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TRANSVERSE EMITTANCE FROM SEM READINGS

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Abstract

The prescription how to compute the transverse emittance from the SEM readings in a beam transfer line is presented.

1. INTRODUCTION

The purpose of this Note is to obtain the formula which relates the SEM readings in a transfer line with the transverse emittances. The calculation is general, the examples are done for the AP1-AP3 antiproton transfer line.

The initial beam can be characterized in TRANSPORT by specifying the beam σ matrix

$$\sigma(0) = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} & \sigma_{16} \\ \sigma_{12} & \sigma_{22} & & \cdots & & \sigma_{26} \\ \vdots & \vdots & & \ddots & & \vdots \\ \sigma_{16} & \sigma_{26} & & \cdots & & \sigma_{66} \end{pmatrix}$$

at the point of injection. Note that the beam matrix is symmetric. The diagonal elements define the coordinates of the beam envelope, the longitudinal beam extent, the momentum spread and the momentum of the central trajectory by the following relations:

$$\sigma_{11} = x_{max}^2, \quad \sigma_{22} = x_{max}'^2, \quad \sigma_{33} = y_{max}^2, \quad \sigma_{44} = y_{max}'^2, \quad \sigma_{55} = \ell^2, \quad \sigma_{66} = \delta^2,$$

where $x_{max}(y_{max})$ is one-half the horizontal (vertical) beam extent, $x'_{max}(y'_{max})$ one half the horizontal (vertical) beam divergence, ℓ one-half the longitudinal beam extent, δ one-half the momentum spread $\Delta p/p$. We also need to specify P_0, the momentum of the central trajectory. Alternately, one could specify the horizontal and the vertical Twiss parameters at the point of injection. The first method is more general for the time being, since TRANSPORT is not equipped to compute the Twiss parameters for transfer lines in which magnet pole faces are rotated. For this reason the present calculation will be done in the σ matrix language. The lattice functions and the emittance can be extracted from the σ matrix at the end.

The following diagonal elements describe the beam at F17:

$$x = 6.557\text{mm} \quad x' = 0.18\text{mr} \quad y = 3.117\text{mm} \quad y' = 0.23\text{mr}$$

$$\ell = 1.0 \quad \delta = 0.1\% \quad P_0 = 8.914\text{GeV/cm}.$$

(ℓ is irrelevant in this calculation.) These elements describe the upright ellipse, the non-diagonal elements are also supplied in order to tilt the ellipse.

The transfer matrix R in general has the form

$$R(s) = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & \eta_x \\ R_{21} & R_{22} & R_{23} & \dots & & \eta'_x \\ R_{31} & R_{32} & & \dots & & \eta_y \\ R_{41} & \dots & & \dots & & \eta'_y \\ R_{51} & \dots & & \dots & & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

It is easy to verify that the product of two such matrices is a matrix of the same form, but they are in general not emittance conserving (unless the dispersion functions are zero). The beam σ matrix at the distance s from the injection point is computed as

$$\sigma(s) = R(s)\sigma(0)R(s)^T.$$

The beam profiles (*i. e.* x_{max} and y_{max} as function of s) are shown in Figs. 1 and 2.

For the horizontal degree of freedom we now have:

$$\begin{aligned} \sigma_{11}(s) &= x_{max}(s)^2 = \epsilon_x(s)\beta_x(s) + \eta_x(s)^2\sigma_{66} && (1) \\ \sigma_{12}(s) &= -\epsilon_x(s)\alpha_x(s) + \eta_x(s)\eta'(s)\sigma_{66} && (2) \\ \sigma_{22}(s) &= x'_{max}(s) = \epsilon_x(s)\gamma_x(s) + \eta'(s)^2\sigma_{66}. && (3) \end{aligned}$$

To obtain the analogous relations for the vertical degree of freedom, augment each index by 2.

