Estimate of Magnetic Forces on Beam Sweeping Kickers

PBAR Note 632

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The beam sweeping magnet kickers are two pairs of conductors placed 90 degrees apart inside a circular magnetic yoke. Each pair has the same axcitation curreent in the opposite directions, and the two currents are a sine and a cosine in phase.
To estimate the magnetic forces on the kickers due to the excitation currents, we make the following simplifications:

1. The four conductors are all parallel to one another.
2. There are no relative motions within the system.
3. the magnetic yoke has $\mu = \infty$, and the fields are zero near the outside inner surface of the yoke.

Fig. 1 shows the cross section of the conductors. Conductors 1 and 3 have equal and opposite currents, conductors 2 and 4 have equal and opposite currents, and the current I_1 leads I_2 by 90 degrees. We can write I_1 as $I_{peak} \cdot \sin\theta$, I_2 as $I_{peak} \cdot \cos(\theta)$, I_3 as -$I_{peak} \cdot \sin(\theta)$, and I_4 as -$I_{peak} \cdot \cos(\theta)$, where I_{peak} is 10,000 amps. We now wish to calculate the net forces on a conductor due to the other three conductors and the magnetic yoke.
We make the simplifying assumption that the magnetic permittivity, μ, is infinite; in reality μ is about 60 and very little B-field escapes to the outside. Assuming that the B-field is zero right outside the yoke, we can then replace the yoke with 4 image currents in the same directions at a distance a so that the B-field does cancel at the outer circumference of the yoke (Fig. 2). The problem is now reduced to summing up all the forces on one conductor due to the other 7 currents. As shown in Fig. 1, for example for conductor 1, we decompose the forces on it into F_x (radially outward) and F_y (tangential). Then,

$$F_{1x} = F_{21(x)} + F_{31(x)} + F_{41(x)} + F_{1i1(x)} + F_{2i1(x)} + F_{3i1(x)} + F_{4i1(x)} =$$

$$\frac{\mu_0 l}{2\pi r} \cdot I_{peak}^2 \cdot (-\sin\theta\cos\theta\sin\beta + \frac{1}{\sqrt{2}}\sin^2\theta + \sin\theta\cos\theta\sin\beta +$$

$$\frac{r}{a}\sin^2\theta - \frac{r}{b}\sin\theta\cos\theta\sin\alpha + \frac{r}{a+\sqrt{2}r}\sin^2\theta + \frac{r}{b}\sin\theta\cos\theta\sin\alpha),$$

and

$F_{1y} = F_{21(y)} + F_{31(y)} + F_{41(y)} + F_{1i1(y)} + F_{2i1(y)} + F_{3i1(y)} + F_{4i1(y)} =$

$\frac{\mu_0 l}{2\pi r} \cdot I_{peak}^2 \cdot (-\sin\theta\cos\theta\cos\beta + 0 - \sin\theta\cos\theta\cos\beta + 0$ -

$\frac{r}{b}\sin\theta\cos\theta\cos\alpha + 0 - \frac{r}{b}\sin\theta\cos\theta\cos\alpha)$,

where $\theta = \omega t$ is the reference phase angle of all currents, l is the length of the conductor, $l = 27.9cm$, $r = 2.00cm$, $a = 4.98cm$, $b = 6.55cm$, $\beta = 45°$ and $\alpha = 12.5°$. Then,

$F_{1x} = 1.37\sin^2\theta$,

and

$F_{1y} = -2.00\sin\theta\cos\theta$,

where a positive F_x indicates an radially outward force, and a positive F_y indicates a force causing a counter-clockwise rotation. Similar analyses for the other three conductors yield

$F_{1x} = F_{3x} = 1.37\sin^2\theta$,

$F_{2x} = F_{4x} = 1.37\cos^2\theta$,

$F_{1y} = F_{3y} = -2.00\sin\theta\cos\theta$,
and
$F_{2y} = F_{4y} = 2.00\sin\theta\cos\theta$.

