P Note 370

SIGNAL-TO-NOISE REDUCTION BY ATTENUATION AT VARIOUS TEMPERATURES

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March 1, 1984

P Note # 370 3/1/84
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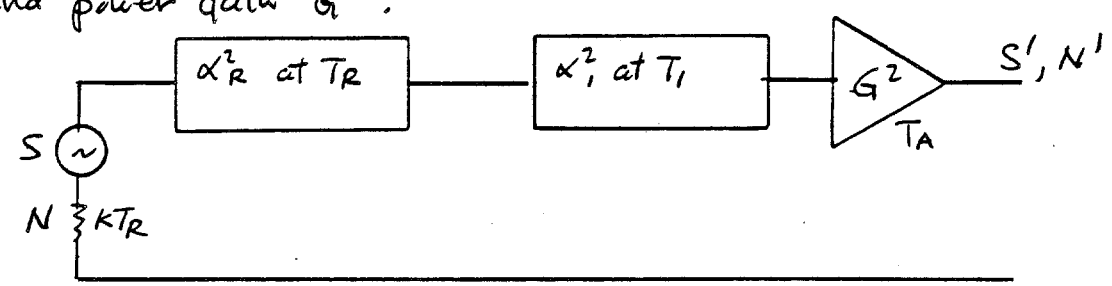
Signal-to-Noise Reduction by Attenuation at Various Temperatures

The purpose of this note is to calculate the signal-to-noise reduction by attenuation preceding the first low noise amplifier stage. In general, the attenuation, besides degrading the signal power, can enhance the noise power if the attenuation is at a higher temperature than the source. This note derives an algorithm which yields an effective equivalent noise temperature for the signal-to-noise ratio which can be useful in comparing various circuit configurations.

We first consider the case where the signal source temperature is T_R, and there is a power attenuation factor α_R^2 at T_R, and an additional power attenuation factor α_1^2 at temperature $T_1 \neq T_R$. We include an amplifier with an equivalent noise temperature T_A and power gain G^2:



The signal power at the amplifier output is

$$S' = S\,\alpha_R^2\,\alpha_1^2\,G^2 \tag{1}$$

The noise power at the output is

$$N' = kT_R\,\alpha_1^2\,G^2 + kT_1(1-\alpha_1^2)\,G^2 + kT_A\,G^2 \tag{2}$$

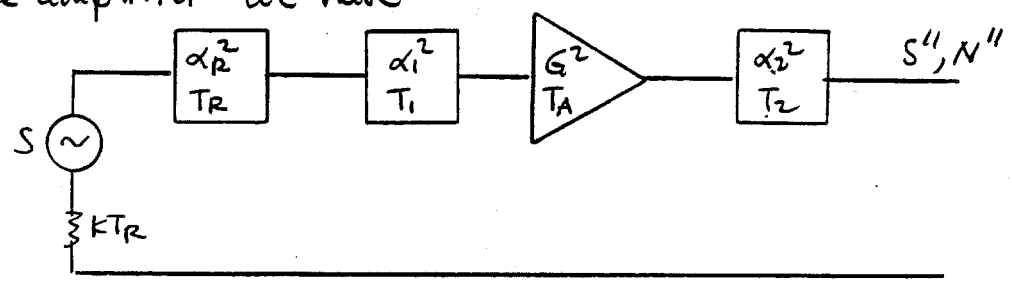
Hence the signal-to-noise ratio at the amplifier output is

$$\frac{S'}{N'} = \frac{S\alpha_R^2\alpha_1^2 G^2}{kT_R\alpha_1^2 G^2 + kT_1(1-\alpha_1^2)G^2 + kT_A G^2} = \frac{S}{kT_{eff}} \tag{3}$$

where

$$T_{eff} = \frac{T_R}{\alpha_R^2} + \frac{T_1(1-\alpha_1^2)}{\alpha_R^2\alpha_1^2} + \frac{T_A}{\alpha_R^2\alpha_1^2} \tag{4}$$

If there is attenuation α_2^2 at temperature T_2 on the output of the amplifier we have



$$S'' = S\alpha_R^2\alpha_1^2 G^2\alpha_2^2 \tag{5}$$

$$N'' = \left[kT_R\alpha_1^2 + kT_1(1-\alpha_1^2) + kT_A\right]G^2\alpha_2^2 + kT_2(1-\alpha_2^2) \tag{6}$$

In this case the effective noise temperature is:

$$T_{eff} = \frac{T_R}{\alpha_R^2} + \frac{T_1(1-\alpha_1^2)}{\alpha_R^2\alpha_1^2} + \frac{T_A}{\alpha_R^2\alpha_1^2} + \frac{T_2(1-\alpha_2^2)}{G^2\alpha_R^2\alpha_1^2\alpha_2^2} \tag{7}$$