The emittance can now be obtained in the following way:

$$\begin{aligned} \epsilon_x^2 &\equiv (\epsilon_x\beta_x)(\epsilon_x\gamma_x) - (\epsilon_x\alpha_x)^2 && (4) \\ &= \sigma_{11}\sigma_{22} - \sigma_{12}^2 - (\sigma_{11}\sigma_{26}^2 + \sigma_{22}\sigma_{16}^2 - 2\sigma_{12}\sigma_{16}\sigma_{26})/\sigma_{66}. && (5) \end{aligned}$$

The horizontal and the vertical emittances for the AP1-AP3 line as computed from this formula are shown in Figs. 3 and 4.

Beta function is now computed as

$$\beta_x(s) = (\sigma_{11} - \sigma_{16}^2/\sigma_{66})/\sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2 - (\sigma_{11}\sigma_{26}^2 + \sigma_{22}\sigma_{16}^2 - 2\sigma_{12}\sigma_{16}\sigma_{26})/\sigma_{66}},$$

and analogous for β_y.

The horizontal and the vertical beta functions for the AP1-AP3 line as computed from this formula are shown in Figs. 5 and 6.

2. EMITTANCE FROM THE SEM READINGS

Denote by σ_x, σ_y the beam widths as read by the SEMs at the position s. The emittances at that point are obtained as

$$\epsilon_x = \frac{\sigma_x^2 - 4\eta_x^2\delta^2}{4\beta_x}$$

and similar for ϵ_y. For $\delta = 0.1\%$ the expression can be written as

$$\epsilon_x = \frac{\sigma_x[\mathrm{mm}]^2 - 4\eta_x[\mathrm{m}]^2}{4\beta_x[\mathrm{m}]} \quad [\pi\mathrm{mm\ mr}].$$

The following table contains the values of the parameters needed in order to compute the horizontal and the vertical emittance for each SEM in the AP1-AP3 line.

Dispersion is obtained as $\eta_x = R16$ etc.

	SEM	s [in]	η_x [m]	β_x [m]	η_y [m]	β_y [m]
1536.06	SM100	~~814.00~~	58.15	24.94	–3.48	77.54
✓	SM101	2499.65	32.80	216.33	–2.67	17.02
✓	SM102	2634.49	34.58	284.65	–1.86	10.71
✓	SM103	3336.31	48.46	756.86	4.88	45.40
✓	SM104	3827.00	95.63	46.78	11.09	210.69
4510.32	SM105	~~4508.32~~	–9.45	15.88	1.77	7.01
5471.94	SM106	~~5467.94~~	32.57	114.24	1.25	35.78
6690.57	SM926	~~6693.96~~	5.40	66.77	0.61	37.51
9548.21	SM921	~~9547.95~~	15.53	201.40	–0.37	20.15
12997.51	SM917	~~12991.34~~	–3.93	8.20	1.48	66.33
14028.73	SM913	~~14027.08~~	1.21	18.77	–0.22	12.15
16[illegible].812	SM909	~~16542.77~~	1.46	73.16	0.62	34.49
18202.84	SM906	~~18216.66~~	–1.15	18.28	2.73	169.37
18894.48	SM900	~~18889.76~~	–2.37	41.62	5.05	9.47