Fig. 3 shows the radial and tangential forces through a complete current cycle. We draw the following conclusion about the behavior of the conductors:

- **Torques caused by the tangential forces about the central Z-axis cancel, hence there is no tendency for the circuit to rotate about the Z-axis.**
- **There is no radially inward force on any conductors during the entire cycle.**
- **Peak radial outward force ≈ 8.0 lb/in on each conductor.**
- **Peak tangential force ≈ 5.2 lb/in on each conductor.**

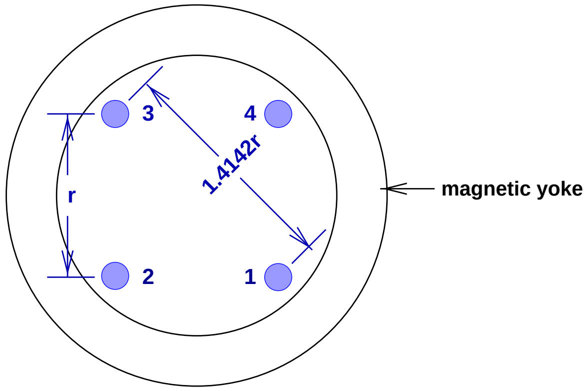

Figure 1: Cross section of conductors; r is 2 cm.

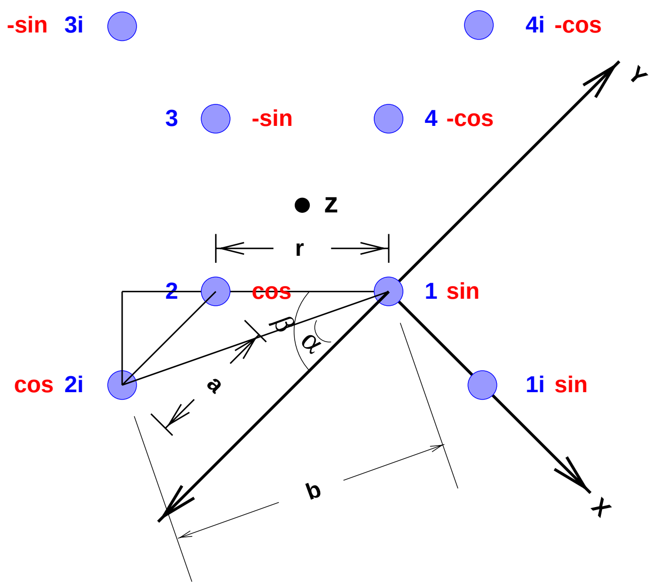

Figure 2: Equivalent current distribution in which the magnetic shield ($\mu = \infty$) is replaced with 4 image currents.

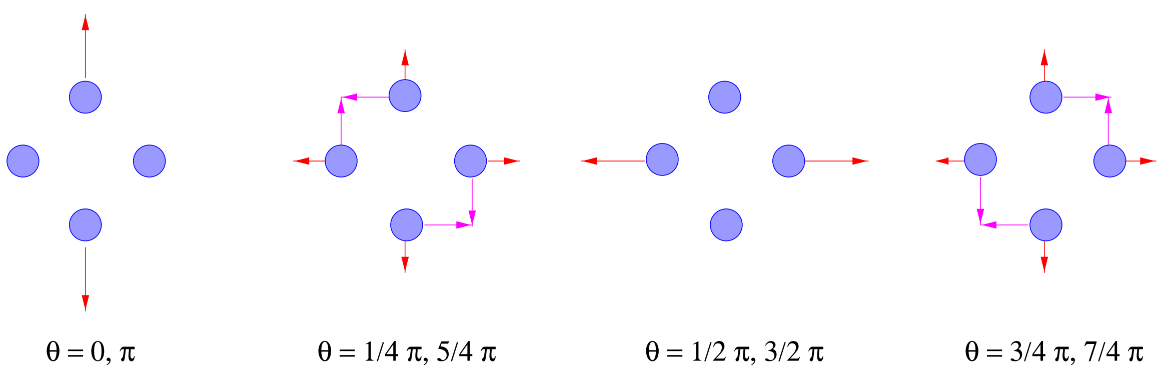

Figure 3: The forces on conductors during a full cycle of current pulse.