On the following page an example is worked out. We use the standard relations:

$$\left.\begin{aligned} (\alpha^2)_{db} &= 10\,\mathrm{Log}(\alpha^2) \\ T_A &= 290^\circ\left[10^{NF/10} - 1\right] \end{aligned}\right\} \text{Tables are attached to look up these algorithms.} \tag{8}$$

use $(\alpha_R^2)_{db} = 1.0\,db \Rightarrow \alpha_R^2 = 0.79$

$T_R = 80°K$

$(\alpha_1^2)_{db} = 0.6\,db \Rightarrow \alpha_1^2 = 0.87$

$T_1 = 223°K$

$NF = 0.8\,db \Rightarrow T_A = 59°K$

$(G^2)_{db} = 30\,db \Rightarrow G^2 = 1000$

$T_2 = 293°C$

$(\alpha_2^2)_{db} = 1.5\,db \Rightarrow \alpha_2^2 = 0.71$

$$T_{eff} = \frac{T_R}{\alpha_R^2} + \frac{T_1(1-\alpha_1^2)}{\alpha_R^2\alpha_1^2} + \frac{T_A}{\alpha_R^2\alpha_1^2} + \frac{T_2(1-\alpha_2^2)}{G^2\alpha_R^2\alpha_1^2\alpha_2^2} \qquad (9a)$$

$$= \frac{80°}{.79} + \frac{223° \times 0.13}{.79 \times .87} + \frac{59°}{.79 \times .87} + \frac{293° \times .29}{1000 \times .79 \times .87 \times .71} \qquad (9b)$$

$$101° + 42° + 86° + 0.17° \qquad (9c)$$

$$\cong 229°K \qquad (9d)$$

In conclusion, we have found that using this T_{eff} for the effective signal to noise temperature is a useful and concise way of evaluating the merits of various amplifier & circuit configurations. If one attempts to discuss db loss, then there is some confusion with regard to the temperature to which the T_{eff} is being compared.

2/19/84

Amplifier Noise Figure vs equivalent noise temperature

$$T_A = 290^\circ \left[10^{NF/10} - 1 \right]$$

NF	T_A	NF	T_A
.05 db	3 °k	3.05 db	295 °k
.10	7	3.10	302
.15	10	3.15	309
.20	14	3.20	316
.25	17	3.25	323
.30	21	3.30	330
.35	24	3.35	337
.40	28	3.40	344
.45	32	3.45	352
.50	35	3.50	359
.55	39	3.55	367
.60	43	3.60	374
.65	47	3.65	382
.70	51	3.70	390
.75	55	3.75	398
.80	59	3.80	406
.85	63	3.85	414
.90	67	3.90	422
.95	71	3.95	430
1.00	75	4.00	438
1.05	79	4.05	447
1.10	84	4.10	455
1.15	88	4.15	464
1.20	92	4.20	473
1.25	97	4.25	482
1.30	101	4.30	491
1.35	106	4.35	500
1.40	110	4.40	509
1.45	115	4.45	518
1.50	120	4.50	527
1.55	124	4.55	537
1.60	129	4.60	546
1.65	134	4.65	556
1.70	139	4.70	566
1.75	144	4.75	576
1.80	149	4.80	586
1.85	154	4.85	596
1.90	159	4.90	606
1.95	164	4.95	617
2.00	170	5.00	627
2.05	175	5.05	638
2.10	180	5.10	648
2.15	186	5.15	659
2.20	191	5.20	670
2.25	197	5.25	681
2.30	202	5.30	693
2.35	208	5.35	704
2.40	214	5.40	716
2.45	220	5.45	727
2.50	226	5.50	739
2.55	232	5.55	751
2.60	238	5.60	763
2.65	244	5.65	775
2.70	250	5.70	787
2.75	256	5.75	800
2.80	263	5.80	813
2.85	269	5.85	825
2.90	275	5.90	838
2.95	282	5.95	851
3.00	289	6.00	865

2/19/84

Attenuation (db) vs attenuation factor

attenuation factor α^2 for power attenuation is related to the db attenuation by

$$(\alpha^2)_{db} = 10 \log(\alpha^2)$$

$(\alpha^2)_{db}$	α^2	$(\alpha^2)_{db}$	α^2
.05	.989	3.05	.495
.10	.977	3.10	.490
.15	.966	3.15	.484
.20	.955	3.20	.479
.25	.944	3.25	.473
.30	.933	3.30	.468
.35	.923	3.35	.462
.40	.912	3.40	.457
.45	.902	3.45	.452
.50	.891	3.50	.447
.55	.881	3.55	.442
.60	.871	3.60	.437
.65	.861	3.65	.432
.70	.851	3.70	.427
.75	.841	3.75	.422
.80	.832	3.80	.417
.85	.822	3.85	.412
.90	.813	3.90	.407
.95	.804	3.95	.403
1.00	.794	4.00	.398
1.05	.785	4.05	.394
1.10	.776	4.10	.389
1.15	.767	4.15	.385
1.20	.759	4.20	.380
1.25	.750	4.25	.376
1.30	.741	4.30	.372
1.35	.733	4.35	.367
1.40	.724	4.40	.363
1.45	.716	4.45	.359
1.50	.708	4.50	.355
1.55	.700	4.55	.351
1.60	.692	4.60	.347
1.65	.684	4.65	.343
1.70	.676	4.70	.339
1.75	.668	4.75	.335
1.80	.661	4.80	.331
1.85	.653	4.85	.327
1.90	.646	4.90	.324
1.95	.638	4.95	.320
2.00	.631	5.00	.316
2.05	.624	5.05	.313
2.10	.617	5.10	.309
2.15	.610	5.15	.305
2.20	.603	5.20	.302
2.25	.596	5.25	.299
2.30	.589	5.30	.295
2.35	.582	5.35	.292
2.40	.575	5.40	.288
2.45	.569	5.45	.285
2.50	.562	5.50	.282
2.55	.556	5.55	.279
2.60	.550	5.60	.275
2.65	.543	5.65	.272
2.70	.537	5.70	.269
2.75	.531	5.75	.266
2.80	.525	5.80	.263
2.85	.519	5.85	.260
2.90	.513	5.90	.257
2.95	.507	5.95	.254
3.00	.501	6.00	.251