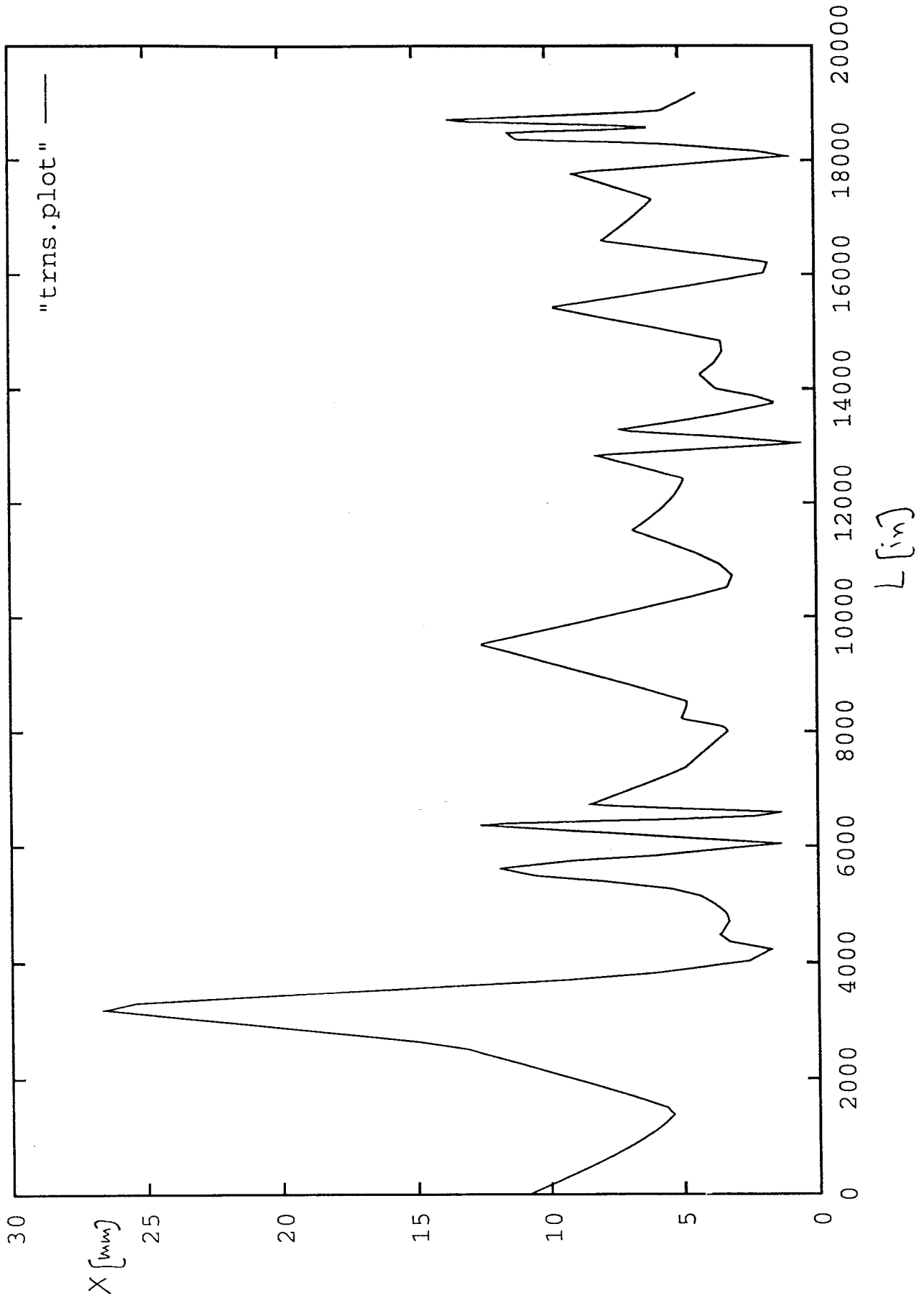
"trns.plot"
X [mm]
30
25
20
15
10
5
0
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
L [in]

