0$ plane to find $g_1(\zeta)$, $g_2(\zeta)$ and, if at all possible, $(3g_3 - g_1''/8)$ and g_2'' .
2. Measure B_Y on $\xi = 0$ plane to find g_1 and $(g_3 + g_1''/8)$.
3. Measure $B_{\hat{\xi}}$ on $\xi = 0$ plane to find g_2 and g_2'' .
4. Measure $B_{\hat{\zeta}}$ on $\xi = 0$ plane to find g_1' and $(g_3' + g_1'''/8)$.

Perhaps I should note here that whatever you can measure would be quite useful for the calculation of tunes. You may have a better way of finding $f_1(Z)$, $f_2(Z)$, etc. than the procedures I am suggesting here.

cc: F. Mills