Fig. 1

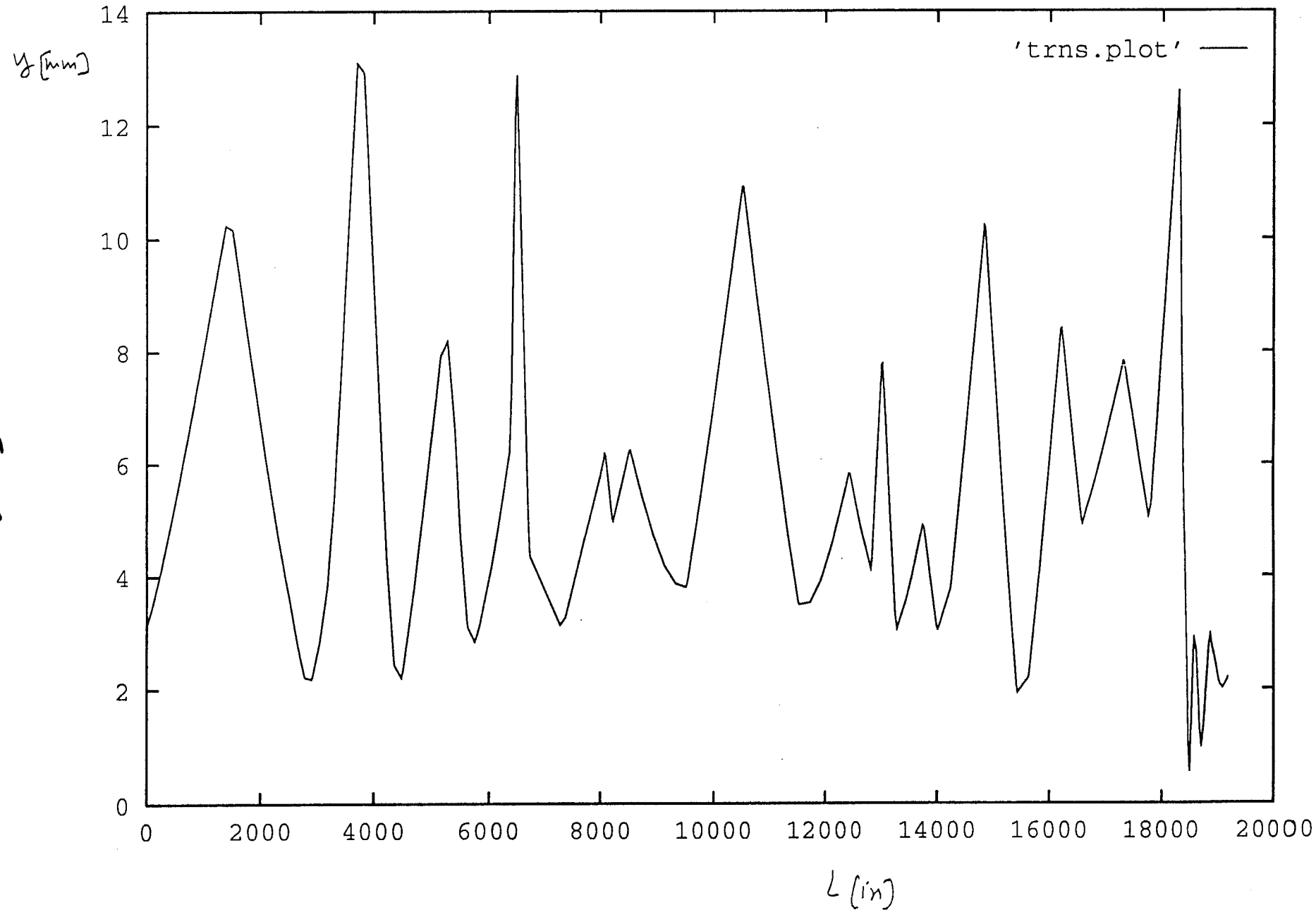
y [mm]
'trns.plot'
0
2
4
6
8
10
12
14
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
L (in)

Fig. 2

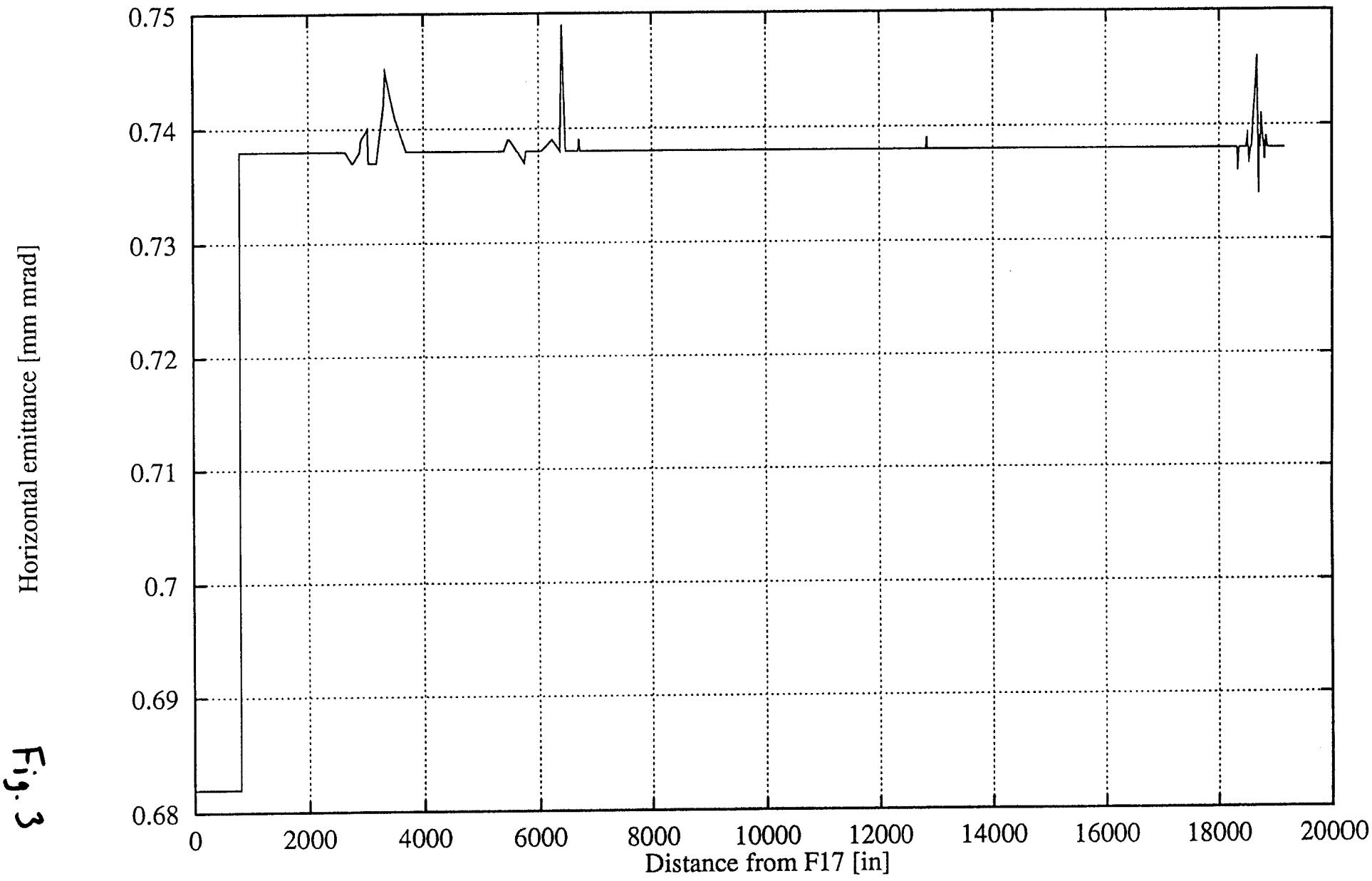
Horizontal emittance [mm mrad]
0.75
0.74
0.73
0.72
0.71
0.7
0.69
0.68
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
Distance from F17 [in]

Fig. 3

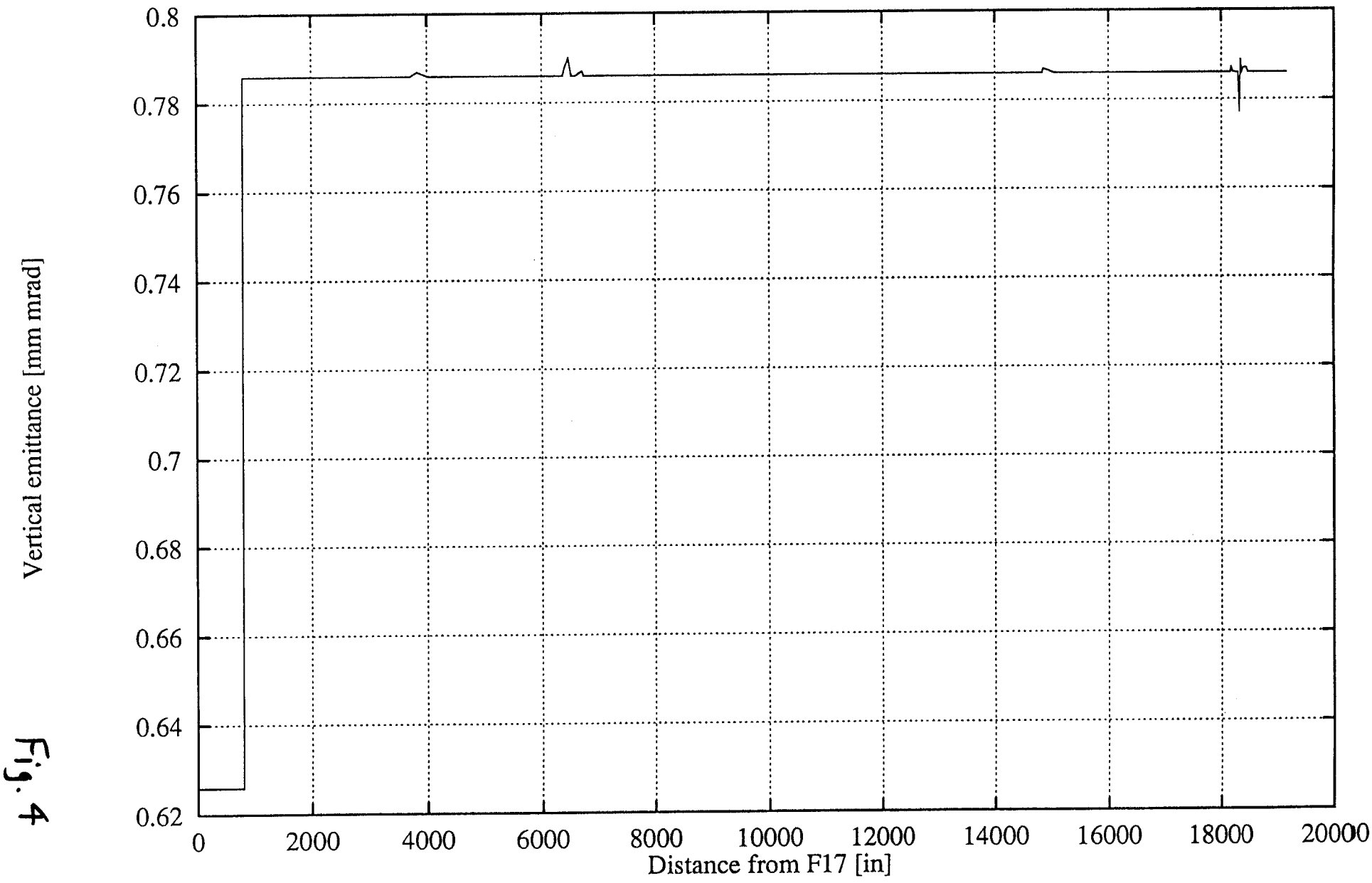
Vertical emittance [mm mrad]
0.8
0.78
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0.62
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
Distance from F17 [in]

Fig. 4

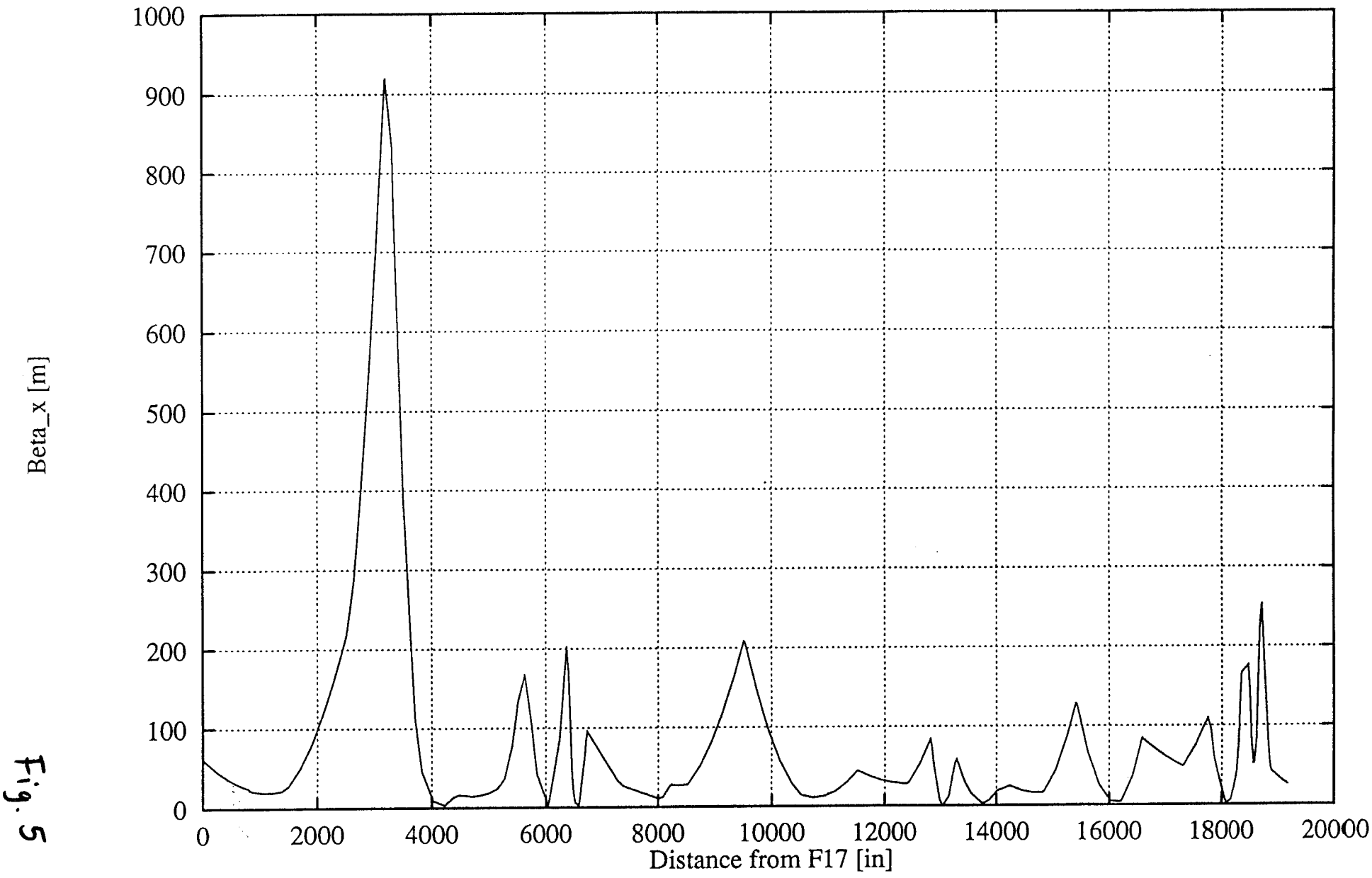
Beta_x [m]
1000
900
800
700
600
500
400
300
200
100
0
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
Distance from F17 [in]

Fig. 5

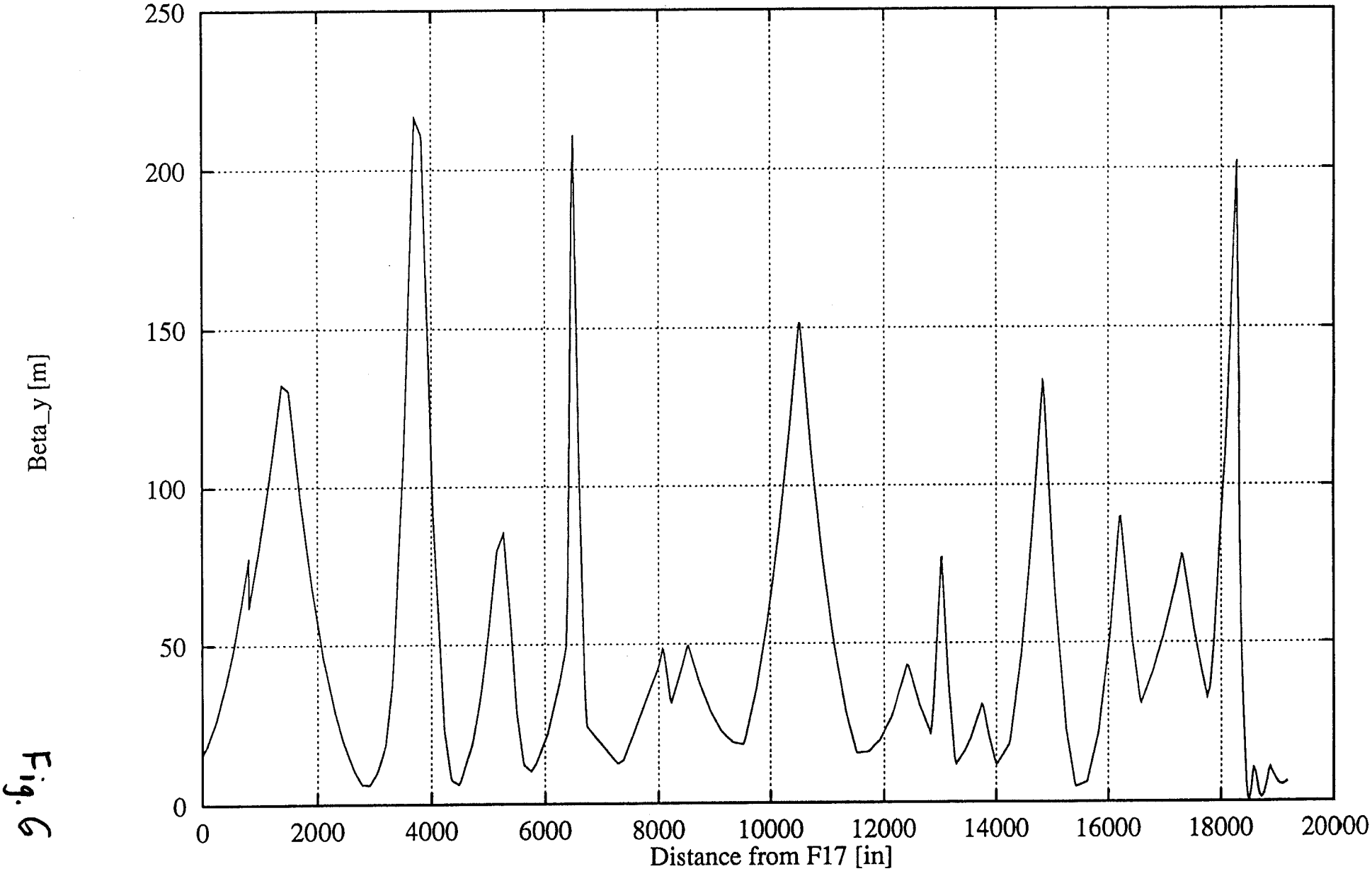
Beta_y [m]
250
200
150
100
50
0
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
Distance from F17 [in]

Fig. 6